International Library of Technology

433

Concrete Design

266 Illustrations

Prepared Under Supervision of

C. K. SMOLEY

DIRECTOR, SCHOOLS OF CIVIL AND STRUCTURAL ENGINEERING
INTERNATIONAL CORRESPONDENCE SCHOOLS

CONCRETE BEAM AND COLUMN DESIGN
ELEMENTS OF STEEL REINFORCEMENT
REINFORCED-CONCRETE BUILDINGS
OFFICE PRACTICE IN CONCRETE DESIGN
DESIGN OF SPREAD FOUNDATIONS
REINFORCED-CONCRETE CANTILEVER
FOUNDATIONS

Published by
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
1928

Printed in U. S. A.

PRESS OF
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
433

93161

PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

INTERNATIONAL TEXTBOOK COMPANY

B

CONTENTS

NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents, the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

REINFORCED-CONCRETE BUILDINGS—(*Continued*)

§ 21

OFFICE PRACTICE IN CONCRETE DESIGN, § 22

DESIGN OF SPREAD FOUNDATIONS, § 23

REINFORCED-CONCRETE CANTILEVER FOUNDATIONS, § 24

CONCRETE BEAM AND COLUMN DESIGN

PLAIN CONCRETE

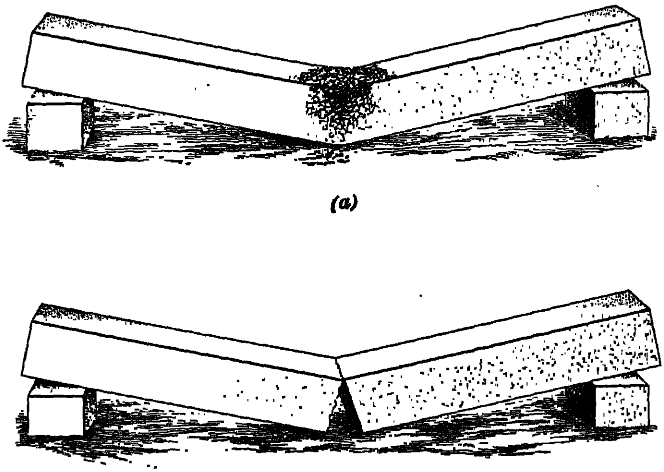
CONCRETE BEAMS

INTRODUCTION

1. As the design of stone beams and columns is like that of plain-concrete beams and columns, they will all be considered in the first part of this Section.

Both stone and concrete, particularly the latter, are weak in resisting bending stresses. This weakness is due not to the crushing at the top of a beam but to the lack of tensile qualities, which allows the beam to start its fracture on the under side. Thus, a beam of concrete or stone never breaks as shown in Fig. 1 (*a*), but always as shown in (*b*). Fortunately, stone beams are seldom used, and plain-concrete beams are of still less frequent occurrence. The use of stone or plain-concrete beams is confined almost exclusively to lintels over door and window openings. In such cases, the size of the beam is governed by the architectural effect that it is desired to obtain, and the beam will usually be

much larger than is necessary for mere strength. Nevertheless, it is sometimes necessary to calculate the strength



(a)

(b)

FIG. 1

of such beams, and for this reason the engineer should understand the methods employed in their design.

METHOD OF DESIGN

2. Stone and plain-concrete beams are designed by exactly the same method as any other kind of beam, as one of wood, steel, or the like. The formula employed is as follows:

$$M = \frac{s\,I}{c} = S\,s,$$

in which M = bending moment;
s = unit stress produced;
I = moment of inertia;
c = half the depth of the beam;
S = section modulus.

In a beam of rectangular section, $S = \frac{b\,d^2}{6}$, in which b is the breadth of the beam and d is its depth, both measured in inches.

3. Stone Beams.—As was mentioned, the formula given in the preceding article can be used in the design of stone beams. The only quantity that remains to be known is the modulus of rupture. This value for various kinds of stones and other materials is given in Table I.

TABLE I

MODULI OF RUPTURE OF VARIOUS MINERALS

Material	Modulus of Rupture
Slate	5,000
Glass	3,000
Bluestone flagging	2,250
Marble, white, Italian	2,090
Marble, white, Vermont	1,850
Marble, gray, Vermont	1,260
Granite, Quincy, Massachusetts	1,800
Granite, New York	1,800
Granite, Connecticut	1,500
Sandstone, Massachusetts	1,080
Sandstone, Middletown, Connecticut	1,000
Sandstone, Ohio	500
Freestone, Little Falls, New York	1,730
Freestone, Belleville, New Jersey	1,440
Freestone, Dorchester, Massachusetts	800
Freestone, Hubeginy	650
Freestone, Caen, Normandy	450
Limestone, average value	1,500
Brick, common or Philadelphia pressed	600
Brick, best hard	800
Rubble masonry in cement	200

With the values in this table, a factor of safety of from 10 to 20 is usually employed when problems dealing with the safe load instead of the ultimate breaking load are being solved.

EXAMPLE 1.—A stone beam is 6 inches broad and 10 inches deep, and it sustains a load of 1,000 pounds in the center of its span. The span is 4 feet. What stress is produced? Neglect the weight of the beam.

SOLUTION.—In this case, $b = 6$ in. and $d = 10$ in. Therefore,

$$S = \frac{b\,d^2}{6} = \frac{6 \times 10^2}{6} = 100$$

Also, $l = 4 \times 12 = 48$ in. and $W = 1{,}000$ lb. Therefore,

$$M = \frac{W\,l}{4} = \frac{1{,}000 \times 48}{4} = 12{,}000 \text{ in.-lb.}$$

Substituting these values in the ordinary formula $M = S\,s$, then $12{,}000 = 100\,s$. Therefore, $s = 12{,}000 \div 100 = 120$ lb. per sq. in. Ans.

EXAMPLE 2.—Design a Quincy granite lintel 6 inches wide on a 5-foot span to carry a total uniform load of 300 pounds per foot with a factor of safety of 10.

SOLUTION.—In this example, $W = 300 \times 5 = 1{,}500$ lb. and $l = 5 \times 12 = 60$ in. Therefore,

$$M = \frac{W\,l}{8} = \frac{1{,}500 \times 60}{8} = 11{,}250 \text{ in.-lb.}$$

According to Table I, the modulus of rupture for Quincy granite is 1,800, and if a factor of safety of 10 is used, the value of s is $1{,}800 \div 10 = 180$ lb. per sq. in. Therefore,

$$S = \frac{b\,d^2}{6} = \frac{6\,d^2}{6} = d^2$$

Substituting these quantities in the formula $M = S\,s$, then $11{,}250 = d^2 \times 180$. Thus, $d^2 = 62.5$ and $d = 7.906$, say 8, in. Ans.

EXAMPLE 3.—A bluestone slab must support 200 pounds per square foot, including its own weight, with a factor of safety of 15. The supports are 10 feet apart. How thick will the slab be?

SOLUTION.—Any width of slab may be assumed. It will be convenient to assume one that is to be 12 in. wide. Then, $W = 10 \times 200 = 2{,}000$ lb. and $l = 120$ in. Therefore,

$$M = \frac{W\,l}{8} = \frac{2{,}000 \times 120}{8} = 30{,}000 \text{ in.-lb.}$$

According to Table I, the modulus of rupture of bluestone is 2,250; then, $s = 2{,}250 \div 15 = 150$ lb. per sq. in. Also,

$$S = \frac{b\,d^2}{6} = \frac{12\,d^2}{6} = 2\,d^2$$

Substituting these values in the formula $M = S\,s$, then $30{,}000 = 2d^2 \times 150$. Therefore, $d^2 = 100$ and $d = 10$ in. Ans.

EXAMPLES FOR PRACTICE

1. A limestone beam is on a span of 4 feet 2 inches. If the beam is 6 inches deep and 4 inches wide, and an allowable stress of 100 pounds per square inch is employed, what total load per foot will it carry? Ans. 92.16 lb.

2. A slate beam is 10 inches wide and 12 inches deep and is on a span of 4 feet. Neglecting its own weight, what load at the center will break this beam? Ans. 100,000 lb.

3. How thick should a bluestone sidewalk be to carry a total load of 300 pounds per square foot with a factor of safety of 10? The stones are supported at the curb and the building line by dwarf walls located 16 feet apart. Ans. 16 in.

4. A glass vault light is 6 inches long and is supported at the ends. It must carry 100 pounds per square inch with a factor of safety of 10. How thick should it be? Ans. 3 in.

---

4. Concrete Beams.—The modulus of rupture of concrete is usually less than that of stone. One should think that this modulus of rupture would have about the same value as the ultimate unit tensile strength of the material since a plain-concrete beam breaks on the tension side. This, however, is not the case, and the modulus of rupture of concrete is about twice the ultimate unit tensile strength. Nevertheless, concrete is still very weak in withstanding bending stresses. For this reason, it is seldom used unreinforced for beams. Few tests that give reliable data have been made on the material so stressed. Another difficulty in regard to determining a value for the modulus of rupture of concrete is that many factors have a decided effect on this quantity. It is affected by the richness of the mixture, the amount of water used in making the concrete, its age, the care taken in mixing, the character of the aggregates, etc.

Table II gives values found by the United States Geological Survey at the government testing plant in St. Louis, Missouri. These tests are probably as complete as have ever been made on plain concrete. Three degrees of wetness are recognized in mixing the concrete, namely, wet, medium, and damp. *Wet concrete* is such that sufficient

TABLE II

MODULUS OF RUPTURE OF CONCRETE

Material	Mixture	Consistency	Modulus of Rupture at Various Ages					
			1 Day	1 Week	4 Weeks	13 Weeks	26 Weeks	1 Year
Neat cement		Water, 21 per cent. weight of dry materials	761	1,512	1,674	1,953	2,023	
Mortar	1–3	Water, 11.5 per cent. weight of dry materials		588	888	1,026	1,008	1,080
Mortar	1–4	Water, 11 per cent. weight of dry materials		390	636	852	864	870
Cinder concrete . .	1–2–5	Wet			175	240	246	
Cinder concrete . .	1–2–5	Medium			198	231	277	
Cinder concrete . .	1–2–5	Damp			198	225	250	
Granite concrete . .	1–2–4	Wet			375	501	539	
Granite concrete . .	1–2–4	Medium			475	536	566	
Granite concrete . .	1–2–4	Damp			499	591	618	
Gravel concrete . .	1–2–4	Wet			391	380	435	
Gravel concrete . .	1–2–4	Medium			451	477	520	
Gravel concrete . .	1–2–4	Damp			426	495	496	
Limestone concrete .	1–2–4	Wet			422	487	507	
Limestone concrete .	1–2–4	Medium			458	541	566	
Limestone concrete .	1–2–4	Damp			537	521	589	

water is added to make it semiliquid; *damp concrete* is decidedly granular, with little tendency to lump; while *medium concrete* is between the other two mixtures. Table II is mostly for 1–2–4 mixtures. A 1–3–6 mixture, which is seldom used, gives values lower than the 1–2–4 mixture. The factor of safety employed is sometimes 6, but usually 10 or higher, as the strength of concrete is uncertain. Instead of using the modulus of rupture for stone and concrete, divided by a factor of safety, sometimes the ultimate tensile strength, properly reduced, is used, as the beam really fails by tension. Either method should lead to the same result, a safe unit bending stress. The best way is to select from experience this stress in the first place. Building laws usually specify working stresses. Many engineers state that plain concrete beams should never be used at all.

EXAMPLE.—Design a concrete beam 6 inches wide on a 6-foot span to carry at its center 800 pounds, which includes the equivalence of its own weight. The safe working stress is to be 30 pounds per square inch.

SOLUTION.—The moment is equal to $\frac{Wl}{4} = \frac{800 \times 6 \times 12}{4} = 14{,}400$ in.-lb. Substituting the correct values, $14{,}400 = \frac{6 \times d^2}{6} \times 30$. Therefore, $d^2 = 14{,}400 \div 30 = 480$ and $d = 21.91$, say 22, in. Ans.

EXAMPLE FOR PRACTICE

1. A limestone concrete beam is made of a 1–2–4 medium mixture. It is 10 inches deep and 8 inches broad. What concentrated load at its center will break it at the end of 26 weeks, provided the span is 7 feet 6 inches and its own weight is neglected? Ans. 3,354 lb.

CONCRETE COLUMNS

5. Plain-concrete and stone columns may be divided into two classes, namely, those which are *centrally loaded* and those which are *eccentrically loaded.* In either case, the height of the column should never be more than twelve times the least dimension of the cross-section.

6. Column Centrally Loaded.—For a **centrally loaded column,** the allowable compressive stress per square inch is multiplied by the area of the cross-section of

the column to find the allowable load. Thus, if it is decided to allow an intensity of stress of 300 pounds per square inch, and the column is of square section 10 inches on a side, the allowable load will be $10 \times 10 \times 300 = 30{,}000$ pounds. The breaking load on columns between two and twelve times as high as the least dimension of their cross-section seems to be independent of their height. A column between these two limits, however, cannot withstand as high an intensity of

TABLE III

ULTIMATE UNIT CRUSHING STRENGTH OF STONE CONCRETE WITH PORTLAND-CEMENT MORTAR

Proportion of Ingredients			Compression Pounds per Square Inch			
Cement	Sand	Stone	7 Days	1 Month	3 Months	6 Months
1	2.0	4	1,600	2,150	2,400	2,500
1	2.5	5	1,430	1,950	2,250	2,350
1	3.0	6	1,250	1,800	2,100	2,200
1	3.5	7	1,100	1,660	1,960	2,080
1	4.0	8	980	1,520	1,820	1,950
1	4.5	9	850	1,400	1,690	1,840
1	5.0	10	750	1,260	1,550	1,720
1	5.5	11	650	1,120	1,420	1,600
1	6.0	12	600	1,000	1,300	1,500

NOTE.—For gravel concrete, use 75 per cent. of the figures given in the table; for cinder concrete, use 65 per cent.

stress as a cube, for it is more likely to break by shearing. For this reason, when employing values taken from Tables III, IV, and V for column calculations, a larger factor of safety should be used than with other concrete work. This factor is at least 6, and for stone and brick it is 10 or 15 or higher.

EXAMPLE 1.—What is the allowable working load on a concrete column that is 10 feet high and 12 inches in diameter and made of 1-2-4 stone concrete 6 months old with a factor of safety of 6?

SOLUTION.—The cross-sectional area of the column is $.7854 \times 12^2 = 113.1$ sq. in. From Table III, the ultimate crushing strength of

1-2-4 concrete 6 mo. old is 2,500 lb. per sq. in. Using a factor of safety of 6, the safe intensity of stress is $\frac{2,500}{6}$. Then the safe total load the column can carry is

$$113.1 \times \frac{2,500}{6} = 47,125 \text{ lb.} \quad \text{Ans.}$$

TABLE IV

ULTIMATE UNIT CRUSHING STRENGTH OF STONES AND STONE MASONRY

Material	Compressive Strength Pounds per Square Inch	Material	Compressive Strength Pounds per Square Inch
Granite, Colorado	15,000	Limestone, Marquette, Michigan	8,000
Granite, Connecticut	14,000	Limestone, Conshohocken, Pennsylvania	15,000
Granite, Massachusetts	16,000	Marble, Montgomery Co., Pennsylvania	11,000
Granite, Maine	15,000	Marble, Lee (dolomite), Massachusetts	22,800
Granite, Minnesota	25,000	Marble, Pleasantville (dolomite), New York	22,000
Granite, New York	16,000	Marble, Italian	12,000
Granite, New Hampshire	12,000	Marble, Vermont	10,000
Bluestone	15,000	Slate	10,000
Sandstone, Middletown, Connecticut	7,000	Bluestone, ashlar piers	2,100
Sandstone, Longmeadow, Massachusetts	10,000	Granite, ashlar piers	2,100
Sandstone, Hudson River, New York	12,000	Limestone, ashlar piers	1,500
Sandstone, Little Falls (brown), New York	10,000	Common sandstone, ashlar piers	1,050
Sandstone, Ohio	8,000	Rubble piers, cement mortar	900
Sandstone, Hummelstown (brown), Pennsylvania	12,000	Rubble piers, lime mortar	480
Limestone, Kingston, New York	12,000		
Limestone, Garrison Station, New York	18,000		
Limestone, Bedford (oolitic), Indiana	8,000		

TABLE V

ULTIMATE CRUSHING STRENGTH OF BRICK MASONRY PIERS

(*Average Age of Brickwork, 6 Months*)

Material	Composition of Mortar	Compressive Strength Pounds Per Square Inch
Wire-cut brick	1 cement, 5 sand	3,000
Dry-pressed brick . .	1 cement, 5 sand	3,400
Dry-pressed brick . .	1 cement, 1 lime, 3 sand	2,300
Repressed brick . . .	1 cement, 5 sand	1,700
Light-hard, sand-struck brick	1 cement, 5 sand	1,900
Light-hard, sand-struck brick	1 cement, 7 sand	853
Hard, sand-struck brick	1 cement, 1 sand	2,100
Hard, sand-struck brick	1 cement, 1 lime, 3 sand	1,500
Hard, sand-struck brick	1 cement, 5 sand	1,200
Sand-lime brick . . .	1 cement, 3 sand	1,100
Sand-lime brick . . .	1 lime, 3 sand	450
Sand-lime brick . . .	Neat cement	1,400
Terra-cotta work . . .	1 cement, 3 sand	2,000

EXAMPLE 2.—Design a square bluestone column, 12 feet high, to carry 1,000,000 pounds. Use a factor of safety of 10.

SOLUTION.—The allowable unit stress, according to Table IV, is $15,000 \div 10 = 1,500$ lb. Therefore, the required area of the column is $1,000,000 \div 1,500 = 667$ sq. in., and the edge of the column is $\sqrt{667} = 26$ in. It will be noted that the load that this column is to carry is enormous, while the dimensions of the column do not seem very large. It is therefore evident that the architect in designing columns for architectural effect usually errs on the side of safety. Ans.

In designing concrete columns, a layer of concrete $1\frac{1}{2}$ inches thick is sometimes added to the outside of the column to act as a fireproof covering. This precaution should always be taken where there is any danger of fire.

7. Eccentrically Loaded Column.—The stress on an **eccentrically loaded column** is computed by the following formulas where the eccentricity of the load is not great enough to cause tension in the column and is in rectangular columns normal to one face.

For circular columns,

$$s = \frac{P}{A} + \frac{8\,e\,P_e}{A\,d} \qquad (1)$$

For rectangular columns,

$$s = \frac{P}{A} + \frac{6\,e\,P_e}{A\,d} \qquad (2)$$

In these formulas,

s = stress, in pounds per square inch, developed in the column;
P = total load on the column, in pounds;
A = area of column section, in square inches;
e = eccentricity of eccentric part of load, in inches;
P_e = eccentric part of load, in pounds;
d = diameter of column, or dimension measured in the plane of the eccentricity, in inches.

If the total load is eccentric, the formulas just given reduce to the following:

For circular columns,

$$s = \frac{P_e}{A}\left(1 + \frac{8\,e}{d}\right) \qquad (3)$$

For rectangular columns,

$$s = \frac{P_e}{A}\left(1 + \frac{6\,e}{d}\right) \qquad (4)$$

According to these formulas, it is necessary first, in designing a column that will stand a given load, to select by inspection the section of column that seems to be about correct, and then to solve the equation for s. This value of s must be less than the allowable working stress it is proposed to use. If it is larger, a larger area of column must be selected, and the problem worked out again; if it is very much smaller, possibly too large a section for economy has

been selected, and in this case a smaller section should be assumed and the equation again solved for s. It should be borne in mind that the height of the column must not be more than twelve times the least dimension of the cross-section, and under certain conditions not more than six times.

EXAMPLE 1.—Design a cylindrical column of 1-2-4 stone concrete, 18 feet high, to carry a central load of 100,000 pounds and an eccentric load of 100,000 pounds, the eccentricity being 4 inches.

SOLUTION.—Since the column is 18 ft. high, it should be at least 18 in. in diameter. The ultimate crushing strength may be taken as 2,500 lb. per sq. in. in 6 mo. The safe working stress would, therefore, be 2,500 ÷ 6 = 417 lb. per sq. in.

A column 28 in. in diameter will be tried first. The area of the cross-section is $.7854 \times 28^2$, or 615.75 sq. in. To apply formula **1**, $A = 615.75$, $P = 200{,}000$, $P_e = 100{,}000$, $e = 4$, and $d = 28$. Substituting in the formula,

$$s = \frac{200{,}000}{615.75} + \frac{8 \times 4 \times 100{,}000}{615.75 \times 28} = 325 + 186 = 511 \text{ lb. per sq. in.}$$

This stress is larger than the allowable stress, which shows that the column section selected is too small. If a section 31 in. in diameter is assumed, then,

$$A = .7854 \times 31^2 = 754.77 \text{ sq. in.}$$

Substituting in the formula,

$$s = \frac{200{,}000}{754.77} + \frac{8 \times 4 \times 100{,}000}{754.77 \times 31} = 402 \text{ lb. per sq. in.}$$

Since this is less than 417 lb., a column of this diameter is safe. Ans.

EXAMPLE 2.—What should be the size of a column of square section, to carry the same load as in example 1, the other conditions being the same?

SOLUTION.—Assuming a column 26 in. square, $A = 26^2 = 676$. Substituting in formula **2**,

$$s = \frac{200{,}000}{676} + \frac{6 \times 4 \times 100{,}000}{676 \times 26} = 433 \text{ lb. per sq. in.}$$

This is larger than 417, therefore a larger section, say 27 in. square, will be tried. $A = 27^2 = 729$. Substituting in the formula,

$$s = \frac{200{,}000}{729} + \frac{6 \times 4 \times 100{,}000}{729 \times 27} = 396 \text{ lb. per sq. in.}$$

Therefore, a column 27 in. square is sufficient. Ans.

EXAMPLES FOR PRACTICE

1. A column 15 inches in diameter and 15 feet high is made of 1–2–4 stone concrete; it carries a central load of 40,000 pounds and an eccentric load of 30,000 pounds. The eccentricity of the eccentric load being 3 inches, what is the intensity of stress produced?

Ans. 668 lb. per sq. in.

2. Design a round Maine granite column 18 feet high to carry 400,000 pounds centrally placed. Use a factor of safety of 10.

Ans. 18.426, say $18\frac{1}{2}$, in. diam.

REINFORCED CONCRETE

BEAMS

INTRODUCTION

8. Authorities Followed.—The design of reinforced concrete is not an exact science. Many engineers and manufacturers have devised excellent formulas to be used in design, and such formulas are usually named after their originator, as Thacher's formulas, Christophe's formulas, etc. It is proposed in this Section to follow somewhat closely the recommendations and formulas embodied in the excellent progress report of the "Joint Committee." This committee was composed of members of the American Society of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering and Maintenance-of-Way Association, and the Association of American Portland-Cement Manufacturers, "for the purpose of investigating current practice and providing definite information concerning the properties of concrete and reinforced concrete."

9. Principal Considerations of Concrete Design. Concrete is a material of low tensile strength. Its valuable properties are durability, fire-resisting qualities, high compressive strength, and relatively low cost. Its strength

ıcreases with age. In the design of reinforced concrète, the ensile strength of the concrete is usually neglected and the teel rods that are inserted into the mass are assumed to take ll the tension. Plain concrete, also, is seldom calculated to arry any tensile strains.

Failures of reinforced-concrete structures are usually due) three causes; namely, bad design, poor materials, and ulty workmanship. Of bad design, little may be said xcept that the utmost care must be exercised. Every detail : a structure must be carefully considered, as it is often on ıe careful and safe design of these details that the strength : a structure depends. Special attention must be paid to ıe connections of the various members of a structure. The ıit stresses employed may be too high. No unit stresses to e used for all cases that may arise are recommended in this ection. The engineer must consult the building laws of the unicipality in which the building is to be erected. These alues are to be reduced, if necessary, according to the xperience and judgment of the designer and according to ıe amount of care used in construction. The unit stresses nployed in the examples in this Section are mostly those commended by the Joint Committee. They are for conete, mixed in the proportions to be specified, "capable of veloping an average compressive strength of 2,000 pounds r square inch at 28 days, when tested in cylinders 8 inches diameter and 16 inches long, under laboratory conditions manufacture and storage, using the same consistency as used in the field." These stresses are higher than are owed in some large cities; but, again, the stresses allowed some cities are considered by many engineers to be too nservative.

Only Portland cement should be used for reinforced conete. Cinder concrete is not suitable, and only suitable st-class steel should be employed. The concrete should properly made out of suitable and sufficient materials. should be properly placed under suitable weather conions, and the forms should be left up a sufficient length time to allow it to attain sufficient strength to support itself.

10. Aggregates.—Concrete, as mentioned elsewhere, is made of cement and **aggregates.** The Joint Committee divides the aggregates into two divisions; namely, *fine* and *coarse.* "**Fine aggregate** consists of sand, crushed stone, or gravel screenings, passing, when dry, a screen having $\frac{1}{4}$-inch diameter holes. * * * **Coarse aggregate** consists of inert material, such as crushed stone or gravel, which is retained on a screen having $\frac{1}{4}$-inch diameter holes. For reinforced-concrete members, a size to pass a 1-inch ring, or a smaller size, may be used." In both fine and coarse aggregates, a gradation of the size of the particles is generally advantageous. In all cases, the aggregates should be of first-class quality, free from soft material and long or flat particles.

In regard to the proportions of aggregates, the Joint Committee states: "For reinforced-concrete construction, a density proportion based on 1–6 should generally be used; that is, 1 part of cement to a total of 6 parts of fine and coarse aggregates measured separately." If twice as much coarse aggregate as fine aggregate is used, this proportion will become 1–2–4, which is a richness of mixture often used for reinforced concrete. For important work, different proportions of fine and coarse aggregates may be made into test pieces of concrete, so that the strength of the concrete can be determined. In columns, a mixture of concrete richer than 1–2–4 is often required. In great masses, where the concrete is used principally for its compressive strength and its weight, poorer mixtures may be employed. In all cases, careful judgment should be exercised in selecting the proportions best suited to the conditions of the problem.

The concrete should be mixed with just enough clean water to permit it to flow into the forms and about the metal reinforcement and yet not cause the broken stone to separate from the cement and sand when the concrete is being conveyed from the mixer. Retempered concrete should not be used.

11. Details of Design.—In regard to stopping off work, the Joint Committee states: "For concrete construction, it is desirable to cast the entire structure at one operation, but

as this is not always possible, especially in large structures, it is necessary to stop the work at some convenient point. This point should be selected so that the resulting joint may have the least possible effect on the strength of the structure. It is therefore recommended that the joint in columns be made flush with the lower side of the girders; that the joints in girders be at a point midway between supports, but should a beam intersect a girder at this point, the joint should be offset a distance equal to twice the width of the beam; that the joints in the members of a floor system should, in general, be made at or near the center of the span. Joints in columns should be perpendicular to the axis of the column, and in girders, beams, and floor slabs perpendicular to the plane of their surfaces." Tongued and grooved expansion joints in plain concrete should be made about every 50 feet.

After concrete has been deposited in column forms it shrinks somewhat. For this reason, girders should not be constructed over columns until at least 2 hours after the columns are placed. If the concrete in the columns has become hard, it should be cleaned and slushed with mortar to insure a good bond.

12. One important factor to be considered in reinforced concrete is the *loads*. In the design of reinforced-concrete beams, columns, and slabs, the weight of the column, beam, or slab must never be neglected, because the dead load in reinforced concrete forms a large proportion of the total load. Therefore, the weight of the beam itself must always be included in the dead load. If a live load is suddenly applied, or is moving or vibrating so that the structure is subject to impact, vibration, or shock, special precautions must be taken. It is customary to add a certain percentage to such a load and thus reduce it to a quiescent live load that has an equivalent effect. Fortunately, such loads are rarely encountered except in railway bridges, which are problems for the bridge engineer.

The Joint Committee states that "the span length for beams and slabs shall be taken as the distance from center to center

of supports, but shall not be taken to exceed the clear span plus the depth of the beam or slab. Brackets shall not be considered as reducing the clear span in the sense here intended."

DESIGN OF SIMPLE RECTANGULAR BEAMS

13. Neutral Axis.—In speaking of beams, the *neutral surface* is a surface above which, in a simple beam, the fibers of which the beam is composed are in compression and below which the fibers are in tension. If a cross-section of the beam is taken, this neutral surface will show as a line, and it is known as the **neutral axis**, as explained in *Theory of Beams*. Now, in ordinary beams made of a single material, as wood or steel or the like, this neutral axis always passes through the center of gravity of the section. With beams composed of concrete and steel, however, this is not the case. Here, the neutral axis is the line where there is no stress, but the center of gravity of the section is not located on it.

14. Assumptions Used in the Derivation of Formulas.—Two assumptions are made in deriving the formulas for beam design. The *first assumption* is that *all the tension is taken up by the steel.* Inasmuch as the concrete, as explained in Art. **9**, cracks easily, no reliance is put on its strength, and the steel rods are assumed to take all the tension. Above the neutral axis, however, the concrete withstands the compressive strains.

The *second assumption* is as follows: *Any plane cross-section of a reinforced-concrete beam remains a plane after the beam is loaded and, therefore, bent.* This matter will be better understood on referring to Fig. 2. In (*a*) is shown an end view of a beam, and in (*b*) a side view. The reinforcement is indicated by *a*. When the beam is loaded, it sags, or deflects, somewhat, and although this sag is usually so small that it cannot be seen, it is there nevertheless. This sag is due to the fact that the fibers in the top of the beam compress and shorten up while the steel in the bottom of the beam stretches.

n (c), a portion of the beam is shown on an enlarged scale. his portion of the beam is the one just at the right of the utting plane $e f$ in view (b). When the beam is loaded, he fiber at g is compressed and pushed over to g'; also, the eel at h is stretched until it extends to h'. Now, g and h re in the cutting plane and, according to the second ssumption, g' and h', which are the new positions of the bers, are also in one plane. Of course, the distances $g g'$ nd $h h'$ are greatly exaggerated in the figure to make this atter clear.

15. Notation Used.—In the formulas about to be condered, the following notation will be used:

s_s = tensile stress, per square inch, in steel;
s_c = maximum compressive stress, per square inch, in concrete;
E_s = modulus of elasticity of steel;
E_c = modulus of elasticity of concrete;
n = ratio of modulus of elasticity of steel to that of concrete $= \frac{E_s}{E_c}$;
M = resisting moment or bending moment, in inch-pounds;
A = area of steel in tension, in square inches;
b = breadth of beam, in inches (see Fig. 2);
d = depth of beam, in inches, from center of steel to top of beam (see Fig. 2);
p = proportion of steel $= \frac{A}{b d}$.

Both k and j are coefficients to be regarded as follows: Let e distance from the neutral surface to the top of the beam called x. Then, the value of k is taken so that $k = \frac{x}{d}$. hen, $x = k d$, which is the value given in Fig. 2 (b). In what llows, it is always more convenient to use $k d$ in place of x. a similar manner, the distance from the center of the eel to the center of compression of the concrete is called $j d$, shown in (b).

16. Derivation of Formulas.—Consider Fig. 2 (c). The angle $h\,N\,h'$ is equal to the angle $g\,N\,g'$. Both the angle $N\,g\,g'$ and the angle $N\,h\,h'$ are right angles; therefore, the triangles $g\,g'\,N$ and $h\,h'\,N$ are similar and

$$\frac{\text{length } h\,h'}{\text{length } g\,g'} = \frac{\text{length } N\,h}{\text{length } N\,g}$$

Now, the length $N\,g = k\,d$ and the length $N\,h = d - k\,d$; also, $g\,g'$ is the deformation in the fiber of concrete at the

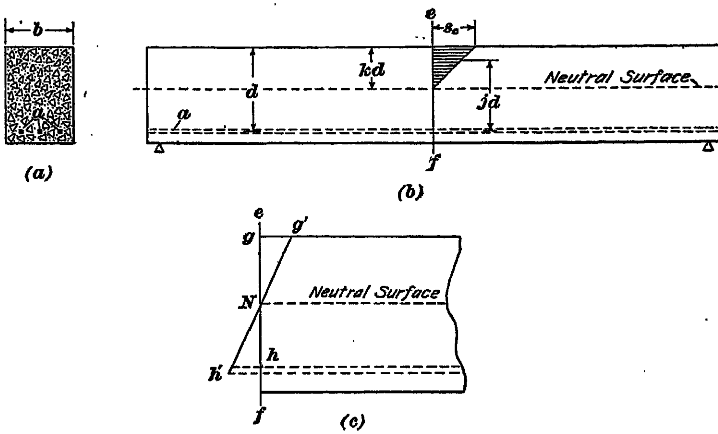


FIG. 2

top of the beam. The modulus of elasticity is equal to the stress divided by the deformation, or strain; that is, $E_c = \frac{s_c}{g\,g'}$, or $g\,g' = \frac{s_c}{E_c}$. Likewise, $h\,h' = \frac{s_s}{E_s}$. Putting these values in the equation just given,

$$\frac{s_s\,E_c}{s_c\,E_s} = \frac{d - k\,d}{k\,d}$$

However, $\frac{E_s}{E_c} = n$, or $\frac{E_c}{E_s} = \frac{1}{n}$. Substituting this value,

$$\frac{s_s}{s_c\,n} = \frac{d - k\,d}{k\,d} = \frac{d(1-k)}{d\,k} = \frac{1-k}{k}$$

Next, consider the section $e f$ in Fig. 2 (b). The concrete above the neutral axis is pressing apart the two parts of the beam on both sides of this section, and the steel below the neutral axis is holding together the two parts of the beam. It is evident that the concrete cannot push harder than the steel can pull, for if it did the two halves of the beam would separate. Therefore, the pull in the steel must just equal the push in the concrete. The amount of pull in the steel is, of course, equal to the area of the steel multiplied by the unit stress in the steel, or $A\,s_s$. The total compression in the concrete can be seen by examining view (b). The greatest unit stress in the concrete is s_c. This stress occurs at the top surface of the beam, and from this point down the stress decreases until at the neutral axis it is zero. The shaded triangle in this view will serve to give an idea of the change of stress. The area on which the compression acts is $b \times k\,d$, and the *average* stress acting on this area is one-half the maximum stress, or $\frac{s_c}{2}$. The total compression is therefore $\frac{b\,k\,d\,s_c}{2}$. Equating this to the tension,

$$A\,s_s = \frac{b\,k\,d\,s_c}{2}$$

Transposing,

$$\frac{s_s}{s_c} = \frac{b\,k\,d}{2\,A}$$

By transposing n in the equation $\frac{s_s}{s_c\,n} = \frac{1-k}{k}$, which has been already derived, there results

$$\frac{s_s}{s_c} = \frac{n\,(1-k)}{k}$$

Equating these two values of $\frac{s_s}{s_c}$,

$$\frac{b\,k\,d}{2\,A} = \frac{n\,(1-k)}{k}$$

But $\frac{A}{b\,d} = p$, or $\frac{b\,d}{A} = \frac{1}{p}$. Substituting this value,

$$\frac{k}{2p} = \frac{n(1-k)}{k}$$

and solving for k,

$$k = \sqrt{2pn + (pn)^2} - pn \qquad (1)$$

The value for j will next be found. The center of compression of the concrete is, of course, at the center of gravity of the shaded triangle which represents the stress in the concrete; that is, one-third of $k\,d$ from the top of the beam. The value of j is therefore found as follows:

$$j\,d = d - \tfrac{1}{3}k\,d = d\,(1 - \tfrac{1}{3}k)$$

Then,

$$j = 1 - \tfrac{1}{3}k \qquad (2)$$

The resisting moment of the beam must now be found. This is equal to the total compression of the concrete multiplied by the distance of the center of pressure from the neutral axis, added to the total tension in the steel multiplied by the distance of its center of tension from the neutral axis. As the total compression in the concrete is equal to the total tension in the steel, the resisting moment is equal to either the total tension in the steel or the total compression in the concrete multiplied by the distance between the center of tension and the center of compression, or $j\,d$. Therefore, the resisting moment may be expressed in terms of either the compression in the concrete or the tension in the steel. Therefore,

$$M = A\,s_s\,j\,d = p\,s_s\,j\,b\,d^2$$

and

$$M = b\,k\,d\,\frac{s_c}{2}\,j\,d$$

Transposing both these equations so as to solve for the stress,

$$s_s = \frac{M}{A\,j\,d} = \frac{M}{p\,j\,b\,d^2} \qquad (3)$$

and

$$s_c = \frac{2M}{j\,k\,b\,d^2} \qquad (4)$$

17. To Design a Simple Beam.—To design a beam, first assume values for p, n, and either d or b, preferably the for-

mer, and find k by the formula $k=\sqrt{2pn+(pn)^2}-pn$. Then find j by the formula $j=1-\frac{1}{3}k$, and calculate M from the loads to be carried. Next, assume a value for s_s and find either b or d by the formula $s_s=\frac{M}{Ajd}=\frac{M}{pjbd^2}$. Then find s_c by the formula $s_c=\frac{2M}{jkbd^2}=\frac{2ps_s}{k}$.

The value found for s_c must not be excessive. If it is, a new value must be assumed for p and the problem reworked or the moment reduced until s_c is safe.

18. As an example, design a beam on a 20-foot span to carry 1,000 pounds per foot. This load includes its own weight. Assume for the example that the total depth of the beam is 30 inches, and as the steel must be kept up a little from the bottom, let $d=28$ inches. Assume that $p=.006$, and as a value for n take 15. Then,

$$k=\sqrt{2pn+(pn)^2}-pn$$
$$=\sqrt{2\times.006\times15+(.006\times15)^2}-.006\times15=.343$$

and $$j=1-\tfrac{1}{3}k=1-\tfrac{1}{3}\times.343=.886$$

Assume that $s_s=16{,}000$ pounds and that $A=pbd=.006b\times28$. Then,

$$\frac{Wl}{8}=\frac{(20\times1{,}000)\times20}{8}=50{,}000 \text{ foot-pounds}$$

Therefore, $M=50{,}000\times12=600{,}000$ inch-pounds.

Substituting these values in the formula $s_s=\frac{M}{Ajd}$, it becomes $16{,}000=\frac{600{,}000}{.006b\times28\times.886\times28}$, which reduces to $b=8.998$, say 9, inches.

Substituting the correct values in the formula $s_c=\frac{2M}{jkbd^2}$,

$$s_c=\frac{2\times600{,}000}{.886\times.343\times9\times28\times28}=559.6, \text{ say } 560,$$

pounds per square inch.

If the designer considers this value of s_c to be safe, the beam may be said to be correctly designed.

19. The objection to the method of design just given is that s_s is assumed and s_c is found. This value of s_c may be too large or too small, and perhaps a new value of p must be assumed and the calculations remade. Of course, a beam is most economical when the maximum allowable unit stress in the steel is realized under the same load that realizes the maximum allowable unit stress in the concrete. If the steel reaches its maximum allowable stress first, then the beam cannot be loaded further until the maximum allowable unit stress in the concrete is reached, because the steel would then be overstressed. In such a condition, not enough steel is used to develop the full stress in the concrete. On the other hand, if too much steel were used, the strength of the beam would be limited not by the maximum stress in the steel, but by the maximum stress in the concrete which would be developed under a smaller load. Often, in practical design, conditions are imposed that fix the value of p or d, when the beam may be designed as just outlined. Its resisting moment is in this case limited either by the stress in the concrete or that in the steel. However, when nothing else interferes, it is often convenient to select such a value of p that the maximum allowable stresses in the concrete and steel are both reached under the same load. Some engineers call such a value of p the *most economical value* and others refer to it as the *critical value*.

To find this amount of steel, the following method is adopted: From the discussion in Art. **16**,

$$\frac{s_s}{s_c n}=\frac{1-k}{k}$$

Solving for k, it becomes

$$k=\frac{n\frac{s_c}{s_s}}{1+n\frac{s_c}{s_s}}$$

Since, at any section, the total compression in the concrete is equal to the total tension in the steel,

$$\frac{s_c\, k\, d\, b}{2} = A\, s_s$$

Transposing, $$k = \frac{2\, A\, s_s}{b\, d\, s_c}$$

But $\frac{A}{b\, d} = p$; therefore,

$$k = \frac{2\, p\, s_s}{s_c}$$

Equating the two values of k thus found,

$$\frac{n\frac{s_c}{s_s}}{1 + n\frac{s_c}{s_s}} = \frac{2\, p\, s_s}{s_c}$$

Solving for p, $$p = \frac{1}{2}\,\frac{1}{\frac{s_s}{s_c}\left(\frac{s_s}{n\, s_c} + 1\right)}$$

20. A beam can now be designed by using the critical value for p. First assume values for s_s, s_c, and n and find p by the formula

$$p = \frac{1}{2}\,\frac{1}{\frac{s_s}{s_c}\left(\frac{s_s}{n\, s_c} + 1\right)} \tag{1}$$

Then find k by the formula

$$k = \sqrt{2\, p\, n + (p\, n)^2} - p\, n \tag{2}$$

and find j by the formula

$$j = 1 - \tfrac{1}{3}\, k \tag{3}$$

Next, assume either d or b and find the other by either the formula

$$s_s = \frac{M}{A\, j\, d} = \frac{M}{p\, j\, b\, d^2} \tag{4}$$

or the formula

$$s_c = \frac{2\,M}{j\,k\,b\,d^2} \qquad (5)$$

If a value for p other than that found by formula **1** is used, then both formulas **4** and **5** must be solved so that the safe value of neither s_s nor s_c will be exceeded.

In actual practice, the designer fixes on values for n, s_s, and s_c and solves formulas **1**, **2**, and **3** once for all. The values assumed in this Section will be $n = 15$, $s_s = 16{,}000$, and $s_c = 650$ for concrete capable of developing a compressive stress of 2,000 pounds in 28 days, tested as previously stated and for steel specified by the committee. Many cities do not allow values so high, and the designer must be governed by the law. However, as some values must be used for illustration, the ones just given will be used in this Section to serve as examples and because they are recommended by the Joint Committee. It is therefore proposed to solve formulas **1**, **2**, and **3** once and for all for this Section.

Using values recommended by the Joint Committee and substituting them in the formula $p = \frac{1}{2} \times \frac{1}{\frac{s_s}{s_c}\left(\frac{s_s}{n\,s_c}+1\right)}$,

$$p = \frac{1}{2} \times \frac{1}{\frac{16{,}000}{650}\left(\frac{16{,}000}{15 \times 650}+1\right)} = .00769$$

Substituting this value in formula **2**, or

$$k = \sqrt{2\,p\,n + (p\,n)^2} - p\,n,$$

$$k = \sqrt{2 \times .00769 \times 15 + (.00769 \times 15)^2} - .00769 \times 15 = .379$$

Substituting this value in formula **3**, or $j = 1 - \frac{1}{3}\,k$,

$$j = 1 - \tfrac{1}{3} \times 379 = .874$$

If s_s is taken at 15,000 to 16,000 and s_c at 600 to 650, the Joint Committee states that for approximate results j may be taken at $\frac{7}{8}$. This would make k equal to $\frac{3}{8}$.

21. As an example, design a concrete beam to carry 900 pounds per foot on a span of 30 feet. This load includes the weight of the beam. The total bending moment is $\frac{900\times30\times30}{8}=101{,}250$ foot-pounds, or $101{,}250\times12=1{,}215{,}000$ inch-pounds. Taking the approximate values $k=\frac{3}{8}$ and $j=\frac{7}{8}$, assume that $d=26$ inches. Substituting the correct values in formula **5**, Art. **20**, $650=\frac{2\times1{,}215{,}000}{\frac{7}{8}\times\frac{3}{8}\times b\times26^2}$; therefore,

$$b=\frac{2\times1{,}215{,}000}{\frac{7}{8}\times\frac{3}{8}\times650\times26^2}=16.85, \text{ say } 16\tfrac{7}{8}, \text{ inches}$$

The area of steel required is, then, $p\,b\,d=.00769\times16\frac{7}{8}\times26$ $=3.374$ square inches. As the economical value for p was used, the work can be checked by formula **4**, Art. **20**; thus,

$$s_s=\frac{M}{A\,j\,d}=\frac{1{,}215{,}000}{3.374\times\frac{7}{8}\times26}=15{,}829 \text{ pounds,}$$

which is a close approximation to 16,000 pounds.

If the value of p is not the critical value, but is near it, the values found for j and k may be used for approximate results, because when used in their formulas under proper conditions both err on the side of safety.

EXAMPLE.—Design a beam on a 27-foot span to carry besides its own weight a load of 3,700 pounds per foot.

SOLUTION.—First, a value for the depth of the beam must be assumed. Thus, assume that the beam is 50 in. deep from the top surface to the bottom surface. As the steel must be up a few inches in the concrete, let it be assumed that $d=48$ in. The weight of the beam is not known and must be assumed. It may be assumed at 1,100 lb. per ft. After the design is completed, the accuracy of this assumption may be tested.

The total load is then $(3{,}700+1{,}100)\times27=129{,}600$ lb. The bending moment is $\frac{129{,}600\times27\times12}{8}=5{,}248{,}800$ in.-lb. For approximate results, assume, as stated by the Joint Committee, that $j=\frac{7}{8}$ and $k=\frac{3}{8}$. Substituting the correct values in formula **5**, Art. **20**, it becomes

$$s_c=\frac{2\,M}{j\,k\,b\,d^2}=650=\frac{2\times5{,}248{,}800}{\frac{7}{8}\times\frac{3}{8}\times b\times48^2}$$

Therefore, $b=21.36$, say $21\frac{3}{8}$, inches. $A=p\,b\,d=.00769\times 21\frac{3}{8}\times 48=7.89$ sq. in.

Checking by formula **4**, Art. **20**,

$$s_s=\frac{M}{A\,j\,d}=\frac{5{,}248{,}800}{7.89\times\frac{7}{8}\times 48}=15{,}839,$$

which is a fairly close approximation.

The weight per foot of the beam is $\frac{50\times 21\frac{3}{8}\times 150}{144}=1{,}113$ lb. This is 13 lb. more than the assumed weight, but for many purposes it is close enough. The problem, therefore, need not be reworked. Ans.

Example 2.—Using the critical value of p, design a floor slab on a 12-foot span to carry safely 500 pounds per square foot besides its own weight.

Solution.—In the design of a slab it is customary to consider a section of the slab 12 in. wide. Assume first that the weight of the slab is 150 lb. The total load per square foot is then 650 lb. and the load on the entire strip is $650\times 12=7{,}800$ lb. The moment is therefore

$$\frac{W\,l}{8}=\frac{7{,}800\times 12\times 12}{8}=140{,}400 \text{ in.-lb.}$$

It is noted that a width of slab of 12 in. is considered. Therefore, the value of b is fixed and that of d is found by formula **5**, Art. **20**. Thus,

$$s_c=\frac{2\,M}{j\,k\,b\,d^2}=650=\frac{2\times 140{,}400}{\frac{7}{8}\times\frac{3}{8}\times 12\times d^2}$$

Therefore, $d=10.48$, say $10\frac{1}{2}$, in.

A, or the area of steel in a strip 12 in. wide, is $A=p\,d\,b=.00769\times 10\frac{1}{2}\times 12=.97$ sq. in. for each foot in width. Checking the design by formula **4**, Art. **20**,

$$s_s=\frac{M}{A\,j\,d}=\frac{140{,}400}{.97\times\frac{7}{8}\times 10\frac{1}{2}}=15{,}754 \text{ lb.},$$

which is a close approximation.

Now, below the steel there should be 1 in. of concrete; therefore, the total depth of the slab will be $10\frac{1}{2}+1=11\frac{1}{2}$ in. A square foot of slab therefore weighs $\frac{11\frac{1}{2}}{12}\times 150=144$ lb., which is close enough to 150, the value assumed. Ans.

EXAMPLES FOR PRACTICE

1. Design to the nearest half inch a concrete slab on a 14-foot span to carry 500 pounds per square foot, besides its own weight. Put 1 inch of concrete below the reinforcement.

Ans. $\begin{cases} 13\frac{1}{2} \text{ in. total thickness} \\ 1.15 \text{ sq. in. of steel per ft.} \end{cases}$

2. Design a beam to carry 1,000 pounds per foot on a 12-foot span. This load includes the weight of the beam. Make the total depth of the beam 17 inches and place the steel 2 inches from the bottom of the beam.

Ans. $\begin{cases} \text{Width of beam, a little over 9 in., say } 9\frac{1}{4} \text{ in.} \\ \text{Area of steel, 1.07 sq. in.} \end{cases}$

3. Design a beam to carry a total uniformly distributed load of 00,000 pounds on a 27-foot span. Make the total depth of the beam 5 inches and raise the steel up 2 inches from the bottom.

Ans. $\begin{cases} \text{Width of beam, a little over } 13\frac{1}{2} \text{ in., say } 13\frac{3}{4} \text{ in.} \\ \text{Area of steel, 5.6 sq. in.} \end{cases}$

---

22. In all the preceding problems, k was taken as $\frac{3}{8}$ and as $\frac{7}{8}$ in order to get uniform results. It must be remembered, owever, that these values are approximate when using alues for s_c, s_s, and n as already given or when using values ear them. These approximate values will often be found ose enough for practical work. Nevertheless, the designer ust always bear in mind that the values are approximate, id if greater accuracy is required, the method explained in e first part of Art. **20** must be followed.

For various reasons the value of p is not always taken at 0769, but is sometimes taken near that value. If it is less an the critical value, then b may be found approximately the formula $s_s = \frac{M}{p\,j\,b\,d^2}$.

In a similar manner, if the value of p is more than the tical value, the formula $s_s = \frac{M}{p\,j\,b\,d^2}$ will not give safe lues, because it is the concrete that will then be overstressed st and b must be found by the formula $S_c = \frac{2M}{j\,k\,b\,d^2}$. If the

value for p is not the critical value, but is near it, the values found for j and k may be used for approximate results, because when they are used in their proper places both err on the side of safety.

23. The problem of design is often presented in this way: The values of b and d are determined and the beam must be designed to withstand a given moment M. According to formula **4**, Art **20**, $s_s = \frac{M}{A\,j\,d}$. Substitute in this formula values for s_s, M, and d. The value of j must be assumed to be $\frac{7}{8}$; its exact value cannot be found because p is not known. Solve this formula for A. As a check, find p from $p = \frac{A}{b\,d}$. If the value thus found is less than the critical value, the beam is safe, because the value of j assumed is for the critical value and is greater for all values of p less than this.

24. As an example, find out how much steel is required in a beam to resist safely a bending moment of 200,000 inch-pounds if $b = 12$ and $d = 20$.

Here, $16{,}000 = \frac{200{,}000}{A \times \frac{7}{8} \times 20}$. Therefore, $A = .7143$ square inch and $p = \frac{.7143}{12 \times 20} = .00298$. As this value is less than the critical value, the beam will not fail on account of crushing the concrete, and the design is therefore safe. The assumed value of j is for a critical value of p; for the actual value of p, it is found to be too small. This inaccuracy, however, errs on the side of safety. If desired, a slightly smaller value of A may be assumed and the problem checked by finding a value for j that corresponds exactly to this value, but such refinement is not often followed.

EXAMPLES FOR PRACTICE

. A beam is 10 inches wide and d is equal to 14 inches. It carries otal load of 1,000 pounds per foot, including its own weight, on a n of 8 feet. How much steel is required? Ans. .484 sq. in.

. How much steel is required in a beam when $M = 500{,}000$ inch-nds, $b = 16$ inches, and $d = 24$ inches. Ans. 1.488 sq. in.

---

25. It is understood that the preceding method is safe y when the concrete is not overstressed. This is taken e of by using the formula only when p comes less than critical value.

he Trussed Steel Concrete Company, manufacturers of the an bars, publish a popular approximate form of the formula d in the preceding problems. It is derived as follows: st the value for s_c is taken at 750 pounds per square inch. s value is high, but is sometimes used for the best rock crete. With s_s equal to 16,000, with the preceding value $_c$, and with n equal to 15, the value of p, from formula **1**, . **20**, is about .01 and that of j is about .86. Substituting se values in the formula $s_s = \frac{M}{A\,j\,d}$ and transposing, it omes $M = .86 \times 16{,}000\,d\,A$, or $M = 13{,}760\,d\,A$. In other ds, to design a beam, assume d and find M, and then e for A by the formula $M = 13{,}760\,d\,A$. The value of A nd must be less than 1 per cent. of $b\,d$. This formula umes stresses in the concrete of 750 pounds, which is high; ower stresses in the concrete are desired, the value of A uld be kept less than 1 per cent of $b\,d$.

he same manufacturers use the same formula for **T**-shaped ms when the neutral axis comes in the slab. If the slab ufficiently thick and sufficiently broad, 2 or more per t. of steel may be used in **T** beams, meaning of course, er cent. of the rectangular part of the beam above the l. For safety, however, the stresses created should be cked by the methods given later or by the tables made that purpose.

26. To Investigate a Simple Beam.—It often happens that a beam is already built, or at least the dimensions are decided on, and it is necessary to decide whether the beam will carry the required load. In such cases, b, d, and A are measured and p is calculated; also, M can be calculated from the load that the beam carries or is to carry. First, a value of n is assumed and k is found by the formula $k=\sqrt{2\,p\,n+(p\,n)^2}-p\,n$. Then j is found by the formula $j=1-\frac{1}{3}k$. Next, s_s is found by the formula $s_s=\frac{M}{A\,j\,d}$ and s_c is found by the formula $s_c=\frac{2\,M}{j\,k\,b\,d^2}$. If p is greater than the critical amount, the strength is limited by the stress in the concrete and $s_s=\frac{M}{A\,j\,d}$ need not be solved. If p is less than the critical amount, the strength is limited by the stress in the steel and the formula $s_c=\frac{2\,M}{j\,k\,b\,d^2}$ need not be solved.

27. As an example, assume that a certain beam is 12 inches wide and 24 inches deep. It is reinforced by two $1\frac{1}{2}$-inch round bars. This reinforcement is 2 inches from the bottom of the beam. It carries a load of 700 pounds per foot, which includes its own weight. The span is 24 feet. Is the beam safe?

The area of a $1\frac{1}{2}$-inch round rod is about $1\frac{3}{4}$ square inches. Therefore, the area of both rods is $3\frac{1}{2}$ square inches. Then,

$$p=\frac{A}{b\,d}=\frac{3.5}{12\times 22}=.0133$$

Assume, as elsewhere, that $n=15$. Therefore,

$$k=\sqrt{2\times .0133\times 15+(.0133\times 15)^2}-.0133\times 15=.4629$$

and

$$j=1-\tfrac{1}{3}\times .4629=.8457$$

Now, the value of p is greater than the critical amount. Therefore, the strength is limited by the stress in the concrete. The maximum moment is $\frac{700\times 24\times 24\times 12}{8}=604{,}800.$

Substituting the correct values, therefore, in the formula $s_c = \frac{2M}{j\,k\,b\,d^2}$,

$$s_c = \frac{2 \times 604{,}800}{.8457 \times .4629 \times 12 \times 22 \times 22} = 532 \text{ pounds,}$$

which is safe. As p is greater than the critical amount, s_s is still safer.

28. Sometimes the problem is varied somewhat from that already given, and is as follows: A beam of a certain size has a certain amount of steel. What safe load will it carry?

In such a case, the method of procedure is almost the same as before. First, find p, j and k, as usual; then assume s_s and s_c and find M by the formula $s_s = \frac{M}{A\,j\,d}$, or $s_c = \frac{2M}{j\,k\,b\,d^2}$. If p is less than the critical percentage, use the first formula; if more than the critical percentage, use the second one.

29. As an example, assume that a beam is 15 inches wide and has a total depth of 30 inches. The steel consists of three 1-inch square rods raised 2 inches off the bottom. What safe uniformly distributed load will the beam carry on a span of 10 feet?

First, find the value of p. In this case it is $\frac{3}{15 \times 28} = .00714$. Then,

$$k = \sqrt{2 \times .00714 \times 15 + (.00714 \times 15)^2} - .00714 \times 15 = .368$$

and

$$j = 1 - \tfrac{1}{3} \times .368 = .877$$

As p is less than the critical amount, the steel limits the strength of the beam because it will exceed its safe value first. Therefore,

$$s_s = \frac{M}{A\,j\,d}, \text{ or } 16{,}000 = \frac{M}{3 \times .877 \times 28}$$

Then, $M = 1{,}178{,}688$ inch-pounds. The moment is equal, of course, to $\frac{W\,l}{8}$; therefore, as $l = 10 \times 12 = 120$ inches, it is

$1,178,688 = \frac{W \times 120}{8}$, and $W = 78,579$ pounds. The load per foot is therefore $78,579 \div 10 = 7,858$ pounds. The beam itself weighs $\frac{15 \times 30}{144} \times 150 = 469$ pounds per foot. Therefore, the total extra load per foot that the beam will safely carry, besides its own weight, is $7,858 - 469 = 7,389$ pounds.

EXAMPLES FOR PRACTICE

1. A beam 7 inches in width and 16 inches in total depth is reinforced with one $1\frac{1}{4}$-inch round rod placed $1\frac{1}{2}$ inches from the bottom. What stress is produced in the concrete if it carries besides its own weight 2,740 pounds per foot on a span of 6 feet 8 inches?

Ans. $s_c = 680$ lb., which is high, but is sometimes used

2. A slab is 4 inches thick and it is reinforced with $\frac{3}{8}$-inch round rods placed 6 inches apart. The rods are up 1 inch from the under surface of the slab. The span is 6 feet. What load besides its own weight will it safely carry? Ans. About 124 lb. per sq. ft.

CONTINUOUS BEAMS

30. The beams so far considered have been simple beams. The bending moments have, of course, been calculated by the ordinary bending-moment formulas. Thus, for a uniformly distributed load, the moment is $\frac{W\,l}{8}$, in which W is the total load and l is the span from center to center of supports. If a beam or a slab is *continuous*, the bending moment between the supports is reduced and a negative moment is created over each support. It is not possible to calculate exactly how much the bending moment is reduced by making the beam continuous, and, besides, the matter is not looked upon in the same light by all engineers.

The Joint Committee recommends that for the inside spans of all continuous beams and floor slabs having more than three supports and of moderate length, the moment be taken as $\frac{W\,l}{12}$ instead of $\frac{W\,l}{8}$, and that the same moment be con-

idered to act in the opposite direction over each support. The end spans of continuous beams are continuous only at one end, the outside end being of course like a simple beam. The Joint Committee recommends that for beams the bending moment at the middle of the end span be considered as $\frac{W\,l}{10}$, nd the negative moment at the adjoining support be taken t a like amount. The bending-moment formulas, of course, re for a uniformly distributed load. No formulas are offered or concentrated loads, and the designer must use his own udgment when reducing moments in such cases. After the ending moment is found, the problem is solved as usual.

31. As an example, design a middle span of a continuous eam to carry a load, including its own weight, of 1,000 ounds per foot; the span is 12 feet.

The total load is $1{,}000 \times 12 = 12{,}000$ pounds, and the aximum moment is $\frac{W\,l}{12} = \frac{12{,}000 \times 12}{12} = 12{,}000$ foot-pounds, r 144,000 inch-pounds. Assume that $d = 12$, and use the itical value of p, which is taken as .00769; then, substituting e values in the equation $s_c = \frac{2\,M}{j\,k\,b\,d^2}$, $650 = \frac{2 \times 144{,}000}{\frac{7}{8} \times \frac{3}{8} \times b \times 12^2}$. Therefore, $b = 9.38$, say $9\frac{1}{2}$, inches and $A = .00769 \times 12 + 9\frac{1}{2}$ = .88 square inch. Checking by $s_s = \frac{M}{A\,j\,d}$, it is $s_s = \frac{144{,}000}{.88 \times \frac{7}{8} \times 12}$ = 15,584 pounds per square inch, which is close. This metal ust be put at the center of the span, and over each support at the top of the beam.

Usually, part of the reinforcement in the bottom of the eam is bent upwards to form the reinforcement over the pports, and additional reinforcement must sometimes be lded to make up the required amount. This reinforcement in the top should extend as far as there is any reverse nding moment, usually one-quarter of the span each way, three-tenths. The rods bent up from the bottom may ually be bent up so that they will cross the neutral surface the point of contraflexure. Care must be taken to insure

that the rods at the top of the beam over the supports are properly anchored so as to prevent them from slipping out of the concrete.

Some engineers claim that the effect of the reverse bending moment is uncertain, owing to the liability of one support settling slightly. Building laws usually cover this practice. If there is any chance of live loads being unbalanced on adjacent spans, it is illogical to consider the beam as continuous so far as the live load is concerned. Whether the beams are designed as continuous or not, the reinforcement should be secured over the supports to take up reverse bending moments.

SLABS REINFORCED IN BOTH DIRECTIONS

32. Sometimes square slabs have been reinforced in both directions. The gain is more apparent than real. Although the bending moment each way is about half the original in square slabs, the exact moments and the method of failure are not known, which makes accurate design impossible. Also, the two extra side beams must be strengthened to carry the slab at the sides. The load on the four supporting beams is not uniform but is a maximum at the center and is assumed to reduce to zero at each beam support, a more severe condition than a uniform load. If the slab is not square, the shorter span carries by far the larger part of the load, and if it is 1½ or more times as long as wide it should never have cross reinforcement. Slabs supported on four sides are not recommended, as they cannot be accurately designed, especially those that are not exactly square.

BEAMS WITH DOUBLE REINFORCEMENT

33. Sometimes, when a beam is used in certain locations, as, for example, a lintel over a window, both its thickness and its width are limited. If there is not sufficient room to build a beam as ordinarily designed to carry the required load, the beam must be strengthened in some manner.

One way of strengthening the beam is to insert a steel

I beam in the concrete. In such a case, the I beam should be designed to carry all the load.

If it is decided to use true reinforced concrete, resort must be had to some other expedient. By increasing the amount of reinforcement in the bottom of the beam, the concrete in the top of the beam will fail first. It is true that increasing the amount of steel reinforcement will lower the position of the neutral axis. Therefore, if the amount of reinforcement is doubled, the beam will hold about 20 per cent. more load than when no excess of reinforcement occurs, although the concrete will fail first. An increase of strength gained by such a method is too expensive.

The usual way of increasing the strength of a beam, the size of which is limited, is by using **double reinforcement;** that is, reinforcement placed at both the top and the bottom of the beam. The reinforcement at the top is in compression and assists the surrounding concrete to withstand the stress put upon it. Although this method is not any too economical, it is the best one to follow when the necessity arises.

34. The derivation of the formulas for such reinforcement will be omitted here, and only the formulas themselves will be given. They are derived along lines similar to those used for simple beams. The same notation is used as with simple beams, but the following additional characters are used:

A' = area of compressive steel, in square inches;

p' = ratio of compressive steel $= \frac{A'}{b\,d}$;

s_s' = unit compressive stress in steel, in pounds per square inch;

d' = depth from center of compressive steel to top of beam, in inches;

z = distance from center of compression to top of beam, in inches.

35. The diagram of the conditions of the beam is shown in Fig. 3. In double-reinforced beams the breadth and the

total depth are known from the conditions of the problem. The value of d is $1\frac{1}{2}$ or 2 inches less than the total depth. Then, virtually, b and d are known. Also, the bending moment is known. Then, from formula **3**, Art. **16**, an approximate value of A may be found. Subtract from this value of A the critical amount of reinforcement. The remainder is the steel that must be taken care of by steel in compression. Usually, it will take about $2\frac{1}{4}$ square inches of steel in compression for each square inch of the calculated excess of steel in tension. The compressive steel is usually so placed that d' equals $1\frac{1}{2}$ to 2 inches. After the beam is thus assumed, it must be investigated to see whether or not the stresses are within safe limits, as follows:

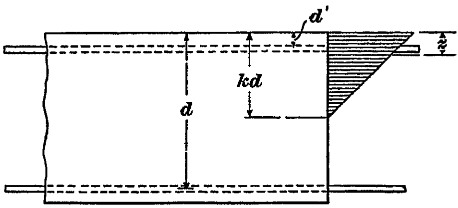


FIG. 3

Find k by the formula,

$$k=\sqrt{2\,n\left(p+p'\frac{d'}{d}\right)+n^2(p+p')^2}-n(p+p') \qquad (1)$$

Then find z by the formula,

$$z=\frac{\frac{1}{3}\,k^3 d+2\,p'\,n\,d'\left(k-\frac{d'}{d}\right)}{k^2+2\,p'n\left(k-\frac{d'}{d}\right)}, \qquad (2)$$

find $j\,d$ by the formula

$$j\,d=d-z, \qquad (3)$$

and find s_c by the formula,

$$s_c=\frac{6\,M}{b\,d^2\left[3\,k-k^2+\frac{6\,p'n}{k}\left(k-\frac{d'}{d}\right)\left(1-\frac{d'}{d}\right)\right]} \qquad (4)$$

Next find s_s by the formula,

$$s_s=\frac{M}{p\,j\,d^2\,b}=n\,s_c\frac{1-k}{k}, \qquad (5)$$

and find s_s' by the formula,

$$s_s' = n\, s_c \frac{k - \frac{d'}{d}}{k} \qquad (6)$$

36. The formulas giving the value of z and $j\,d$ need not always be solved, because formula **4,** Art **35,** does not include these terms and formula **5,** Art **35,** in one form, does not include them. Therefore, as a rule, the equations may be solved in the following order:

$$k = \sqrt{2\,n\left(p + p'\frac{d'}{d}\right) + n^2(p+p')^2} - n(p+p') \qquad (1)$$

$$s_c = \frac{6\,M}{b\,d^2\left[3\,k - k^2 + \frac{6\,p'\,n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right]} \qquad (2)$$

$$s_s = n\, s_c \frac{1-k}{k} \qquad (3)$$

$$s_s' = n\, s_c \frac{k - \frac{d'}{d}}{k} \qquad (4)$$

37. As an example, assume that a beam is on a 20-foot span. It is limited in total depth to 20 inches and in width to 10 inches. It carries 1,000 pounds per foot besides its own weight. Design the beam.

First, the bending moment must be found. The beam weighs $\frac{10 \times 20 \times 150}{144} = 208$ pounds per foot. The bending moment is therefore $\frac{(1{,}000 + 208)\ 20 \times 20 \times 12}{8} = 724{,}800$ inch-pounds. The approximate value of A is found by the formula $s_s = \frac{M}{A\,j\,d}$. If d is taken as 18 inches, it becomes $16{,}000 = \frac{724{,}800}{A \times \frac{7}{8} \times 18}$. Therefore, $A = 2.876$ square inches, which is the amount of steel to be used at the bottom of the beam.

The critical value is $p=.00769$. This area is $p\,d\,b=.00769\times18\times10=1.384$ square inches. The excessive area is, then, $2.876-1.384=1.492$ square inches. The approximate amount of steel required at the top of the beam is then $1.492\times2\frac{1}{4}=3.357$ square inches. The beam is now approximately designed, but it must be investigated to see whether or not it is safe. The values are $b=10$, $d=18$, $A=2.876$, $A'=3.357$, and $d'=2$. Then, $p=\frac{2.876}{10\times18}=.01598$; also, $p'=\frac{3.357}{10\times18}=.01865$.

Substituting the correct values in formula **1**, Art. **36**,

$$k=\sqrt{2\times15\,(.01598+.01865\times\tfrac{2}{18})+15^2\,(.01598.+01865)^2}\\-15\,(.01598+.01865)=.3813$$

Substituting the values in formula **2**, Art. **36**, $s_c=$

$$\frac{6\times724{,}800}{10\times18^2\left[3\times.3813-.3813^2+\frac{6\times.01865\times15}{.3813}(.3813-\tfrac{2}{18})(1-\tfrac{2}{18})\right]}\\=653$$

This value is 3 pounds higher than that recommended by the Joint Committee. However, the calculations do not have to be remade, as the value is close. When the beam is drawn on the plans, a little extra steel can be put in the top to reduce this compression if desired.

The value of s_s may be found by substituting the correct values in formula **3**, Art. **36**. Thus,

$$s_s=15\times653\,\frac{1-.3813}{.3813}=15{,}893 \text{ pounds per square inch,}$$

which is slightly low, but practically correct.

The value of s_s' is found by substituting values in formula **4**, Art. **36**. Thus,

$$s_s'=15\times653\,\frac{.3813-\frac{2}{18}}{.3813}=6{,}941 \text{ pounds per square inch}$$

This value is low, but it will always be found this way, and is the reason that double reinforced beams are not economical. This value cannot be raised in any convenient manner.

EXAMPLE FOR PRACTICE

A certain beam is built on a span of 30 feet. It is 12 inches wide and 20 inches deep. It contains 4.09 square inches of steel 2 inches from the bottom and 3.38 square inches of steel 2 inches from the top. It carries besides its own weight a load of 500 pounds per linear foot. There is doubt as to whether the stresses in the beam are too high. Determine the stresses in the concrete and in the steel in compression and tension.

$$\text{Ans.}\begin{cases} s_c = 775 \text{ lb. per sq. in.} \\ s_s = 15{,}770 \text{ lb. per sq. in.} \\ s_s' = 8{,}581 \text{ lb. per sq. in.} \end{cases}$$

DESIGN OF T BEAMS

38. Very often, in reinforced-concrete work, a beam and a floor slab are cast together. In such cases, the slab takes part of the compressive stresses caused by the bending moment, and the beam is no longer a rectangular beam, but one of **T** section.

Particular care must be exercised in the construction of T beams. The beam and the slab must be placed at one time so that there will not be a line of weakness between them. The stirrups must extend up into the slab, and the slab reinforcement should be tied to them. Short rods are sometimes run through the ends of the stirrups and made to extend into the concrete slab a short distance on each side of the beam. In short, every precaution possible must be used to insure that the slab and the beam will act together. If there is the slightest doubt about this unison, the beam must be figured as an ordinary rectangular beam underneath the slab.

39. The Joint Committee in discussing **T** beams states that "in beam and slab construction, an effective bond should be provided at the junction of the beam and slab. When the principal slab reinforcement is parallel to the beam, transverse reinforcement should be used extending over the beam and well into the slab.

"Where adequate bond between slab and web of beam is provided, the slab may be considered as an integral part of

the beam, but its effective width shall be determined by the following rules:

"(*a*) It shall not exceed one-fourth of the span length of the beam.

"(*b*) Its overhanging width on either side of the web shall not exceed four times the thickness of the slab.

"In the design of **T** beams acting as continuous beams, due consideration should be given to the compressive stresses at the support."

This last point is a matter of importance that is likely to be overlooked. At the supports of continuous beams there is a reverse bending moment, which is considered equal to the positive moment at the center of the span. Now, at the center of the span, the compression is at the top of the beam, and is taken up partly by the adjoining slab. At the support, however, inasmuch as the bending moment is reversed, the same amount of compression must be taken up by the narrow, or stem, part of the **T** at the lower side of the beam. As the total compression here is considered to be the same as at the middle of the span, the unit compression at the support is greater because the resisting area is less. The Joint Committee allows stresses 15 per cent. higher here than elsewhere; that is, if 650 pounds per square inch is allowed elsewhere, 748, or practically 750, pounds per square inch is allowed in compression of continuous beams near the supports. If this pressure is exceeded or if it is desired to reduce this value—some authorities consider it too high—then the beam must be made either wider or deeper in the stem of the **T** or else sufficient steel must be left in the bottom of the beam near the supports, making it practically a double reinforced beam at this point. This last method is the one usually followed.

40. In designing **T** beams, the same notation will be used as before, with the following changes:

b = width of flange, in inches;
b' = width of stem, in inches;
t = thickness of flange, in inches.

It is to be noted that the term b means the total width, and not the width of the beam proper, as in the preceding cases. This change is made so that some of the formulas already given will apply to **T** beams, as will be explained.

In the first place, the span of the beam and the thickness of the slab are known. Then, the depth of the beam and the width of the stem are usually assumed. From these conditions and the rules given, b may be assumed, but care should be taken not to consider the same part of the slab as acting for both of two adjacent beams. Then b, t, d, and b'

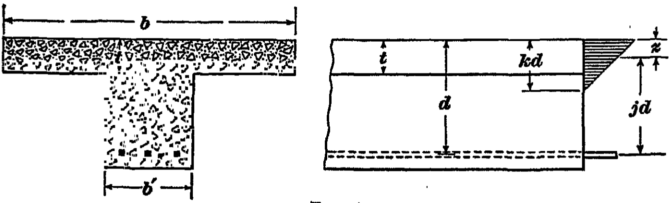


FIG. 4

are known. Reference to Fig. 4 will give a clear idea of where these dimensions are located.

41. First find an approximate value for A by the following formula:

$$A = \frac{M}{(d - \frac{1}{2}t)\, s_s} \qquad (1)$$

This formula will usually give values of A that err slightly on the side of safety. Some engineers, after A is thus found, consider the beam as designed. This method of procedure, however, is liable to cause trouble, because the value of A found by formula **1** may be such that the stress in the concrete is excessive. In such cases, the beam should be reassumed in larger proportions. Therefore, for a safe analysis of the beam, the following procedure must be carried out. This method neglects the compression in the stem.

After A has been assumed from formula **1** find the value of $k\,d$ from the formula

$$k\,d = \frac{2\,n\,d\,A + b\,t^2}{2\,n\,A + 2\,b\,t} \qquad (2)$$

From here on, the work divides itself into two cases. Case I is when $k d$ is less than t; that is, when the neutral axis comes in the slab. Case II is when $k d$ is greater than t; that is, when the neutral axis comes in the stem of the **T**. If the problem is in Case I, the beam is investigated by the formulas used for a rectangular beam. It is to be remembered that formula **2** just given is meant for use in Case II. If the problem is in Case I, it simply indicates that fact, and the value of k must be assumed or found by the formulas for rectangular beams. If the problem falls in Case II, the following procedure is employed: Find z by the formula,

$$z = \frac{3 k d - 2 t}{2 k d - t} \times \frac{t}{3} \qquad (3)$$

Then find $j d$ by the formula,

$$j d = d - z; \qquad (4)$$

find s_s by the formula,

$$s_s = \frac{M}{A j d}, \qquad (5)$$

and find s_c by by the formula,

$$s_c = \frac{M k d}{b t (k d - \frac{1}{2} t) j d} = \frac{s_s}{n} \times \frac{k}{1-k} \qquad (6)$$

The values of s_s and s_c must be safe. As previously stated, these formulas neglect the compression in the stem. For approximate results, the formulas for rectangular beams are sometimes used.

42. The design of **T** beams is best illustrated by an example as follows: A beam is on a span of 16 feet. It carries a load of 5,000 pounds per foot, which includes its own weight. It is cast solid with a slab that is 10 inches thick. The total depth of the beam is 31 inches, and the width of the beam proper is 16 inches. Design the beam.

First, let it be decided to keep the steel up 2 inches from the under side of the beam. Then, $d = 31 - 2 = 29$ inches. The maximum bending moment is $\frac{5{,}000 \times 16 \times 16 \times 12}{8}$

=1,920,000 inch-pounds. Substituting the correct values in formula **1**, Art. **41**,

$$A=\frac{1,920,000}{(29-\frac{10}{2})\ 16,000}=5 \text{ square inches}$$

The span is 16 feet. According to the recommendations of the Joint Committee, the value taken for b must not exceed $\frac{1}{4}\times 16=4$ feet, or 48 inches. Also, according to the Joint Committee, the overhang must not exceed $4\ t$, or $4\times 10=40$ inches. This second condition would make b equal to $40\times 2+16=96$ inches. As the former condition imposes the smaller value, it must be used. Some engineers also impose a third condition, namely, that b must be less than $5\ b'$; but as $5\ b'=5\times 16=80$, the conclusion arrived at above would not be altered. Therefore, b will be taken at 48 inches.

Substituting the correct values in formula **2**, Art. **41**,

$$k\ d=\frac{2\times 15\times 29\times 5+48\times 10^2}{2\times 15\times 5+2\times 48\times 10}=8.243 \text{ inches}$$

This value of $k\ d$ is less than the value of t. Therefore, the neutral axis lies in the flange and the problem falls in Case I, which means that it should be solved by the formulas for rectangular beams. In this solution, it is better to solve for k and j rather than assume one to be $\frac{3}{8}$ and the other to be $\frac{7}{8}$, because it is probable that the stresses in the concrete will be so low that these values will not be very close. Following this method,

$$p=\frac{A}{b\ d}=\frac{5}{48\times 29}=.003592$$

$$k=\sqrt{2\ p\ n+(p\ n)^2}-p\ n=\sqrt{2\times .003592\times 15+(.003592\times 15)^2}$$
$$-.003592\times 15=.2788$$

$$j=1-\tfrac{1}{3}k=.9071$$

and

$$s_s=\frac{M}{A\ j\ d}=\frac{1,920,000}{5\times .9071\times 29}=14,597,$$

which is more than safe. As was stated before, the formula $A=\dfrac{M}{(d-\frac{1}{2}\ t)\ s_s}$ usually gives values that err on the side

of safety, and this fact is evident from the result just obtained.

$$s_c = \frac{2\,M}{j\,k\,b\,d^2} = \frac{2 \times 1{,}920{,}000}{.9071 \times .2788 \times 48 \times 29^2}$$
$$= 376 \text{ pounds per square inch}$$

This value is low, but it is always best to have this value low in **T** beams.

43. As an example, to illustrate Case II, the following problem is proposed. It is very similar to the problem just given. A **T** beam is on a span of 16 feet. It carries a load of 5,000 pounds per foot, which includes its own weight. It is built solid with a slab 6 inches thick and the total depth of the beam is 31 inches. The value of d is taken at 29 inches, and b' is equal to 16 inches. Design the beam.

The bending moment is the same as before, namely, 1,920,000 inch-pounds. The value of A may be found approximately from the formula $A = \frac{M}{(d - \frac{1}{2}\,t)\,s_s}$. Thus, it is

$$A = \frac{1{,}920{,}000}{(29 - 3)\,16{,}000} = 4.6154 \text{ square inches}$$

The value of b is the same as before. Substituting the correct values in formula **2**, Art. **41**,

$$k\,d = \frac{2 \times 15 \times 29 \times 4.6154 + 48 \times 6^2}{2 \times 15 \times 4.6154 + 2 \times 48 \times 6} = 8.0388 \text{ inches,}$$

which is greater than the given value for t.

Substituting the proper values in formula **3**, Art. **41**,

$$z = \frac{3 \times 8.0388 - 2 \times 6}{2 \times 8.0388 - 6} \times \frac{6}{3} = 2.4 \text{ inches}$$

Substituting the proper values in formula **4**, Art. **41**,

$$j\,d = 29 - 2.4 = 26.6 \text{ inches}$$

Substituting the proper values in formula **5**, Art. **41**,

$$s_s = \frac{1{,}920{,}000}{4.6154 \times 26.6} = 15{,}639 \text{ pounds per square inch}$$

which is safe.

Substituting the proper values in formula **6**, Art. **41**,

$$s_c = \frac{1,920,000 \times 8.0388}{48 \times 6(8.0388 - \frac{6}{2})26.6} = 400 \text{ pounds per square inch,}$$

ich is safe.

EXAMPLES FOR PRACTICE

1. A T beam is on a span of 24 feet. It carries a load of 8,000 inds per foot, which includes its own weight. The following values re been decided on: $t = 10$ inches; $d = 41$ inches; $b' = 20$ inches; 15; and $A = 12$ square inches. Find the stress in the concrete and steel.

Ans. $\begin{cases} s_s = 15,502 \text{ lb. per sq. in.} \\ s_c = 438 \text{ lb. per sq. in.} \end{cases}$

2. A T beam is on a span of 18 feet. It carries a load of 4,000 inds per foot, which includes its own weight. The following values re been decided on: $t = 6$ inches; $d = 30$ inches; $b' = 16$ inches; 15; and $A = 4\frac{1}{2}$ square inches. Find the stress in the concrete and steel.

Ans. $\begin{cases} s_s = 15,630 \text{ lb. per sq. in.} \\ s_c = 357 \text{ lb. per sq. in.} \end{cases}$

---

44. As was stated, the formulas for Case II already given, glect the compression in the stem of the **T**. They will ially be found to give satisfactory values. However, en the slab, or flange part, of the beam is small, evidently ood part of the compressive resistance is taken up by the m. To neglect this compression would make the concrete pear more heavily stressed than it really is; also, when a ge part of the compression is taken by the stem, the value $j\,d$ is really less than that calculated by the formulas eady given and the stress in the steel will be somewhat her than was supposed. The Joint Committee recommnd that the following formulas be used if the flange is all compared with the stem:

$$k\,d = \sqrt{\frac{2\,n\,d\,A + (b-b')\,t^2}{b'} + \left(\frac{n\,A + (b-b')\,t}{b'}\right)^2} - \frac{n\,A + (b-b')\,t}{b'} \qquad (1)$$

$$z = \frac{(k\,d\,t^2 - \frac{2}{3}t^3)\,b + (k\,d - t)^2\left[t + \frac{1}{3}(k\,d - t)\right]b'}{t\,(2\,k\,d - t)\,b + (k\,d - t)^2\,b'} \qquad (2)$$

$$j\,d = d - z \tag{3}$$

$$s_s = \frac{M}{A\,j\,d} \tag{4}$$

$$s_c = \frac{2\,M\,k\,d}{[(2\,k\,d - t)\,b\,t + (k\,d - t)^2\,b']\,j\,d} \tag{5}$$

These formulas are rarely used; nevertheless, when employing the formulas under Case II, it must be borne in mind that the compression in the stem is neglected.

45. As an example of the use of the formulas in the preceding article, the problem given in Art. **43** will be solved by them. The following values are the same in both cases:

$M = 1{,}920{,}000$ inch-pounds;
$t = 6$ inches;
$d = 29$ inches;
$b' = 16$ inches;
$A = 4.6154$ square inches;
$b = 48$ inches.

Therefore, $k\,d$

$$= \sqrt{\frac{30 \times 29 \times 4.6154 + (48 - 16)6^2}{16} + \left(\frac{15 \times 4.6154 + (48 - 16)6}{16}\right)^2} - \frac{15 \times 4.6154 + (48 - 16)6}{16}$$

$$= \sqrt{322.962375 + 266.568888} - 16.3269 = 7.955 \text{ inches}$$

Then,

$$z = \frac{(7.955 \times 6^2 - \frac{2}{3}\,6^3)48 + (7.955 - 6)^2[6 + \frac{1}{3}(7.955 - 6)]16}{6(2 \times 7.955 - 6)48 + (7.955 - 6)^2\,16}$$

$= 2.484$ inches

$$j\,d = 29 - 2.484 = 26.516 \text{ inches}$$

and $s_s = \dfrac{1{,}920{,}000}{4.6154 \times 26.516} = 15{,}689$ pounds per square inch

This is slightly larger than the value found in Art. **43**, but is not dangerously so.

$$s_c = \frac{2 \times 1{,}920{,}000 \times 7.955}{[(2 \times 7.955 - 6)48 \times 6 + (7.955 - 6)^2\,16]\,26.516}$$

$= 395.2$ pounds per square inch,

which is a little less than the value in Art. **43.** On the whole, for cases like those given, it can be seen that the formulas that neglect the compression in the stem are sufficiently accurate unless the beam proper is large in proportion to the slab.

DETAILS OF DESIGN

46. Fireproofing.—So far, the quantity of concrete below the reinforcement in the lower part of the beam has not been discussed. This concrete serves two purposes. It holds the reinforcement in place and protects it from fire and moisture. The Joint Committee recommends that to protect the steel from fire the thickness of concrete under the reinforcement of girders should be 2 inches; under the reinforcement of beams, $1\frac{1}{2}$ inches; and under the reinforcement of floor slabs, 1 inch. These recommendations are for ordinary conditions, and must be changed as circumstances require. All sharp corners of beams should be chamfered.

47. Splices.—In splicing bars that take tension, the splice must be such that the tension of the entire bar is taken by the bond between that bar and the concrete, unless, of course, the bars are securely fastened together.

A very simple rule can be derived for the amount of lap necessary. Its derivation will be readily understood by referring to Fig. 5. Here, two round bars are lapped, and

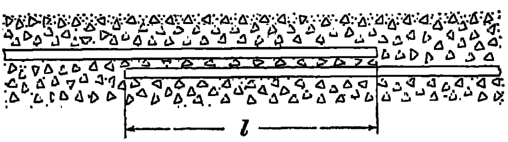


FIG. 5

the value of l must be found to make the section *good*, as it is called. Let the diameter of the bar, in inches, be called d. The circumference of the bar is then $d\times3.1416$, and its surface at the lap is $d\times l\times3.1416$. If the safe adhesion is 80 pounds per square inch, the total adhesion will be $d\times l\times3.1416\times80$. The distance l should be such that the

bar is liable to break before it will slip; that is, the resistance to slipping must at least equal the resistance to rupture. The area of the bar is $d^2 \times .7854$ square inch. If the unit tensile stress is taken at 16,000 pounds per square inch, the resistance to rupturing will be $d^2 \times .7854 \times 16{,}000$. Equating these two values, it is found that $d \times l + 3.1416 \times 80 = d^2 \times .7854 \times 16{,}000$, which reduces to $l = 50\,d$. Therefore, in lapping rods in reinforced-concrete work, the lap should be made fifty times the diameter of the rod. If the rods are made of drawn wire, the lap should be twice this amount. These values of safe bond are the ones recommended by the Joint Committee for concrete; that will develop a strength of 2,000 pounds in 28 days, tested as previously stated. Of course, for deformed bars, the lap does not have to be so great.

48. Value of n.—The value of n in the preceding formulas was taken as 15. This value of n is known to increase somewhat with the stress in the concrete, but many designers consider it to be constant. If the value of E_s is 30,000,000, then the value of E_c, to make n equal to 15, will be 2,000,000. The value to be used for n varies with different building laws and in the opinion of different engineers. The value employed in the examples is the one frequently used, and the one that the Joint Committee recommends to be used under ordinary conditions for the grade of concrete just mentioned.

49. Size and Spacing of Bars.—In the preceding formulas, the value of A has been taken as exact. In actual practice, this area of rods must be made up of commercial stock that is rolled in standard sizes. It will seldom be found possible to make up the exact area required. If, for this reason, slightly less steel than is specified is used, the calculations should be remade in order to make sure that the stresses in the steel will not be excessive.

In placing the bars in the concrete, it should be remembered that they must be protected from fire at the sides of the beam as well as underneath. In other words, if 1½ or 2 inches of concrete is allowed below the steel, these rods should be so placed that the same quantity of concrete will be between

m and the sides of the beam; also, they should be placed enough apart to allow the concrete to flow easily around m.

he general recommendations of the Joint Committee are part as follows: "The lateral spacing of parallel bars

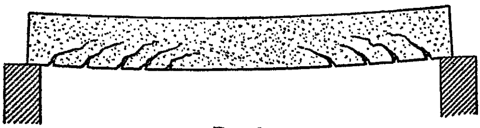


FIG. 6

uld not be less than two and one-half diameters, center to ter, nor should the distance from the side of the beam the center of the nearest bar be less than two diameters. e clear spacing between two layers of bars should not be than ½ inch."

50. **Shear in Beams.**—One of the most frequent thods of failure in concrete beams is illustrated in Fig. 6. s form of breaking is called *failure by diagonal shear*, but s generally due to *failure by diagonal tension*. The theory this form of failure will not be discussed here, but it may said to be due to the facts that all the reinforcement is the bottom of the beam and that no truss rods or stirrups used. The use of *truss rods* and *stirrups* will help prevent s form of failure, and both are used in beams of considere depth.

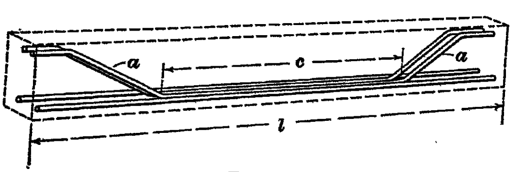


FIG. 7

51. **Truss rods,** besides preventing the diagonal crack- just mentioned, serve to take up at least part of the ative bending moment that occurs in continuous beams r the supports; also, if bent up high enough, they help the beam and slab together.

Fig. 7 shows the main rod reinforcement for an ordinary beam that is either carrying a uniformly distributed load or a load concentrated at its center. The maximum bending moment occurs at the center of the span; therefore, all the reinforcement is found at the bottom of the beam. However, near the ends of the span, the bending moment becomes less and the tendency to diagonal tension becomes greater. Therefore, some of the reinforcement can be spared from the bottom of the beam and may be bent up to form truss rods, as shown at *a*. These truss rods run directly across the place where the tension cracks would appear; and, thus, for the beam to fail here, the truss rods would have to be ruptured.

The distance designated by *c* in the illustration, is often made such that some of the steel will pass through the neutral surface at the point of contraflexure; that is, when the bending moment changes from positive to negative. It usually varies so that *c* is from one-fourth to one-half of *l*. It is understood that the truss rods, although shown for convenience as stopping short at the end of the span, extend into the adjoining beam or are bent over or secured at the ends so as not to slip. They also are preferably in smaller unit bent up, not all at once, but in pairs at intervals from the bottom of the beam to distribute their effect.

52. Stirrups, besides preventing diagonal tensile cracks, serve two other purposes, namely, to tie the reinforcement into the body of the beam in case the concrete below should fail from fire or any other cause, and to tie the beam to the slab, for they not only enter the slab, but may also be made to tie up to its reinforcement.

There is no rigid analysis governing the size and placing of stirrups. The matter is governed largely by experience. Stirrups are often made of $\frac{3}{8}$-inch square or $\frac{3}{16}'' \times 1''$ iron. They are spaced closer at the ends of the beam than at the center of the span. The Joint Committee recommends a maximum spacing of three-quarters of the depth of the beam. This maximum spacing occurs at the center of the span. At the ends of the span, the stirrups are often spaced

one-sixth or one-seventh of the depth of the beam. These stirrups must be spaced with extreme care, as the strength of the structure depends on them. Designs of successful similar buildings will often serve as models, not to be directly copied, perhaps, but to indicate at least about what the size and the spacing of stirrups should be. Short, deep beams and **T** beams must be designed with especial care.

If the stirrups are inclined to the vertical, they should be securely fastened to the main reinforcement. Care should be taken to see that the stirrups will not pull out of the concrete. Often, they are bent over at the ends or securely fixed to the slab reinforcement. Deformed bars also give a better grip on the concrete.

53. In regard to web resistance, the Joint Committee states that calculations should be made on a basis of maximum shearing stress; that is, although the beam fails by diagonal tension, it is designed to withstand shear. Therefore, the formulas, although using shear, must really limit the diagonal tension. This would indicate that exact methods for calculating these stresses are not yet devised. The Joint Committee states that, "Experiments bearing on the design of details of web reinforcement are not yet complete enough to allow more than general and tentative recommendations to be made."

The formulas referred to by the Joint Committee employ the following notation:

V = total shear, in pounds, to be found as explained in *Forces Acting on Beams;*
v = shearing unit stress, either horizontal or vertical, in pounds per square inch;
b = breadth of beam, in inches;
$j\,d$ = lever arm of resisting couple, as explained before, in inches;
P = stress in one stirrup, in pounds per square inch;
a = horizontal spacing of stirrups, in inches;
b' = width of stem of **T** beam, in inches.

The formulas for rectangular beams, whether reinforced at the bottom only or double reinforced, are:

$$v=\frac{V}{b\,j\,d} \qquad (1)$$

For vertical stirrups:

$$P=\frac{V\,a}{j\,d} \qquad (2)$$

For stirrups inclined at 45°:

$$P=.7\,\frac{V\,a}{j\,d} \qquad (3)$$

For **T** beams:

$$v=\frac{V}{b'\,j\,d} \qquad (4)$$

54. As an example, assume that the main reinforcement in a certain beam has been designed and that it remains to look into the web reinforcement. The beam is on a 20-foot span and carries 1,500 pounds per foot, which includes its own weight. Assume that $b=12$ inches and that $d=21$ inches.

First, the maximum shear is equal to either reaction, or

$$\frac{1{,}500\times 20}{2}=15{,}000 \text{ pounds}$$

The unit shear, assuming that $j=\frac{7}{8}$, is

$$v=\frac{V}{b\,j\,d}=\frac{15{,}000}{12\times\frac{7}{8}\times 21}=68 \text{ pounds per square inch}$$

The Joint Committee states that for concrete alone the web shear should not exceed 40 pounds per square inch, with truss rods properly placed, 60 pounds per square inch and that, even with truss rods and stirrups, the shear should never be over 120 pounds. As the value found is less than 120 pounds, the beam may be used; but as it is more than 40 pounds, stirrups and truss rods must be employed.

The size of the stirrups is next selected, and the allowable stress in it is decided on. Thus, the value of P, the allowable stress in one total stirrup, may be determined, bearing

in mind that either the stirrup must have some mechanical bond or else the value of P must not be great enough to disengage the stirrup from the concrete by either tension or shear. Let it be assumed in this case that the allowable stress in each stirrup is 6,000 pounds. Now, it is generally assumed that the concrete itself can take one-third of the shear; therefore, the stirrups have to resist only two-thirds of the shear; that is, $\frac{2}{3}\times 15{,}000 = 10{,}000$ pounds. Substituting the current values in formula **2**, Art. **53**, it becomes $6{,}000 = \dfrac{10{,}000\ a}{\frac{7}{8}\times 21}$, or $a = 11$ inches, about. This is the spacing for vertical stirrups near the supports where the shear is greatest. Of course, near the center of the beam, this spacing will increase as the shear decreases, but it should never be greater than $\frac{3}{4}\ d$.

It must be remembered that this method of designing web reinforcement is not a complete analysis of the case, and the Joint Committee does not state that it is more than tentative. However, it serves as a guide, and its use should be supplemented by good sound judgment and experience.

One point recommended by the Joint Committee to assist in preventing diagonal tension cracks is to arrange the horizontal reinforcement so that its unit stresses will be relatively low at points of high shear.

55. Another detail of design that is generally considered at the same time that the question of stirrups is taken up, is the bond stress of the concrete to the reinforcement. It is not alone necessary that the bar be long enough to offer bond to withstand the total tension in the reinforcement; in addition, the unit bond at any one point must not be exceeded. It is evident that if the tension in the main reinforcement at the center of the span is, say T, and the tension in it at another section near it is T', the difference in tension between these two points will be $T-T'$. This extra tension, which the bar looses between these two points, is of course taken up by the bond between the bars and the concrete, and this bond must not be exceeded. Rather than find the tension

at two points by finding the bending moment, a simpler method has been devised.

Let u = unit bond at section under consideration, in pounds per square inch;

O = sum of perimeters of horizontal reinforcement bars at the section under consideration;

$d\,j$ = the lever arm, as before;

V = external shear at the section under consideration.

Then, $$u = \frac{V}{j\,d\,O}$$

The value of u thus found must not exceed safe limits, which, according to the Joint Committee, are 80 pounds for plain bars and 40 pounds for drawn wire for the grade of concrete recommended. Of course, for deformed bars, higher bond values may be used.

COLUMNS

CONCENTRICALLY LOADED COLUMNS

56. There are two general methods of reinforcing concrete columns with steel. One method is known as *straight reinforcement* and the other as *hooped reinforcement.* These two styles of reinforcement are illustrated in Fig. 8. In (*a*) is shown straight reinforcement. It consists of steel rods that stand vertically in the concrete. Sometimes, the rods are placed directly in the middle of the column, but as a rule they are arranged around the outside of the column at least 2 inches from the surface. These steel rods are tied together with wire ties, as shown at *a*. The distance between two ties should not exceed the width, or diameter, of the column. If the ties are spaced too far apart, the column is liable to fail by the reinforcement bulging out, as shown in Fig. 9.

In Fig. 8 (*b*) is shown a column reinforced with hooped reinforcement. This type of reinforcement consists of either

a steel spiral or a separate steel hoop that is at least 2 inches from the surface of the column, as shown at *b*.

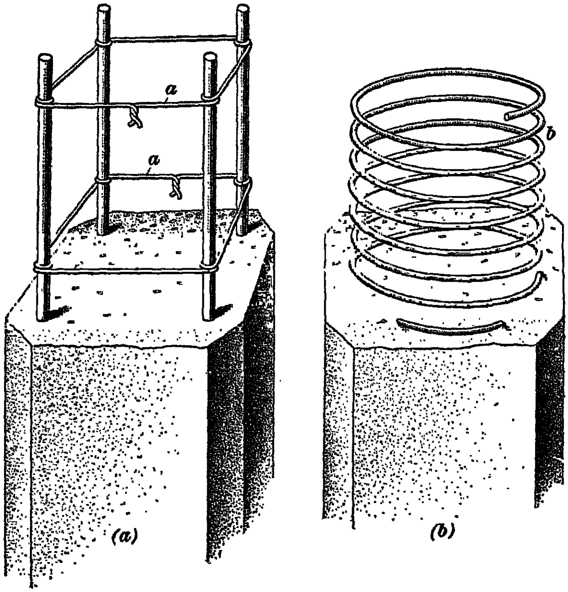


FIG. 8

57. In addition to the two types of reinforcement just mentioned there are many styles, more or less indeterminate, that engineers must examine with care. Thus, Fig. 10 (*a*) shows a column in which the reinforcement is straight. The netting around the reinforcement serves the twofold purpose of holding the reinforcement in place and of holding the surface of the concrete so that it will neither crack nor crumble off during a fire.

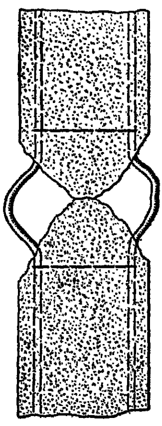


FIG. 9

In view (*b*) is shown a hooped column. The hooping is shown at *b*, and at *a* are shown light steel wires that hold the hooping in place while the concrete is being poured. These wires *a* are too thin to be considered as vertical reinforcement.

In (*c*) is shown a column that has two distinct kinds of reinforcement, the hooped reinforcement at *b* and the vertical, or straight, reinforcement at *a*.

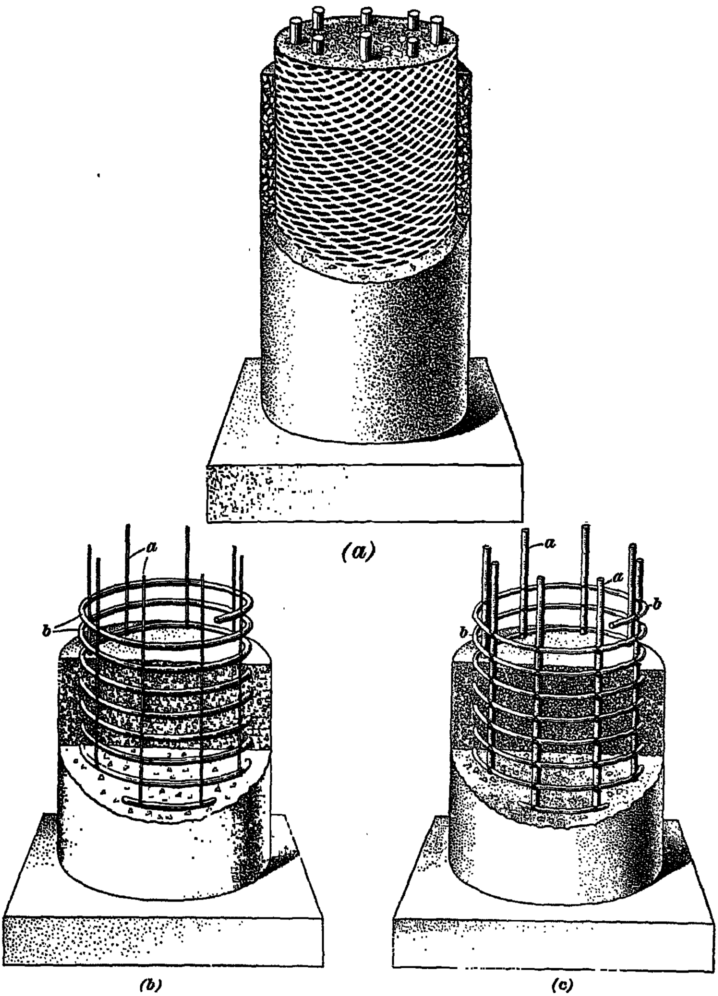


FIG. 10

58. Column Fireproofing.—The Joint Committee recommend that in concrete columns the outside $1\frac{1}{2}$ inches

of concrete be not considered in the design, as it may be injured in a fire. This coating, therefore, is simply a *fireproof covering*. After a column is designed, provided the recommendations of the Joint Committee are followed, $1\frac{1}{2}$ inches is added to all sides; that is, if the column is rectangular, it is made 3 inches broader and 3 inches deeper. All sharp corners should be chamfered off. Some engineers do not add this fireproof covering, although it is safe to do so.

59. Straight Reinforcement.—The design of **straight reinforcement** in columns will be taken up first.

Let A_c = effective area of cross-section of concrete, in square inches;

A_s = area of cross-section of steel, in square inches;

E_c = modulus of elasticity of concrete;

E_s = modulus of elasticity of steel;

s_c = compressive stress per unit area of concrete;

s_s = compressive stress per unit area of steel;

m_c = compressive unit strain in concrete;

m_s = compressive unit strain in steel;

$n = \frac{E_s}{E_c}$;

W = total load on column.

The load on the column equals, of course, the total compressive stress on the concrete plus the total compressive stress on the steel. Expressed as a formula, this would be

$$W = A_c s_c + A_s s_s \qquad (1)$$

From the principles given in *Stresses and Strains*,

$$E_c = \frac{s_c}{m_c} \text{ and } E_s = \frac{s_s}{m_s}$$

These equations may be written

$$m_c = \frac{s_c}{E_c} \text{ and } m_s = \frac{s_s}{E_s}$$

Now, as the concrete and steel are in direct contact and there is no slip of the steel—that is, the bond between the

steel and concrete is good—it follows that if the concrete in the column is compressed, say $\frac{1}{50}$ inch, the steel will also be compressed a like amount. As the same unit compression is exerted on both the steel and the concrete throughout the entire column, and the steel is bonded all the way, the unit strain in the concrete must equal the unit strain in the steel; that is, m_c must be equal to m_s. Therefore,

$$\frac{s_s}{E_s}=\frac{s_c}{E_c},$$

or

$$s_s=s_c\frac{E_s}{E_c}=s_c n \qquad (2)$$

Substituting the value just found for s_s in formula **1**,

$$W=A_c\, s_c+A_s\, s_c\, n,$$

or

$$W=s_c\,(A_c+n\, A_s) \qquad (3)$$

This is the equation used to design columns with straight reinforcement.

The Joint Committee recommends a value for s_c of 450 for concrete that develops a unit compressive strength of 2,000 pounds in 28 days, tested as previously stated. Some engineers do not use a value so high, but it will be used throughout this Section in order to get uniform results. The value of n is taken at 15, although sometimes considered less.

60. As an example, assume a column 1 foot square with 4 square inches of steel. What safe load will it carry?

First, a 1½-inch covering must be deducted. This makes the column $9\times 9=81$ square inches. Deducting the area of the steel, $A_c=81-4=77$ square inches. Also, $A_s=4$ square inches. Substituting these values in formula **3**, Art. **59**,

$$W=450\times(77+15\times 4)=61{,}650 \text{ pounds}$$

61. It will be well to investigate the theory of straight steel reinforcement in columns in order to obtain some idea of the economy of the system used. As was stated, the safe value for s_c is taken at 450 pounds per square inch; also, n is considered to be equal to 15. Substituting these values in formula **2**, Art. **59**,

$$s_s = s_c\, n = 450 \times 15 = 6{,}750 \text{ pounds per square inch}$$

Now, the safe unit compressive stress for steel may be taken as high as 16,000 pounds. Therefore, in a column with straight reinforcement, and the grade of concrete mentioned, the steel cannot be stressed up to its safe limit without overstressing the concrete. It cannot be said, therefore, that this method of placing the steel is economical. The use of steel in the column has advantages. In the first place, the formulas given in Art. **59** do not take into account the length of the column; the longer a column is, the more it is likely to bend or bulge, and the steel helps materially to resist this tendency. Then, again, the introduction of steel permits the column to be made much smaller in size, and this is often of great advantage. The most important advantage of the steel, however, is that there are usually certain slight eccentricities of load that are impossible to predetermine, and the steel more or less takes care of them.

The length of a column of this kind, according to the recommendations of the Joint Committee, should be less than 15 times its diameter or least width.

62. Having shown how to investigate a column, the method of designing one to carry a certain load will now be considered. Thus, assume that it is desired to design a column to carry a load of 40 tons.

In the first place, three things must be decided: (1) The value to be used for s_c; (2) that to be used for n; and (3) that to be used for the percentage of reinforcement. The values to be used for s_c and n will be chosen at 450 and 15, respectively. The percentage of reinforcement may be any reasonable amount. Let it be supposed that it is desired not to have the column too large; therefore, assume 2 per cent. In the examples for practice that follow, to obtain uniform results, the three values just given will also be used. To return to the problem, let A_c = area of concrete. Then, A_s = 2 per cent. of $A_c = \frac{1}{50} A_c$. Substituting the correct values in formula **3,** Art. **59,** it will be found that

$$W = s_c\,(A_c + n\,A_s)$$
$$= 450\,(A_c + 15 \times \tfrac{1}{50}\,A_c)$$
$$= 450\,A_c\,(1 + \tfrac{3}{10}) = 450 \times \tfrac{13}{10} \times A_c = 585\,A_c$$

But $W = 40 \times 2{,}000 = 80{,}000$ pounds; therefore.

$80{,}000 = 585\,A_c$ and $A_c = 80{,}000 \div 585 = 137$ square inches

The amount of steel required is $137 \times \frac{2}{100} = 2.74$ square inches. Therefore, the total area of the column will be $137 + 2.74 = 139.74$ square inches. A 12″ × 12″ column would therefore be ample for the conditions imposed. To this column, however, 1½ inches must be added on each surface for fireproofing purposes. Therefore, the column will be 15 inches square.

EXAMPLES FOR PRACTICE

1. Design a square column to carry a safe load of 48 tons.

Ans. { Size of col., approx. 16 in. on a side; Area of steel required, 3.3 sq. in. }

2. A column is 14 inches in diameter and contains 4 square inches of steel. What safe load will it carry? Ans. 67,965 lb.

3. Design a round column to carry 62 tons safely.

Ans. { Diam., about 19.5 in.; Area of steel, 4.24 sq. in. }

4. Design a rectangular column 10 inches wide to carry 32 tons safely.

Ans. { Depth, about 19 in.; Steel required, 2.19 sq. in. }

---

63. Empirical Rules for Straight Reinforcement. There are one or two more or less empirical formulas that are used in designing concrete columns with straight reinforcement. These formulas mostly originate from the building laws of various cities. One rule stipulates that the safe allowable load shall be 500 pounds per square inch of column section properly reinforced, counting both concrete and steel the same. Suppose it is desired to design a column to carry 25 tons. Now, 25 tons equals 50,000 pounds, and the required area would be $50{,}000 \div 500 = 100$ square inches, which would

mean a 10″×10″ column. The $1\frac{1}{2}$-inch fire protection is not included in the preceding calculations. Either sufficient steel must be inserted in the column to prevent the concrete from being overstressed in case $1\frac{1}{2}$ inches is removed from the outside by fire, or the $1\frac{1}{2}$ inches must be added afterwards, or, as is sometimes done, it must be neglected altogether.

64. In a large building, the loads on the columns in the lower floors become very great, and as it is usually not desirable to increase the outside size of the column abnormally, the percentage of reinforcement is increased. A style of reinforcement sometimes employed under such conditions is shown in Fig. 11. It consists of four angles riveted back to back in the center of the column to form a steel core. The area of this steel core can be increased if desired by using packing plates between the angles. Some engineers assume that the steel core takes all the vertical load and that the concrete prevents it from bending sidewise. The rule for designing such a column, therefore, is to allow a safe compressive unit stress on the steel and nothing on the concrete. Thus, if 20 square inches is required and four angles are to be used, each one will have to have an area of $20 \div 4 = 5$ square inches. Angles 4 in. × 4 in. × $\frac{11}{16}$ in. would be large enough. The stress in the steel should not be so high as to cause the concrete around it to break loose. In construction of this kind, steel rods should be used in conjunction with the steel core. These rods should be spaced as shown in the figure and securely wired together. They are not considered in calculating the safe load, but are inserted to stiffen the concrete, which is under high stress, and to prevent it from splitting away from the steel core in the center.

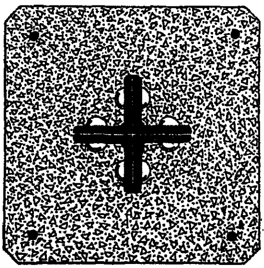


FIG. 11

65. Hooped Columns.—In regard to the effect of **hooped columns**, the Joint Committee states that: "The

general effect of bands or hoops is to increase greatly the 'toughness' of the column and its ultimate strength, but hooping has little effect upon its behavior within the limit of elasticity. It thus renders the concrete a safe and more reliable material, and should permit the use of a somewhat higher working stress." That is to say, in hooped concrete columns the hooping itself is not used in calculating the strength, but on account of the hooping a higher unit stress in the concrete may be used.

The Joint Committee recommends a stress 20 per cent. higher than that used for longitudinal reinforcement. Thus, if 450 pounds is used for longitudinal reinforcement, the stress for hooped columns will be 540 pounds. The Committee specifies, however, that to use this increase of stress the volume of the steel hooping must be at least 1 per cent. of the volume of the enclosed concrete. Some engineers use stresses lower than this, but these mentioned are used in this Section to make results uniform. One point of importance, however, to be remembered is that the concrete outside of the hooping is assumed to take no stress, all the stress being taken by the enclosed concrete. As the steel is embedded in the concrete at least 2 inches, this amount, instead of $1\frac{1}{2}$ inches, must be deducted to get the effective area of the column.

66. As an example, design a round-hooped concrete column to carry 100 tons. The load to be carried is $100 \times 2{,}000 = 200{,}000$ pounds. The required effective area is then $200{,}000 \div 540 = 370.37$ square inches. The diameter of the hooping, therefore, is approximately 22 inches. As there must be 2 inches of concrete outside of the hooping, the outside diameter must be at least 26 inches. The quantity of steel required per inch should be at least $370.37 \div 100 = 3.7$ cubic inches. In all cases, the hooping should be arranged in hoops, or spirals, so that the vertical distance from hoop to hoop, or from one loop of the spiral to another, will not be greater than one-fourth the diameter of the enclosed column. In this case, this maximum distance

would be $22 \times \frac{1}{4} = 5\frac{1}{2}$ inches. If hoops are used, the volume of the steel in each hoop is $5\frac{1}{2} \times 3.7 = 20.35$ cubic inches. As the circumference of the hoop is 69 inches, the sectional area of each hoop must be $20.35 \div 69 = .2949$ square inches. The nearest size of round rod that can be easily obtained on the market is the $\frac{5}{8}$ rod. The values given for the spacing and the size of the hoops are the limiting values. Safer values may be used if considered necessary.

67. Combined Reinforcement.—Sometimes, both hooped and longitudinal reinforcement is used. The hooping must not be considered as before, but if more than 1 per cent. and less than 4 per cent. of longitudinal bars is used, the Joint Committee will allow stresses in the concrete 45 per cent. greater than when no hooping is used; that is, a unit stress of about 650 pounds.

68. As an example, design a round column to carry 75 tons. Use 3 per cent. of longitudinal bars. The total load on the column is $75 \times 2{,}000 = 150{,}000$ pounds. Substituting correct values in formula **3**, Art. **59**, it becomes $150{,}000 = 650\ (A_c + 15 \times A_s) = 650\ A_c\ (1 + .45)$. Therefore, $A_c = 159.2$ and $A_s = 4.776$. The total area inside the hoop reinforcement, then, is about 164 square inches. This corresponds to a diameter that is a trifle less than 14.5 inches. By adding 2 inches outside the reinforcement, the diameter of the column is 18.5 inches. The rings must have a volume of at least 1.64 cubic inches for every inch in the height of the column.

The values used throughout these column problems are recommended by the Joint Committee. They are based on concrete that will gain an ultimate strength of 2,000 pounds per square inch in 28 days, when tested as specified before. They are somewhat high and may be reduced to suit the building laws of various cities or the judgment of the engineer.

EXAMPLES FOR PRACTICE

1. Design a hooped column that has no longitudinal reinforcement, to carry $62\frac{1}{2}$ tons.
Ans. { Diam., $21\frac{1}{4}$ in.
Metal per in., 2.31 cu. in.

2. Design a hooped column having in addition to the hoops $2\frac{1}{2}$ per cent. of vertical reinforcement, to carry 80 tons.
Ans. { Diam., $19\frac{3}{8}$ in.
Area of longitudinal reinforcement, 4.48 sq. in.
Hooping per in. of height, 1.79 cu. in.

ECCENTRICALLY LOADED COLUMNS

69. The methods of design just considered are for columns centrally loaded. Sometimes the column is not centrally loaded, in which event extra precautions must be taken. In ordinary building construction, where beams frame in opposite sides of a column, the column will be centrally loaded if both beams are loaded evenly. However, if one beam carries a large live load and the other beam does not, the loaded beam will deflect. This deflection will cause the column to bend. If the beams on each side and the columns have about the same rigidity, the bending moment in the column both above and below the beam will be about $\frac{W\,l}{48}$, in which W is the live load and l is the span. In the lower stories of a building, there will probably be such heavy loads from the stories above that the bending caused by the floorbeam is insignificant when compared with the concentric load from the superimposed columns. In the upper stories, however, this effect will be more noticeable, and should therefore always be borne in mind. The wall columns of a building offer still greater difficulties in calculation. Sufficient steel should be used and low unit stresses employed. If doubt exists about the safety, it should be investigated for bending.

70. The wind also causes stresses in columns. If the building is comparatively low and wide, or is protected, these stresses are either small or absent; but if the building is unpro-

tected from the wind and is narrow, the wind stresses must be considered. Brackets under beams and girders are largely used to stiffen the joints of a structure against wind.

There is probably no exact way to calculate the stresses caused by the wind. The following formulas, however, which are based partly on an analogy of conditions to those in a steel building, will serve at least to indicate that wind stresses are present and of importance:

$$C=\frac{P\,x}{2\,w} \qquad (1)$$

$$M=\frac{P\,y}{4} \qquad (2)$$

A diagram of the building is shown in Fig. 12. At *a* is shown the column under investigation. *P* is the total wind pressure over a surface whose height is *x* and whose width is the distance between two bays; *y* is the clear height of one story; and *w* is the distance from center to center of columns. *C* is the total concentric tension or compression put in the column by the wind, and *M* is the bending moment in the column. The wind also causes in the cross-girders a bending moment of about the same amount.

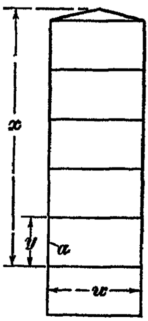


FIG. 12

71. An approximate method of design that will probably give safe results for a reinforced column that is eccentrically loaded is as follows: First, assume a column section. Then find the stress in the concrete and the steel due to compression, and next the stress in the concrete and the steel due to bending. Add the two stresses in the concrete thus found, and also add algebraically the two stresses in the steel thus found. The total stresses in the concrete and the steel thus obtained must be below the safe allowable stress for these materials. As this method is approximate, it will be well always to use low unit stresses. It is evident that hooped columns cannot resist tension on the side of the column opposite the eccentric load.

72. An example will serve to bring out these points. The column shown in Fig. 13 (*a*) carries two loads, one concentric and one eccentric, as shown. The column is of the size and design shown in the section (*b*). What stresses are produced?

First, the direct pressure will be found. It must be remembered that the outside $1\frac{1}{2}$ inches of concrete is for fireproofing purposes only. Therefore, the total effective area of the column is $22 \times 39 = 858$ square inches. For the reinforcement shown, $A_s = 6 \times 2.25 = 13.50$ square inches. Therefore, $A_c = 858 - 13.50 = 844.5$ square inches. The total load W is $(50 + 25) \times 2{,}000 = 150{,}000$ pounds. Substituting correct values in formula **3**, Art. **59**, it becomes $150{,}000 = s_c\,(844.5 + 15 \times 13.50)$. Therefore, $s_c = 143$ pounds per square inch, compression, and $s_s = n\,s_c = 15 \times 143 = 2{,}145$ pounds per square inch, compression.

The moment caused by the eccentric load may be taken at $36 \times 25 \times 2{,}000 = 1{,}800{,}000$ inch-pounds. Using the notation employed in connection with beams, and remembering to deduct for the fireproofing, $p = \dfrac{4 \times 2.25}{37.5 \times 22} = .01091$; $p' = \dfrac{2 \times 2.25}{37.5 \times 22} = .00545$; $d = 37.5$ inches; and $d' = 1.5$ inches. Therefore, according to formula **1**, Art. **36**,

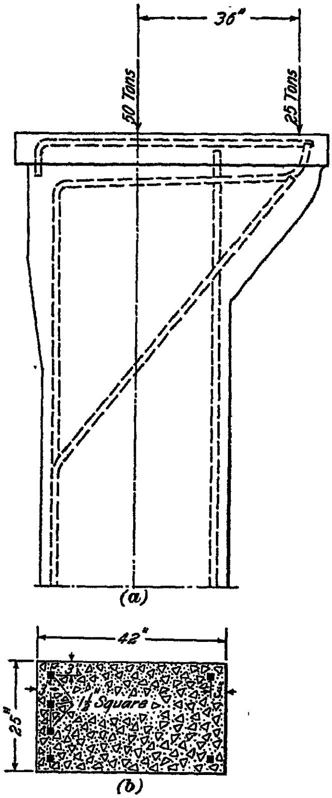


FIG. 13

$$k = \sqrt{2 \times 15\left(.01091 + .00545 \times \frac{1.5}{37.5}\right) + 15^2(.01091 + .00545)^2} - 15(.01091 + .00545) = .3824$$

From formula **2**, Art. **36**, $s_c =$

$$\frac{6\times 1{,}800{,}000}{22\times 37.5^2\left[1.1472-.3824^2+\frac{90\times .00545}{.3824}\left(.3824-\frac{1.5}{37.5}\right)\left(1-\frac{1.5}{37.5}\right)\right]}$$

$$=245 \text{ pounds per square inch}$$

From formula **3**, Art. **36**,

$$s_s=15\times 245\times\frac{1-.3824}{.3824}=5{,}935 \text{ pounds per square inch}$$

From formula **4**, Art. **36**,

$$s_s'=15\times 245\times\frac{.3824-\frac{1.5}{37.5}}{.3824}=3{,}291 \text{ pounds per square inch}$$

Thus, the greatest compression in the concrete is 143+245 =388 pounds per square inch, and the greatest tension in the steel is 5,935−2,145=3,790 pounds per square inch. The greatest compression in the steel is 2,145+3,291=5,436 pounds per square inch. These stresses are low and therefore the column is apparently safe. If the stresses are considered too high, a larger section of column must be assumed, and the stresses must be refound. If the stresses are considered too low, a smaller section may be investigated.

73. There are some inaccuracies in the foregoing investigation that must not be overlooked. In the first place, owing to the fact that there is more steel on one side of the column than on the other, the central load is not absolutely central with respect to the resisting column surface, because the column is more rigid on one side than on the other. Often, the column is designed with the same quantity of steel on both sides. This tends to reduce this defect. Then, the moment of the eccentric load was taken about the center of the column. Bending really occurs about the neutral axis. This neutral axis, however, is not in the same place as was found for beams, because the neutral axis is the line along which there is no stress and the direct compression in the column would shift this line. A third inaccuracy is

caused by the column bending out of plumb. This would have the effect of increasing the lever arm of the eccentric load. However, the column should be so short in comparison to its length that this deflection is negligible. Still another defect is that the space between the rods on the compression side is so wide that it is doubtful whether the column would act exactly as a double reinforced beam under bending. Lastly, the exact distribution of stress in the column is unknown. For all these reasons, eccentrically loaded columns cannot be investigated exactly and should not be used if they can be avoided. In any case, low unit stresses should be employed. The design of columns so loaded depends a great deal on experience, and they are not recommended to be used in important work.

In eccentrically loaded columns, particular attention must be paid to the wiring together of the rods. Here, the wires are used to prevent the bars from bulging, and they also act as stirrups to take up shear and diagonal tension. A reasonable amount of hooping will often answer the purpose. This hooping in such a case should not be considered as enabling the concrete to carry any more load.

EXAMPLES FOR PRACTICE

1. A column carries a concentric load of 10 tons and an eccentric load of 30 tons. The distance from the eccentric load to the center of the column is 3 inches. The column is 28 inches square and is reinforced by four $1\frac{3}{4}$-inch square bars. One of these bars is placed at each corner so that the center of each bar is $2\frac{1}{2}$ inches from the two nearest column faces. Find, approximately, the stresses produced, considering that the outside $1\frac{1}{2}$ inch of the column is simply a fireproofing.

$$\text{Ans.}\begin{cases}\text{No tension} \\ s_c = 145 \text{ lb. per sq. in.} \\ s_s' = 1{,}566 \text{ lb. per sq. in.}\end{cases}$$

2. A column is 20 inches square. It contains four rods $1\frac{1}{2}$ inches square. These rods are placed one at each corner, $2\frac{1}{2}$ inches from each face. It carries a load of 10 tons placed 20 inches from the center of the column. The outside $1\frac{1}{2}$ inches is considered to be effective only as fireproofing. What approximate stresses are produced?

$$\text{Ans.}\begin{cases}\text{Compression in concrete, 302 lb. per sq. in.} \\ \text{Compression in steel, 3,912 lb. per sq. in.} \\ \text{Tension in steel, 5,411 lb. per sq. in.}\end{cases}$$

In columns where the eccentricity is so small as to cause no tension in the concrete, the following approximate method will give probably closer results than the one just given. The same notation is used as with double reinforced beams. Also, h equals the width of the column from face to face after the fireproofing has been deducted, measured in the direction of the eccentric loads. By *transformed section* is meant the column section in which the steel is replaced by n times its area in concrete. First the *centroid* or center of gravity of the transformed column section must be located. If u is the distance from the centroid to the compression face after the fireproofing is removed, then $u=\frac{\frac{h}{2}+n\,p\,d+n\,p'\,d'}{1+n\,p+n\,p'}$. The moment of all the loads, whether in the geometrical center of the column or not, must be taken about the axis through this centroid. Let this moment be called M. Let A_t be the area of the transformed section, and I the moment of inertia of this section. Then $A_t = b\,h + n\,(A + A')$ and $I=\frac{1}{3}\,b(u^3+(h-u)^3]+n\,A\;(d-u)^2+n\,A'\;(u-d')^2$. The total maximum compression in the concrete is $s_c=\frac{W}{A_t}+\frac{M\,u}{I}$ where W is the total load. When $\frac{M\,u}{I}$ is greater than $\frac{W}{A_t}$ tension exists and the previous method must be employed.

As an example, assume a column with the fireproofing coat removed. It is then 21 inches square and carries 8 square inches of steel 1 inch from the face farthest from the eccentric load and 2 square inches of steel 1 inch from the other face. It carries 20,000 pounds placed in the center of the column, that is, $10\frac{1}{2}$ inches from each face, and 10,000 pounds 3 inches nearer the face where the 2 square inches of steel are located. Find the stresses produced. In this case $n=15$, $A=8$, $A'=2$, $d'=1$, $d=20$, $h=21$, and $b=21$. Then $p=\frac{A}{b\,h}$, not $\frac{A}{b\,d}$, as in beams, since the entire section is considered. $p=\frac{8}{21\times 21}$

$=.01814059$ and $p' = \dfrac{2}{21 \times 21} = .00453515$. Therefore,

$$u = \frac{\frac{21}{2} + 15 \times .01814059 \times 20 + 15 \times .00453515 \times 1}{1 + 15 \times .01814059 + 15 \times .00453515} = 11.947 \text{ in.}$$

20,000 pounds is $10\frac{1}{2}$ inches from the compression face. Its moment arm is therefore $11.947 - 10.5 = 1.447$ inches. The other load is $10\frac{1}{2} - 3 = 7\frac{1}{2}$ inches from the compression face. Its arm is therefore $11.947 - 7.5 = 4.447$ inches. The value of M is therefore $1.447 \times 20{,}000 + 4.447 \times 10{,}000 = 73{,}410$ inch pounds. Then $A_t = 21 \times 21 + 15\ (8+2) = 591$. Also, $I = \frac{1}{3} \times 21\,[11.947^3 + (21 - 11.947)^3] + 15 \times 8\ \ (20 - 11.947)^2 + 15 \times 2\ (11.947 - 1)^2 = 17130.12 + 7783.30 + 3595.10 = 28508.52$. Therefore, $s_c = \dfrac{30{,}000}{591} + \dfrac{73{,}410 \times 11.947}{28508.52} = 50.76 + 30.76 = 81.52$ pounds. As 50.76 is greater than 30.76, no tension exists. Since the entire column is in compression, the steel will always be safe. The above method, as was stated, is only applicable to cases where there is no tension and is in that case even only approximate, so low stresses should be used.

ELEMENTS OF STEEL REINFORCEMENT

USE OF STEEL IN REINFORCED CONCRETE

INTRODUCTION

1. Definitions.—*Reinforced concrete* is concrete in which is embedded steel in the form of rods, bars, shapes, or netting for the purpose of aiding the concrete in carrying loads. The metal used with the concrete is called *reinforcement;* the reinforcement is so proportioned and distributed within the concrete that the concrete and the steel act together as one unit in carrying the load.

2. Purpose of Reinforcement.—The necessity of reinforcing concrete arises from the fact that concrete is weak when subjected to tension. The compression strength of concrete is high enough to make concrete a very desirable building material; by adding steel reinforcement in such manner as to overcome the weakness in tension, concrete becomes available for many types of construction otherwise impossible of execution in concrete.

3. Applications of Reinforced Concrete.—Concrete reinforced with steel is used in many different types of structural elements. It is, however, in the construction of beams and similar elements subject to bending that the fundamental idea of reinforced concrete is most strikingly brought out, since when beams are bent one side is extended and the other com-

pressed; the steel is therefore placed in the tension side in order to strengthen it. Reinforced concrete is, however, also used in the construction of structures entirely in tension, such as tanks and certain types of pipes, as well as in structural elements mainly or entirely in compression, such as columns.

4. **Bond.**—The strength of a reinforced-concrete struc-tural member depends not only upon the quantity and quality of the concrete and steel. There is another very important element which greatly influences the strength, namely, the so-called *bond*. The bond is a cementing force that binds the steel and the concrete together and enables the concrete and the steel to act together in one unit in carrying the load. In rein-forced-concrete construction the features influencing the bond are of special importance because this method of construction s conditioned upon the bond.

5. **Distribution of Steel.**—Another important factor is he distribution of the steel reinforcement and especially its ocation and shape. Consider, as an illustration, a beam. Since n a bent beam one side is extended, the reinforcement must be ocated near the tension side; but this reinforcement is not suffi-ient, because tension stresses occur also in other parts of the beam, and reinforcement is needed to take care of these. The listribution of the tension stresses in reinforced-concrete struc-ures is not sufficiently known, and it is therefore not always possible to distribute the reinforcement to best advantage. A great many experiments have been made with reinforced con-rete in order to disclose the laws governing the combination of steel and concrete so that the steel reinforcement may be placed in the correct location, and this subject will now be dis-ussed.

6. **Tests of Reinforced Concrete.**—The tests made on einforced concrete are of two kinds; namely, *field tests* made on completed structures, such as buildings, bridges, or tanks, nd *laboratory tests* made on specially prepared specimens, usually of moderate size and so designed as to emphasize the ffect of some one particular feature. It is not the purpose ere to render a detailed account of tests and test results, but

certain selected tests will be used as examples illustrating and explaining the action of the reinforcement. In studying such tests, it must be borne in mind that the tests are frequently carried to complete destruction of the test piece; the phenomena exhibited in tests to destruction are not those likely to be encountered in daily practice, since the test loads are made much greater than the structure is called upon to sustain in practice. One purpose of such tests is to discover the weakest point of the structure and the manner of failure, in order to devise proper methods of preventing such failures.

7. The Joint Committee. — Through tests such as described and also through experience in the field, there has been created a vast store of knowledge relating to the theory and practice of reinforced-concrete construction. As a result, a certain uniformity of practice has been obtained, although there is still considerable difference of opinion among engineers in regard to many of the details. It is of course desirable that uniformity of design and erection methods be obtained so far as practicable, and in order to facilitate such uniformity a joint committee was appointed by the American Society of Civil Engineers, the American Society for Testing Materials, and other technical societies, for the purpose of creating a standard for both field and office methods. The final report published by this joint committee contains much valuable information; it is considered authoritative by most engineers and is frequently referred to in literature dealing with reinforced concrete. The instruction in this and following Sections is in accordance with the recommendations of the committee, which for brevity is referred to as the Joint Committee. The final report of the Joint Committee is printed in the Transactions of the societies named; copies of these transactions are on file in most public libraries and may be consulted there by any one interested.

TESTS OF REINFORCED CONCRETE

BOND AND BOND TESTS

8. Bond Resistance.—As already stated, the bond between concrete and steel is a cementing force existing between surfaces of concrete and of steel in contact; the strength of the bond between two such surfaces is called the *bond resistance*. The bond resistance is measured as the force required to pull a bar *b*, Fig. 1, out of a concrete block *a*, which in the diagram is shown cylindrical, but may be of any section, such as square. A bond test consists in pulling such a bar out of a block of concrete. One of three things must happen in an experiment of this kind: (1) The rod may break, in which event the tensile strength of the rod, and not the bond, is tested; (2) the block may split, as in Fig. 1, in which event the strength of the concrete and not the bond is the factor tested; or (3) the rod may pull out, both rod and block being otherwise intact. Only in the latter case can the bond existing between the steel and the concrete be measured.

The bond resistance is measured in pounds per square inch. The resistance per unit area, called the *intensity of bond stress*, equals the total force required to pull out the rod divided by the area of contact between the steel and the concrete.

9. Classification of Bond Resistances.—Experiments indicate that when a certain load is reached the bond resistance of the embedded rod is overcome and the rod begins to slide, or, as it is usually termed, *to slip*. Even after the initial slip takes place a considerable force is required to pull the rod through its concrete envelope, and distinction must therefore be made between the original, or so-called *adhesive resistance*, and the subsequent or so-called *sliding resistance*. It is the adhesive resistance upon which the strength of reinforced concrete depends, because if once sliding takes place, cracks are developed in the structure which lead to its ultimate destruction from other causes, and the primary purpose of bond tests is therefore to determine the adhesive resistance.

10. Adhesive Bond.—The amount of adhesive resistance developed between a steel bar and a concrete block depends mainly upon the following five factors: (1) *Area of contact between concrete and steel,* (2) *character of surface of rod,* (3) *shape of cross-section of rod,* (4) *strength and density of concrete,* (5) *size of the blocks,* and (6) *character of the reinforcement of the concrete block.*

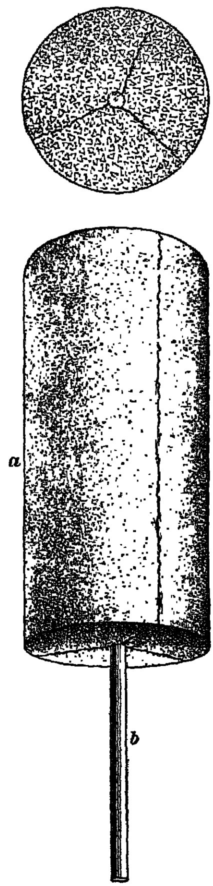


FIG. 1

11. Area of Contact.—The bond resistance increases in direct ratio with the length of embedment, so that a rod embedded for a length of 2 feet in a concrete block is twice as firmly held as is a similar rod embedded only 1 foot. In all kinds of reinforced-concrete construction it is of extreme importance that there shall be sufficient adhesion between the bars or rods and the concrete. A number of smaller rods will have stronger adhesion, or bond, than will one or a few larger bars containing an equivalent amount of metal. Thus, a bar 1 inch square measures $1\times4=4$ inches around, and if embedded to a depth of 10 inches the contact area is $4\times10=40$ square inches. So far as tensional strength is concerned, four $\frac{1}{2}$-inch square bars are the equivalent of one 1-inch bar, because their cross-sectional areas aggregate 1 square inch. But if the contact areas be compared, each $\frac{1}{2}$-inch bar has a circumference of $4\times\frac{1}{2}=2$ inches, or, for the four, $4\times2=8$ inches, so that if embedded to a depth of 10 inches, the four $\frac{1}{2}$-inch bars have a total contact area of $8\times10=80$ square inches, or twice as much as the 1-inch bar which they replace. For this reason, in reinforced-concrete construction it is usually preferable to use many light bars rather than a few heavy bars of the same total

cross-sectional area, because, as indicated in the example, the light bars have greater surface than the heavy bars, and therefore more contact area. For the same reason it is never permissible in reinforced-concrete construction to substitute one or a few heavy bars where many light bars are called for by a plan or specification.

12. Effect of Character of Surface of Rod.—Slight irregularities in the surface increase the bond resistance. Rusted bars develop about 15 per cent. more bond resistance than similar bars with ordinary so-called mill surface, and these again develop much more resistance than polished bars. Bars that are oily or greasy have a very small bond resistance and should not be used in reinforced-concrete construction.

Since experiments have shown that slight accidental irregularities in the surface of the steel improve the bond resistance, bars manufactured with irregularities are being produced and used. Such bars are known as *deformed bars* in distinction from the smooth round or square bars, which are termed *plain bars*. Deformed bars of several types will be described in this Section; their common feature is that knobs, projections, or depressions are formed in or on their surface for the special purpose of affording a better grip in the concrete. Authorities differ as to the efficiency of such devices. In America, deformed bars are very extensively used, in some localities almost to the exclusion of plain bars. In Europe, deformed bars are practically unknown and plain rods are used entirely. Laboratory experiments comparing plain and deformed bars do not always agree, although it seems to be an established fact that in the best-designed types of deformed bars the bond resistance is considerably greater than in plain bars. Probably at least 25 per cent. greater bond resistance is developed by well-designed deformed bars than by plain bars; the term *plain bars* here means ordinary rolled bars and not the kind of light bars known as *drawn wire,* which is manufactured by pulling the metal through dies and which is very smooth. Only one-half the bond resistance of rolled bars is developed by drawn wire.

13. Effect of Shape of Cross-Section.—Square bars have a bond resistance equal to 75 per cent. of that of round rods of the same cross-sectional area; flat bars are less firmly gripped than square bars. Structural steel shapes such as T's, L's, and I's have comparatively large surface areas but are not counted upon to develop great bond strength, owing to the irregular shape of the cross-section, which often prevents the concrete from flowing into a full contact all over the surface.

14. Effect of Strength and Density of the Concrete.—Tests indicate that the bond resistance of the embedded rod is nearly proportional to the amount of cement used in the concrete. Better and stronger concrete invariably gives greater bond resistance, so that any precautions taken to insure well-proportioned, well-mixed, well-laid concrete increase the efficiency not only of the concrete but of the reinforcement as well. More especially, the importance of proper curing of the concrete cannot be too strongly emphasized, since concrete which is allowed to harden under water or under damp sand—that is, concrete which is furnished with sufficient water for its proper hardening—will develop a bond resistance 10 to 45 per cent. in excess of that of concrete hardening in air.

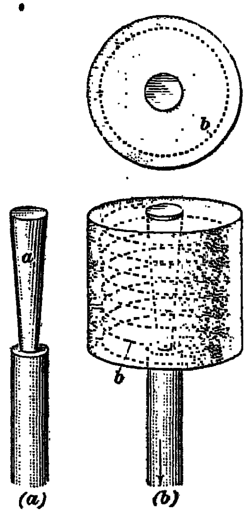


Fig. 2

15. Bond Resistance Affected by Size and Reinforcement of Block.—The size and reinforcement of a concrete block or specimen affects the bond resistance in a manner sometimes overlooked. For example, a rod of the shape shown at *a* in Fig. 2 (*a*), embedded in a concrete block reinforced with a spiral coil as shown in (*b*), will be difficult to pull out; for not only must the adhesive resistance be overcome, but on account of the taper, the rod must make a larger hole for itself in order to pass through. This will cause a wedgelike action

that will tend to split the block; but this tendency is counteracted by the spiral reinforcement, or hooping, which must be broken before the block can split. Therefore the bond resistance is increased by the hoops. Increasing the size of the block has a similar effect of overcoming the tendency to split. In practice, hoops are rarely used for increasing the bond resistance, but the important lesson is that it is useless to try to increase the bond resistance by using deformed bars having a so-called *wedging-taper* shape unless there is sufficient concrete or reinforcement around the rod to prevent the concrete from bursting open. Since the concrete cannot be forced to burst until there has been some slight motion of the rod, it is the sliding resistance rather than the adhesive resistance that is increased by the use of tapering deformed bars.

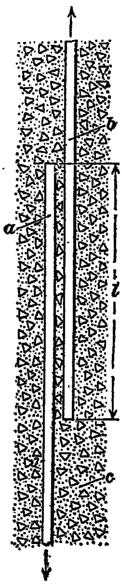


FIG. 3

16. Splicing of Rods.—The adhesion between concrete and steel makes it feasible to splice steel rods by means of concrete. Thus, in Fig. 3 are shown two rods *a* and *b* embedded in a concrete block *c*. Since both of these can be so firmly embedded that they will break rather than pull out, a joint can be made that is stronger than either of the rods; it is only required that the block be strong enough not to burst and that the *length of embedment*, or *lap*, *l* be great enough to prevent the rods from pulling out. This length of embedment is usually made 40 or 50 times as great as the diameter of the rod. Joints of the type described are much used in reinforced-concrete column construction; but in the construction of beams they are not looked upon with favor, because if any defects should occur in the joint the beam would necessarily break.

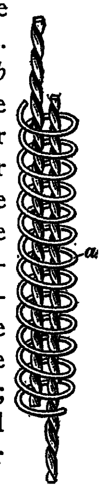


FIG. 4

Wherever splices have been made in beam reinforcement some form of positive mechanical lock has been employed, or, if the reinforcement consisted of deformed bars, these have

been spliced by surrounding the joint with a spiral coil as at *a* in Fig. 4. The effect of such a coil upon a joint of deformed bars is similar to the effect of a coil surrounding a single deformed bar embedded in concrete; that is, it tends to prevent motion of the rods in the concrete.

COLUMNS AND COLUMN TESTS

17. Columns.—Concrete is extensively used in the construction of columns in both engineering and architectural

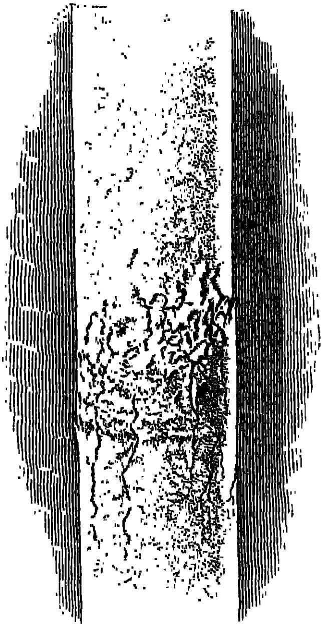


FIG. 5

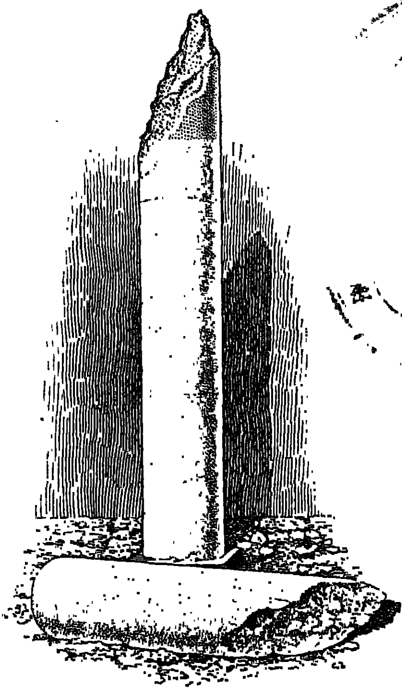


FIG. 6

works. The columns may be of plain concrete or of reinforced concrete, but plain-concrete columns are now rarely used and then only for short and heavy members, so that the reinforced-concrete column is of far greater practical impor-

tance. It is, however, necessary to investigate the action of plain-concrete columns in order to understand the action of reinforced-concrete columns, and for this reason tests have been made on plain-concrete columns.

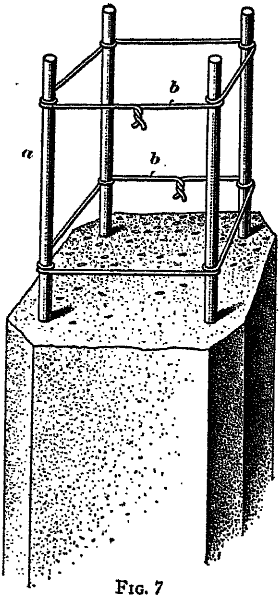


FIG. 7

18. Tests of Plain-Concrete Columns.—A plain-concrete column tested to destruction fails in one of two ways: The way shown in Fig. 5 is known as a *direct-compression failure;* that illustrated in Fig. 6 is called *diagonal-shear failure.*

A **direct-compression failure** results when the ultimate compressive strength of the concrete has been reached, the crushing of the concrete being manifested in a flaking or scaling of the outside of the concrete and a gradual shortening or telescoping of the column. This type of failure is likely to occur in columns made from concrete of lean mixtures, while columns made from concrete mixtures rich in cement are more likely to fail by diagonal shear. A **diagonal-shear failure** is caused by shear stresses, and occurs, as the name implies, along a sloping plane. This type of failure is always very sudden and is accompanied by a loud report, while the compression failure is more gradual.

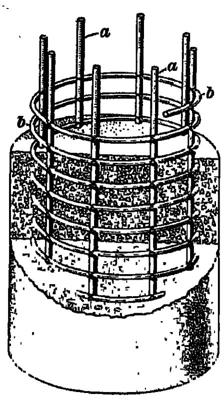


FIG. 8

19. Reinforced-Concrete Columns.—Three main types of concrete columns having steel reinforcement are in common use; these are: (1) The *rodded column,* Fig. 7, provided with vertical rods *a* and widely spaced horizontal ties *b*; (2) the *hooped column,* Fig. 8, provided with vertical rods *a*

and closely spaced spirally wound hoops *b*; and (3) the *cored column,* Fig. 9, having so-called cores or vertical structural shapes embedded in the concrete. The cored column cannot properly be called a reinforced-concrete column, because in it the steel carries the load and the concrete serves chiefly as protection for the steel, therefore no further reference will be made to it in this Section.

20. Rodded Columns.—Tests on rodded columns usually result in failures of two kinds similar in appearance to those of plain columns. One type of failure is shown in Fig. 10, where the column failed by diagonal shear, and the other is shown in Fig. 11, where the column failed by compression. In both of these illustrations the vertical rods, exposed by the failure of the column, are marked *a*. These rods, perfectly straight before the test started, have buckled under the load; and, as it takes considerable force to buckle steel rods embedded in concrete, it is evident that the rods carried considerable load, and therefore added to the strength of the column and made it stronger than a plain-concrete column of the same size. But it is also evident that the column could have carried even more load if the rods had remained straight, and they could have been kept so by the use of ties to prevent the rods from buckling. As shown in Fig. 11, the buckling begins right above and below the ties *b*, so that, by spacing the ties more closely, the rods would be forced to carry more load before buckling, and the entire column would there-

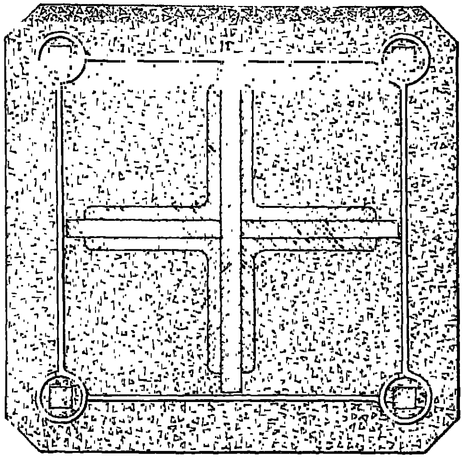


FIG. 9

fore be strengthened. In practice the ties should not be more than 12 inches apart, and preferably less.

21. Tests on Hooped Columns. — The difference between hooped and rodded columns is in the spacing of the ties, the ties being very close together in hooped columns. This close spacing not only increases the usefulness of the vertical rods, but, in addition, the hoops also increase the resistance of

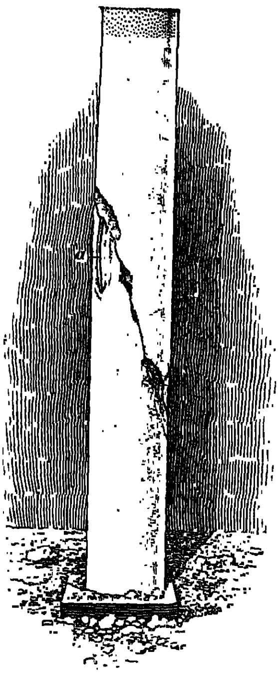


FIG. 10

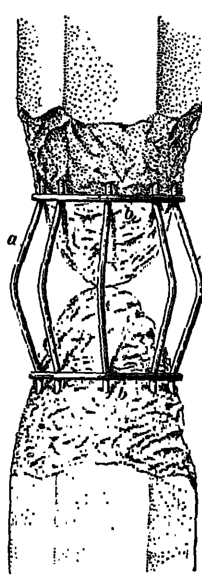


FIG. 11

the concrete. In tests of plain-concrete columns, it has been found that the compression failure is due to a sidewise yielding of the concrete; consequently, anything that will prevent this sidewise yielding will increase the strength of the column. Thus, in the test of the plain column, as shown in Fig. 5, it can be seen how the concrete disintegrated in a lateral direction; closely spaced hoops tend to prevent such disintegration. A

hooped column tested to destruction is shown in Fig. 12; this column, on account of the hoops, carried much greater loads than would a plain column or a rodded column. A plain or rodded column usually breaks suddenly. A hooped column yields slowly and under much greater loads, but unfortunately it is not possible to utilize all of this increase in resistance because of the great shortening of hooped columns under load. When tested to destruction some hooped columns have shortened as much as 2 inches in a length of 10 feet before they finally collapsed. The shortening is especially great in columns reinforced with hoops only or with hoops in conjunction with very light verticals; to prevent shortening, plenty of vertical steel must therefore be used. Columns so reinforced are so much stronger than rodded columns that 55 per cent. more load may safely be carried on a hooped column than on a rodded column of the same section and having the same vertical reinforcement.

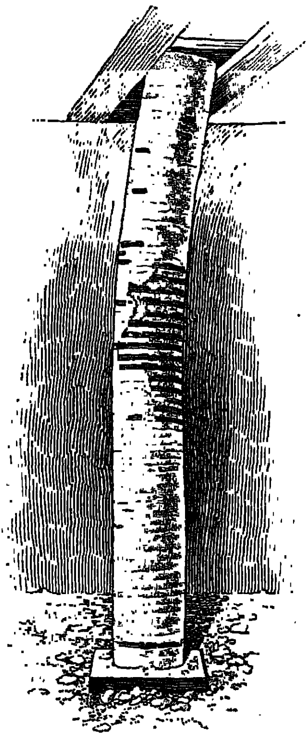


Fig. 12

One special feature of hooped columns is that they give warning before they collapse. This warning consists in a scaling of the surface as indicated in Fig. 12. In this column the hoops are plainly visible although they were covered by the concrete before the test began. The flaking or scaling takes place long before the column actually collapses, and it serves in practical construction as a danger signal should the column inadvertently become overloaded.

BEAMS AND BEAM TESTS

22. Use of Concrete in Beam Construction.—Plain concrete is rarely used in beams for construction purposes, although many beams of plain concrete have been made for

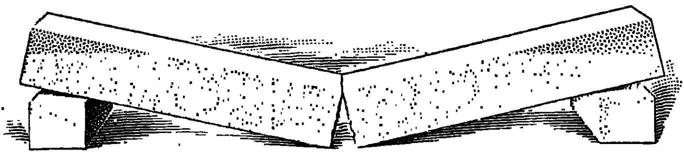


FIG. 13

test purposes. Plain-concrete beams when loaded to the breaking point invariably fail in the manner shown in Fig. 13, by pulling apart of the bottom fibers, because the under side of the beam is stressed in tension and concrete is weak in

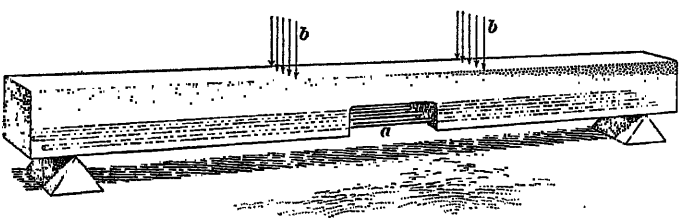


FIG. 14

tension. Concrete is, however, well suited for constructions in which tensile resistance is not to be furnished by the concrete itself, because concrete is strong in compression and in direct shear. Therefore, if means be introduced into the beam

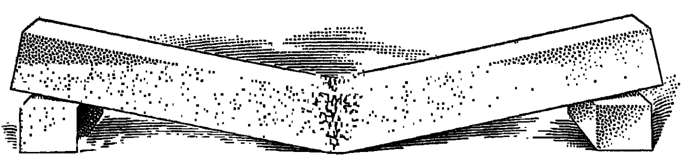


FIG. 15

to take care of the tension stresses, a strong and reliable beam results. The simplest way of taking care of the tension is by introducing plain straight bars *a* near the bottom surface, as in Fig. 14, where a part of the concrete has been broken away at

the center of the span in order to show the bars, or reinforcement, more clearly. A beam so reinforced is found by test to have entirely different properties than the previously described plain beam. Thus, in the best-designed reinforced-concrete beams failure results by compression of the top layers of the concrete, as shown in Fig. 15, under much higher test loads than plain beams could carry.

23. Methods of Testing.—Tests of many different kinds have been made for the purpose of establishing the laws governing the behavior of reinforced-concrete beams under load. There are two essentially different methods of testing reinforced-concrete beams; in one method, much used in the early days of reinforced-concrete construction, beams were

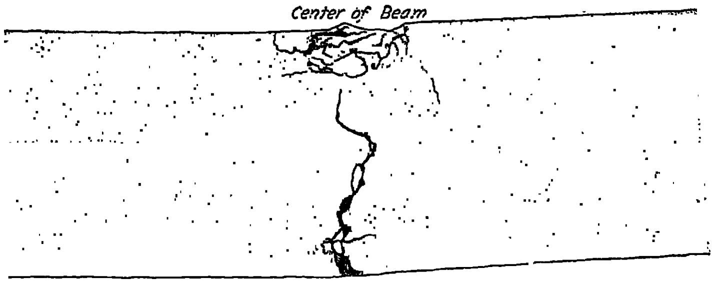


FIG. 16

loaded to destruction, and the total carrying capacity as well as the cause of failure were noted. In the other method, the minute shortenings and lengthenings are measured at various points of the beam; and since it is possible to compute the stress when the deformation is known, this method teaches what intensity of stress actually exists at any point of the beam under a certain load. These two methods do not necessarily preclude one another, since the beam can be tested to destruction after the desired loadings and corresponding measurements have been completed.

The load in **field tests** is usually in the shape of piles of brick, pig iron, sand or cement in bags, or merchandise in boxes, and the load is uniformly distributed over the length of the beam or over the entire floor panel if a floor is tested.

In **laboratory tests,** the load is applied by means of a machine, usually in the form of two concentrated loads acting against the beam as indicated by the arrows *b* in Fig. 14. These concentrated loads are the same distance from each end and frequently divide the span into three equal parts, in which case the beam is said to be loaded at the *third points.* If the middle part of the beam between the loads is twice as long as the end parts, the beam is said to be loaded at the *quarter points.* In the following, wherever beam tests are referred to, loads at the third points are utilized unless otherwise stated.

24. Beam Types Developed Through Testing.—It is not the intention here to describe the conclusions drawn from

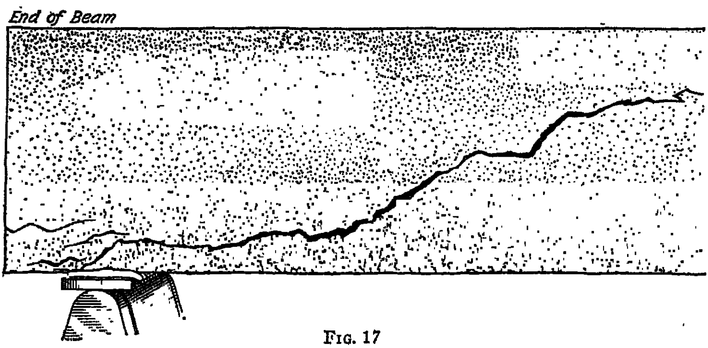


FIG. 17

analysis of carefully measured deformations. For the present purpose it suffices to say that reinforced-concrete beams when tested crack, that certain well-defined modes of cracking may be distinguished, and that all tests without exception indicate that the most efficient method of reinforcing is to place the reinforcing steel rods at right angles to the cracks. There are two distinct types of cracks in reinforced-concrete test beams; the one shown in Fig. 16 is caused by **direct tension,** the other, shown in Fig. 17, is caused by **diagonal tension.** Direct-tension cracks are vertical, and call therefore for horizontal reinforcement known as *main tension rods,* whereas diagonal tension cracks are inclined, running at times almost horizontal, and call for vertical or inclined reinforcement known as *web reinforcement.*

Examples of three commonly used methods of web and tension reinforcement are shown in Fig. 18. In (*a*) the two main tension rods *a* are straight, and the web reinforcement consists of lighter rods *b* bent to **U** shape and circling the tension rods, therefore usually referred to as **U** *bars* or *stirrups*. In (*b*) the single main tension rod *c* and the web reinforcements *d* are in one piece; this reinforcement is manufactured and sold as a unit. In (*c*), the main tension rods are four in number; two of these, marked *e*, continue straight from end to end of the beam, while two, marked *f*, are straight and horizontal in the middle of the beam only, the ends being bent up at an angle to form a so-called *truss*, for which reason these rods are often referred to as *truss rods* or *trussed rods*. Any one of these three types contains two essential elements, namely, horizontal tension rods and vertical or inclined web reinforcement, and any one of these types may be, and frequently is, used in practical construction.

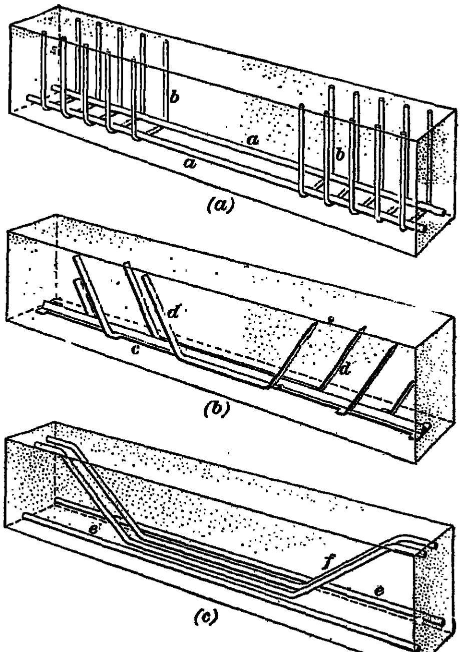


Fig. 18

A much more efficient type of reinforcement is, however, obtained by combining the **U** bars and the truss rods in one beam, as shown in Fig. 19, which is typical of modern reinforced-concrete beam construction. In this drawing the beam is shown in elevation in (*a*) and in section in (*b*); the tension rods are marked *a*, the truss rods *b*, the **U** bars *c*. If tested to

destruction, a well-designed beam of this kind may fail by compression of the concrete. An illustration of a beam failing

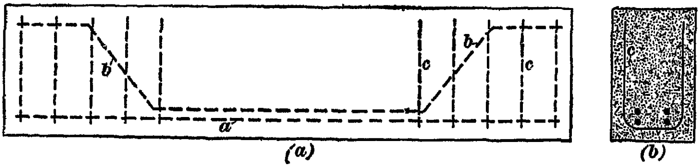


FIG. 19

by compression is shown in Fig. 20. The point of failure by compression is at the top of the beam and the failure is manifested by a slight scaling or flaking of the top layers.

25. T Beams.—All the beams referred to in the preceding articles have a rectangular cross-section such as shown in (*b*)

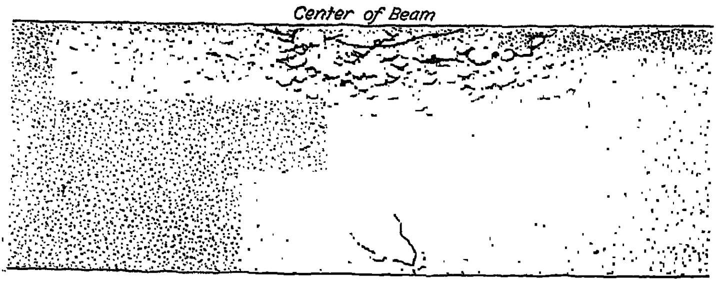


FIG. 20

in Fig. 19. It is, however, usually more economical to use a **T** beam, the shape of which is shown in Fig. 21. The **T** beam is a natural development in reinforced concrete; for, as already stated, the top fibers are in compression and the bottom fibers

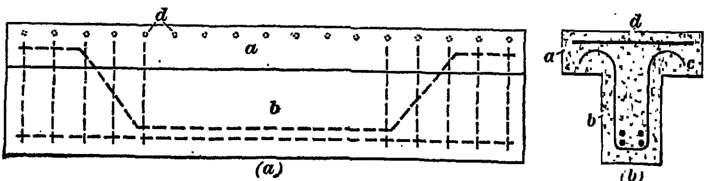


FIG. 21

are in tension in all beams, whatever their material may be. In reinforced-concrete beams, steel has been introduced for

the specific purpose of taking the tension at the bottom of the beam, so that the concrete is no more required for that purpose. If then, the part of the concrete that is no more needed be eliminated, a **T** beam results, which will be just as efficient as the rectangular beam, since the portions removed are of no practical utility. Of course, a small portion of the concrete must be retained in order to form a connection between the tension steel and the top, or compression, part *a*, called the *flange*. This connecting portion *b* is called the *web* or *stem*.

26. The thickness of the flange is often so arranged that the bottom of the flange coincides with the *neutral axis*. The **neutral axis,** *m n* in Fig. 22 (*a*), is the line in the section which forms the boundary between the compression stresses at the top and the tension stresses at the bottom; its exact location depends upon the ratio of steel area to concrete area, but is usually about one-third of the total depth below the top of the beam. The location of the neutral axis with reference to the bottom of the flange in a **T** beam is of importance, since if the neutral axis falls within the flange, two beams like those shown in (*a*) and (*b*) in Fig. 22 will be of the same bending strength, provided they have the same depth and breadth and have an equal amount of tension reinforcement; but if in a beam like that in (*a*) the flange is made of such small depth that the neutral axis falls in the web, the beam will be weaker than the rectangular beam shown in (*b*) because part of the compression area has been removed.

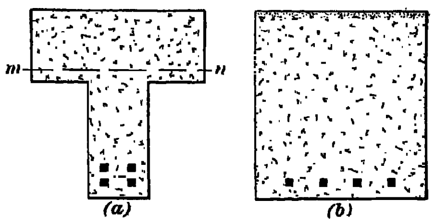


FIG. 22

27. The tension reinforcement in a **T** beam may be arranged as in Fig. 21; that is, exactly as for the rectangular beam shown in Fig. 19. But, even though the neutral axis is located within the flange, so that the **T** beam and the rectangular

beam will have the same amount of tension reinforcement, there is nevertheless an important difference in the action of the reinforcement in the two beams, because the steel is more scattered in rectangular beams than in T beams, as is shown in the two sections in Fig. 22 (*a*) and (*b*). It has already been explained that the bond resistance of steel embedded in concrete is affected by the quantity of concrete surrounding the steel, and it may therefore be expected that a better bond is developed between steel and concrete in rectangular than in T beams, since in the rectangular beam more concrete surrounds the steel than in the T beam. This point has not been fully investigated by test, although the question of bond stresses in reinforced-concrete beams is a very important one.

28. Bond Resistance in Beams.—It has been proved by test that under a small load a reinforced-concrete beam behaves exactly as if there were no reinforcement present in it. This cannot be verified with the naked eye, but only by microscopic measurements. As the load is increased, the beam begins to show very fine cracks; these first appear when the intensity of load reaches the point at which the plain concrete would fail. These minute cracks have no influence upon the strength of the beam; they simply mark the stage at which the reinforcement begins to act. At this stage the bond between the steel and the concrete is called into play. In a beam which is being tested to destruction, at first only a small portion of the contact surface of the rod near the center of the beam is stressed in bond; but, because it is only a small portion that is so stressed, the bond resistance at this point is all the sooner exceeded and the bond is destroyed, so that, as the load is gradually being increased, the bond is progressively destroyed from the center of the beam toward the ends. If at the end of the beam there is insufficient bond resistance, the reinforcement will eventually slip and the beam will ultimately fail by what is commonly termed a *bond failure*. By its appearance, a bond failure cannot be distinguished from the diagonal tension failure illustrated in Fig. 17; the only way of telling them apart is by constant measurements made under increasing loads.

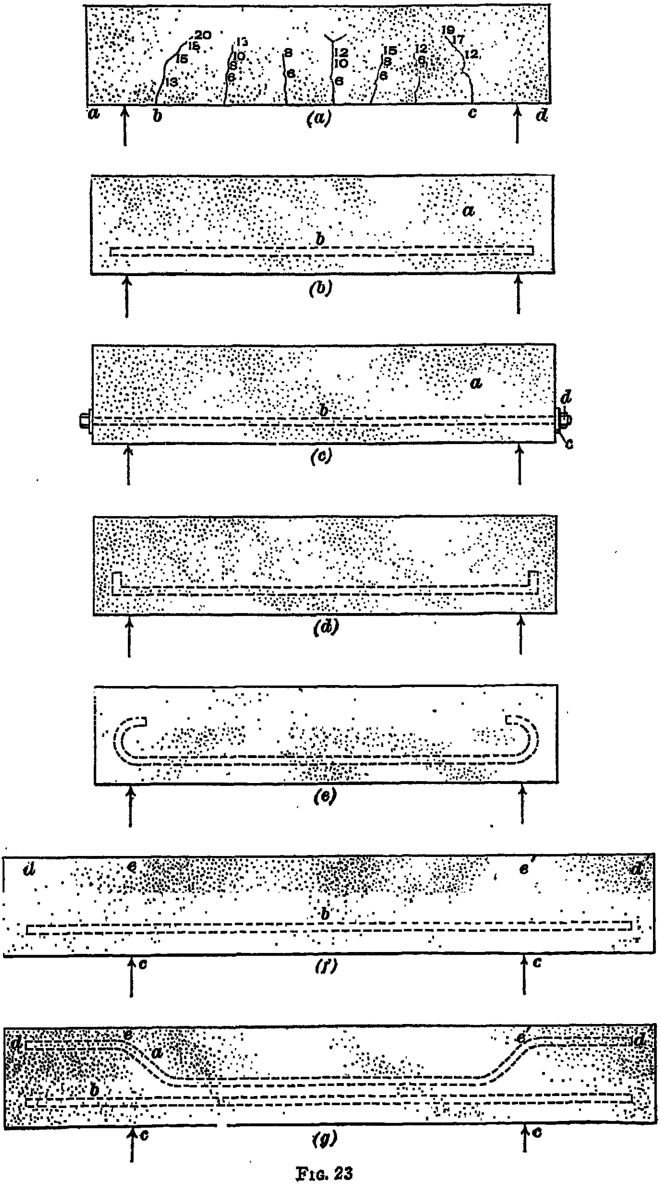

20
15
15
13
13
10
8
6
8
6
12
10
6
15
8
6
12
6
19
17
12
a
b
(a)
c
d
a
b
(b)
a
b
d
c
(c)
(d)
(e)
d
e
e'
d
b
c
(f)
c
d
e
a
e'
d
b
c
(g)
c

Fig. 23

If during the progress of a test of the kind just mentioned a diagram is made of the cracks in the beam, a drawing results of the typical appearance shown in Fig. 23 (*a*), where the fine cracks are indicated by black lines. In general, the cracks at the center appear first and with increasing loads grow deeper and become more numerous toward the ends of the beam, as indicated by the figures inscribed upon the diagram. These figures are written at the deepest penetration of the crack at the time of observation and indicate the load then on the beam in thousands of pounds. Thus, at a load of 6,000 pounds, the five cracks marked *6* extended from the bottom of the beam up to the several numerals *6*, indicating that the reinforcement was then relieving the concrete of its tension stress. At 12,000 and 13,000 pounds load, respectively, further slips of the reinforcing rod occurred, which caused the two cracks nearest the ends to open. After that time and up to final failure, the entire bond resistance was developed in the spaces *a b* and *c d* near the ends of the beams, and when this bond resistance was overcome the beam failed.

29. The beam of Fig. 23 (*a*) was reinforced with one 1-inch plain round rod as indicated in Fig. 23 (*b*). Since this type of arrangement does not always furnish the necessary bond resistance at the end, and since the bond resistance at the end often determines the ultimate strength of the beam, various means have been tried in order to increase the end anchorage of reinforcing rods. Thus, in (*c*) the rod is provided with washers *c* held by nuts *d* for the purpose of preventing sliding, but this design is not satisfactory, because the washers do not become effective until the adhesion between concrete and steel is entirely destroyed, permitting dangerously large cracks to form. A better anchorage is obtained by hooking the ends as in (*d*), which greatly increases the bond resistance, especially if the hook is made semicircular and of large diameter, as in (*e*), where the diameter of the hook is 6 times the diameter of the rod. Still another method is to let the beam overhang the support, as in (*f*), where the rod *b* extends beyond the supports *c*. This method is often used in certain floor construc-

tions to be described hereafter, especially in the form shown in (*g*), which comprises horizontal straight rods *b* and trussed rods *a* extending beyond the supports *c*.

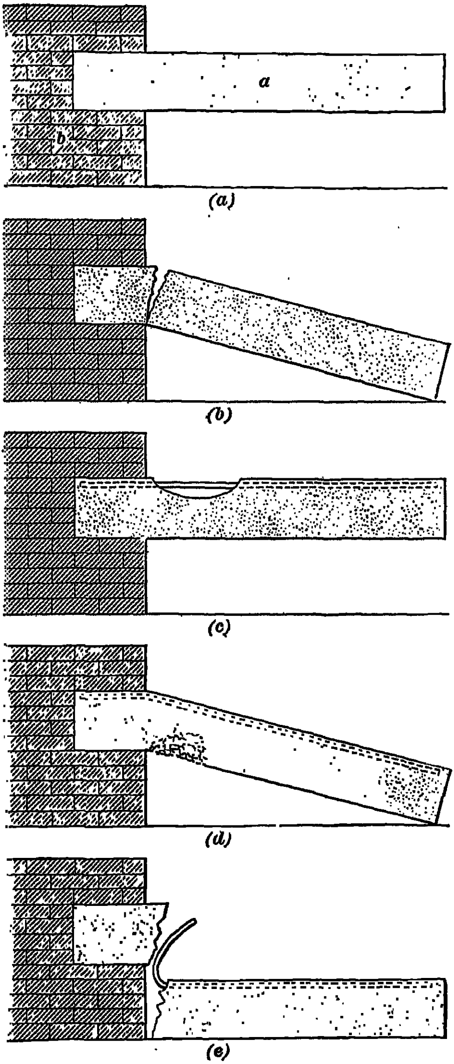


FIG. 24

The necessity for anchoring reinforcing rods is not confined to the main tension rods. Thus, the stirrups used in T beams serve the purpose of holding flange and stem together and on account of the stresses thereby caused in them have a tendency to pull out. One commonly used way of anchoring the stirrups into the T-beam flange is to bend their ends into semicircular shape *c* as shown in Fig. 21 (*b*).

30. Reinforced-Concrete Cantilever Beams.—The beams so far considered have all been so-called *simple beams*, by which is meant beams freely supported at each end. Another form of beam frequently used is the *cantilever beam*, which is a beam

overhanging at one end as indicated in Fig. 24 (*a*), where *a* is a plain-concrete beam fixed at one end in a wall *b*. If tested to destruction, such a beam fails suddenly by cracking across the top at the support, as indicated in Fig. 24 (*b*), because in cantilever beams the bottom is in compression and the top is in tension, and the greatest tension stress occurs at the supported end. The beam may be greatly strengthened by the introduction of tension reinforcement near the top where the tension stresses occur. A beam so reinforced is illustrated in Fig. 24 (*c*), where a small portion of the concrete has been broken away to exhibit the steel. If tested to destruction, a reinforced-concrete cantilever may fail in various ways; one kind of failure is as shown in (*d*), where the concrete near the support has failed in compression; another kind of failure is as shown in (*e*), where the bond stress of the steel has exceeded the safe limit and the rods have pulled out of the wall. The proper anchorage of the rods into the wall is very important in cantilever beams; however, if the cantilever is short, failure may be caused by sliding of the rods in the projecting part rather than in the wall part, because of lack of anchorage of rods in the projecting beam. Cantilevers may of course also fail because of excessive tension stresses in the steel.

Cantilevers do not always project from walls. Cantilever beams may be continuations of simply supported beams overhanging their supports, as indicated in Fig. 23 (*f*) and (*g*); in both of these diagrams the cantilever portion is marked *d e* at one end and *d′ e′* at the other end. Since cantilevers need reinforcement in the top, the construction shown in (*g*) is better than the one in (*f*), the distance *d e* being reinforced at the top in (*g*) but not in (*f*). Cantilevers may also be of the type known as wall footings, to which reference is made in the following article.

31. Tests of cantilevers have received comparatively scant attention although the problems involved are of great importance in the construction of footings. Tests on footings have been made at the University of Illinois by Professor A. N. Talbot; an example of a footing such as tested is shown

in Fig. 25, where *a* is a short block representing a wall and *b c* is a transverse block representing the footing under the wall. The load is brought on the footing *b c* by the wall *a*, and other pressures act upwardly from the soil under the footing, causing the footing *b c* to act as a cantilever projecting on both sides of the wall *a*. In other words, in a footing the pressures act in a direction opposite to the loads acting on most beams, since usually the loads on beams act downwardly, whereas in footings the pressures act upwardly. Since the pressure acts upwardly, the cantilever projections *b* and *c* must have the reinforcement disposed at the bottom where the tension stresses occur; one reinforcing rod is shown at *d*. Several fine cracks developed

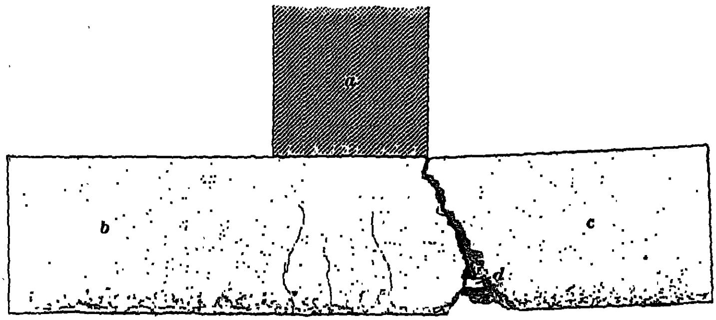


FIG. 25

early in the test are indicated; these mark the initial slipping of the reinforcement which takes place in cantilevers as in the simply supported beams already described. The large crack which finally wrecked the cantilever was caused by the tension rods pulling through the concrete.

32. Shear Stresses in Reinforced-Concrete Beams. The problems connected with the shear stresses actually existing in reinforced-concrete beams are complicated and have not been satisfactorily solved at the present time, it being impossible to determine the actual shear stresses in a beam containing web reinforcement. The problem is still further complicated by the difficulties encountered in determining the shearing strength of concrete; these difficulties arise from the

impossibility of eliminating tension stresses in the test specimens, so that, since concrete is weak in tension, the shear-test specimens are in reality broken by tension. Experiments seem to indicate that the shear strength of concrete is almost equal to the compression strength, and that a beam having sufficient strength to withstand diagonal tension is amply strong against shear.

FLOORS AND FLOOR TESTS

33. Reinforced-Concrete Floors.—In both engineering and architectural work, two kinds of floors are in common use, both of which are supported at intervals upon columns. In one type of floor, known as a *ribbed floor,* a system of flat concrete plates is carried on reinforced-concrete beams, referred to as *ribs,* which transfer the floor load to the columns. In the other type of floor, known as a *flat-slab floor,* flat concrete plates span from column to column without ribs. Both of these types embrace a number of variations.

34. Ribbed Floors.—In Fig. 26 (a) is shown a perspective view of a panel of a ribbed floor; by *panel* is here meant the space of floor included between four columns. This panel consists of a flat plate S of reinforced concrete called a *slab;* the slab is supported upon parallel beams B called *ribs.* Some of the beams B are supported at each end upon larger ribs A called *girders,* while other beams are supported at each end upon the columns C. The girders A are also supported at each end upon the columns.

35. Slabs, beams, girders, and columns are all reinforced, the reinforcement being indicated by dotted lines in the elevation and by black specks in the cross-section. The **slabs** are rectangular beams, of comparatively thin and very wide section; the tension rods d in these wide beams extend from support to support, that is, from beam to beam. In addition to this so-called *principal reinforcement,* the slabs contain rods running at right angles to the principal rods; these rods are called *secondary reinforcement* or *distributing rods,* and serve the purpose of transferring the loads to the principal reinforce-

ment, and also of preventing cracks due to temperature shortening and to shrinkage of the concrete in setting, for which reason these rods are sometimes referred to as *temperature reinforcement* or *shrinkage rods.*

36. The **beams** are reinforced as already explained in connection with beam tests, the reinforcement consisting of

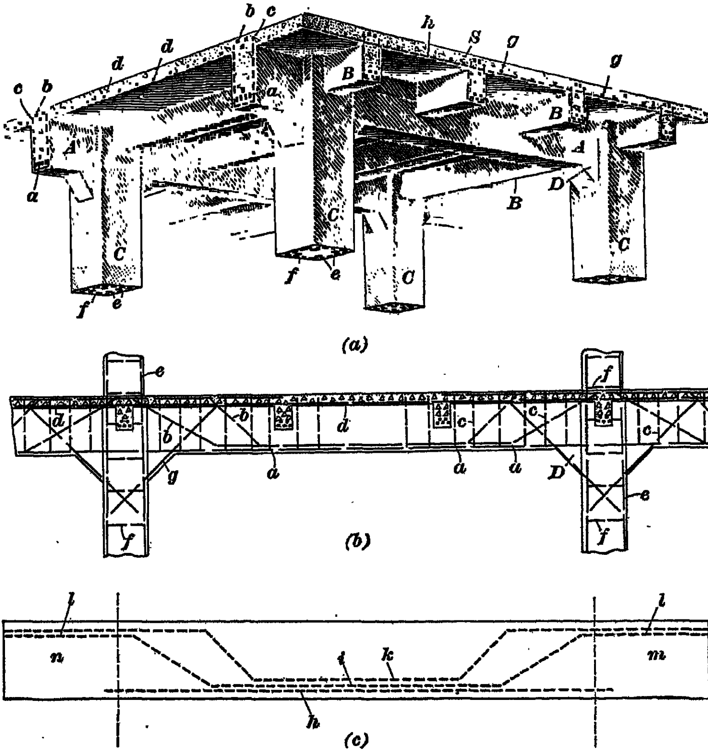


FIG. 26

straight main tension rods, trussed main tension rods, and stirrups. The reinforcement of the **girders** is shown by the dash lines in Fig. 26 (*b*), which is a side elevation of the girder *A*; the main tension rods of the girder are marked *a*, the bent-up ends of the trussed rods are marked *b*, and the stirrups *c*. At *d* is indicated the principal reinforcement of the slab; *e* indi-

cates the vertical rods of the columns and f the column ties. The girders are provided with *brackets* D at their intersection with the columns; in (b) is shown how the brackets are reinforced with diagonal rods g. Brackets are not always used; their purpose when used is to stiffen the building, to reduce the span of the girders, and to increase the depth of the girders at the columns.

37. The principle of construction of ribbed floors is an elaboration of the principles already described in connection with beams, with the important difference, however, that no slab, beam, or girder of a ribbed floor can ever act as an individual, but the entire structure is so knit together and the component parts are so united that they all act inseparably together. Thus, the slab acts as a compression flange to the beams and girders, so that each beam and each girder becomes part of a **T** beam, in which, as previously explained, the flange acts in compression and thereby furnishes an essential portion of the strength of the beam or girder. In order to obtain the necessary unity of slab and beam or of slab and girder, the concrete of the slab, beams, and girders is usually cast in one continuous process, so that the slab and the ribs are virtually one piece. Not only is the concrete of each panel placed entirely in one operation, but the concrete of many panels is placed so as to extend in virtually one unbroken sheet over the largest possible floor area, the object being to pour the structure as far as possible in one operation, or to make it *monolithic*, as it is usually called. In monolithic structures, the beams of one panel are continuations of the beams of another panel, not merely as regards location but each beam is actually an extension of another beam. By this so-called *continuity of construction* certain stresses are caused to exist in the concrete, which require particular attention; they are described in the following articles.

38. Continuity of Ribbed Concrete Floors.—Where each beam is a monolithic, integral continuation of another beam, the beams are no more simply supported but each beam becomes in part a cantilever beam for that portion of the span

which is nearest the support, and requires reinforcement of the type previously described for cantilevers, in addition to reinforcement of the type required for a simply supported beam. This means that each beam or girder must have some straight reinforcement in the bottom at the middle of the span, as explained for beams, and some straight reinforcement in the top at the end of the span, as explained for cantilevers, and shown for the girder in Fig. 26 (*b*). In order to make plain the arrangement of the steel in this girder the main tension steel of the girder is again shown in Fig. 26 (*c*), but this time alone. The main tension steel comprises two distinct kinds of rods, as shown, straight rods *h* and truss rods *i* and *k*. The truss rods project several feet into the adjoining span on each side, as at *l*, thereby forming a cantilever reinforcement for the girders *m* and *n* of the adjoining spans in accordance with the principle illustrated in Fig. 24 (*c*) and explained in connection therewith. Since in Fig. 24 (*d*) it has been shown that a cantilever beam may fail by compression in the lower fibers at or near the support, it is sometimes desirable to strengthen the concrete at that point. This may be done by adding a bracket at *D*, as shown in Fig. 26 (*a*) and (*b*).

While in the foregoing explanation particular reference was made to the girder, the cantilever reinforcement, or, as it is more often called, the *continuity reinforcement*, is by no means confined to the girders, but is used in beams and slabs as well, being always in the form of a reinforcement in the top over the supports. In girders, continuity reinforcement is placed over columns; in beams, it is placed over girders; in slabs, it is placed over beams, its function being to take care of the tensile stresses existing at these points. The presence of such tensile stresses has been proved by tests.

39. Tests on Ribbed Concrete Floors.—Reinforced-concrete floors are often tested after completion, either because building authorities insist upon a test load being placed on the floor for the purpose of demonstrating its safety or because the owner desires to satisfy himself of the safe and satisfactory condition of the work done by the contractor. It is customary

in such tests (which are often referred to as *acceptance tests*) to measure the deflection of the beams and girders tested and also to examine the structure for cracks; in all ordinary cases the deflections are very small and the cracks are invisible, because the load placed on the floor is small compared with the ultimate carrying capacity of the structure. In rare instances glaring defects in the structure have been discovered and remedied but usually such tests serve no useful purpose except to assure the owner of the safety of the structure.

However, elaborate scientific tests of great value have been made on reinforced-concrete floors. In some cases floors built for test purposes were loaded to destruction; but in the majority of cases only small loads were used, because the floors were parts of commercial buildings which could not be destroyed. In these tests, measurements were made on the structural members to determine the deformations and thereby the stresses actually existing. An instrument called an *extensometer* is used for the measurements, and the deformations existing at many hundreds of points of a floor panel are determined by its aid with such accuracy that the stresses actually existing in the reinforcement may be calculated. As a result, it is known that tensile stresses occur in all continuous members over the supports, and that these stresses are at the top side of the member and require in many cases as much reinforcement over the supports, for the upper fibers, as is required at the center of the span for the lower fibers; this is true for all slabs, beams, and girders.

40. Flat-Slab Floors.—Floors made of flat, reinforced-concrete slabs *a*, Fig. 27 (*a*), are supported upon the flaring heads, or caps, *b* of the reinforced-concrete columns *c*. Tests on completed buildings, as well as on structures erected for the purpose, indicate that great tension stresses occur in the top of the flat slab over the column heads, which require a large amount of reinforcement over the columns near the top of the slab. At the same time correspondingly large compression stresses occur at the bottom of the slab in the region adjacent to the column, for which reason the concrete is often strength-

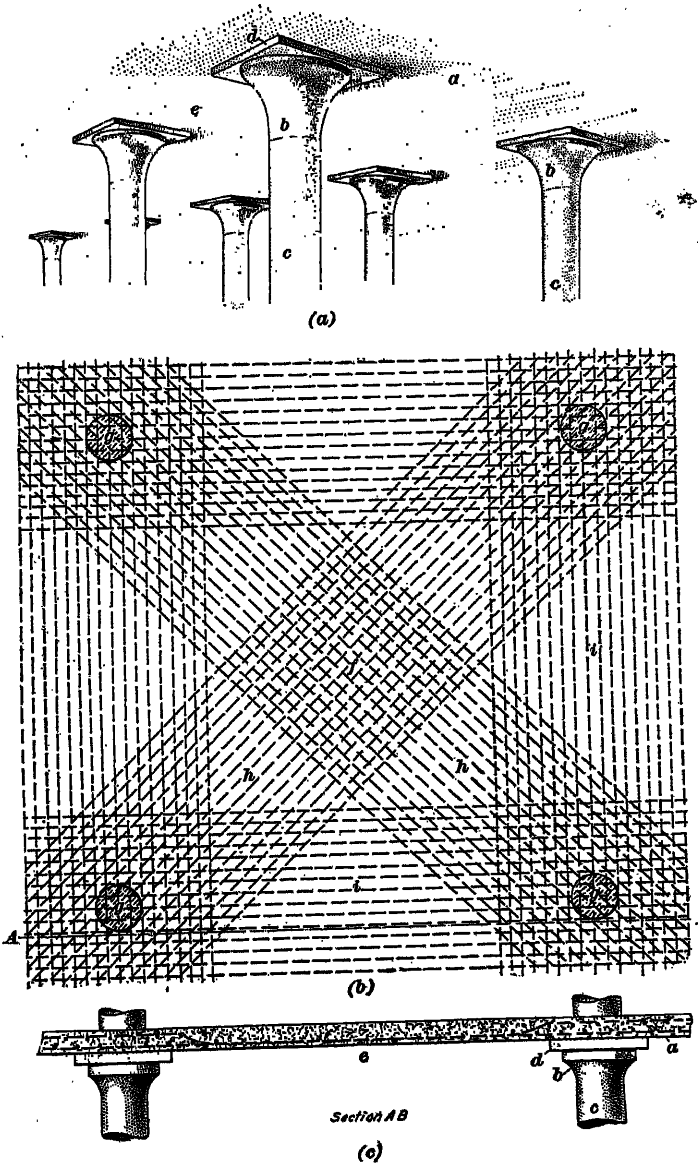


Fig. 27

ened by adding the concrete *d*, called the *drop*, to the thickness of the slab. Midway between columns at *e* and at the center of the panel, the greatest tension stresses occur at the bottom of the slab and the greatest compression at the top of the slab, for which reason the reinforcement must be close to the bottom of the slab at the center. In practical construction the reinforcement may be arranged in a number of different ways. One method in quite common use is shown in Fig. 27 (*b*), which is a plan view of part of the floor shown in (*a*). The columns are indicated by the circles *g*; between columns the reinforcing rods extend in *diagonal belts h* and *direct belts i*. At the columns, the rods are raised in the concrete to near the top surface; between columns the rods sag to near the bottom surface of the slab, in both direct and diagonal belts.

PIPES AND PIPE TESTS

41. In the structures heretofore considered, the concrete was utilized chiefly in compression and the steel chiefly in tension; but in pipes subjected to an internal pressure both concrete and steel are in tension. A reinforced-concrete pipe *a*, Fig. 28, subjected to the internal pressures indicated by the arrows *b*, will burst because of the weakness of the concrete unless strengthened by a suitable reinforcement. The reinforcement usually consists of circumferential bands *c* and *d* that carry the tension stresses and of longitudinal rods *e* that transfer the pressures to the bands.

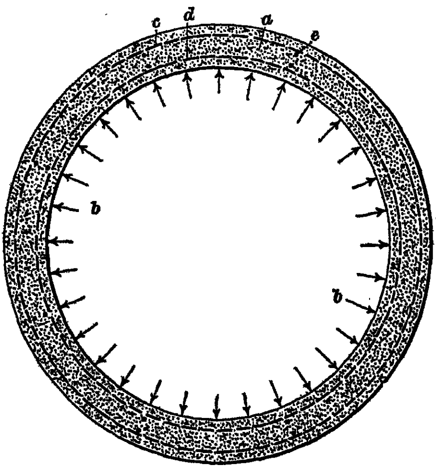


FIG. 28

42. The principle of the method described for reinforcement of pipes subjected to internal pressure is so obvious that tests are not needed to establish the correctness of the theory; the tests made on reinforced-concrete pipes have, therefore, usually been directed toward finding the strength of pipe subjected to external pressure and determining the waterproofness of reinforced-concrete pipes subjected to high internal pressures. It has been found that well-made concrete pipes with plenty of reinforcement will withstand very large pressures with but insignificant leakage, and that the leakage will constantly decrease until it ceases altogether because of precipitation of small particles which fill the minute fissures through which the water escapes.

REINFORCEMENT

METALS USED FOR REINFORCEMENT

43. Standard Grades of Steel.—The physical characteristics of the metals used for reinforcement of concrete must receive careful attention, if an efficient and economical design is to be produced. Steel is the best material for reinforcement, as it has great tensile strength and ductility and can be obtained readily in the market.

The bars used for reinforced-concrete construction are either new bars manufactured from raw materials or are the product of worn-out railroad rails rerolled. Bars of the first-named kind are called *billet-steel bars* because they are rolled from blocks called billets; those of the second kind are named *rerolled bars* because they are rerolled from old steel rails after suitable treatment.

The American Society for Testing Materials has formulated standard specifications for "billet-steel concrete reinforcing bars" which it classifies in three standard grades, called *structural grade, intermediate grade,* and *hard grade.* **Structural-grade steel** was formerly known as *medium steel* and is the

steel generally used for reinforcement. The **intermediate** and **hard-grade** steel have higher percentages of carbon, which gives them greater ultimate tensile strength and higher elastic limit.

There are soft grades of steel made that differ very little from iron, except in ductility, but soft steel is not generally employed for reinforcement.

44. Properties of Billet-Steel Reinforcement.—In order to insure delivery of the proper grade of steel, certain tests are called for in the standard specifications. These tests are of two kinds; namely, *tension tests* and *bending tests.* In the **tension tests,** the tensile strength and the elongation of test specimens are determined. For both plain and deformed bars (except cold-twisted bars, which will be considered separately later on) the following ultimate tensile strengths are required:

Structural-steel grade. . . 55,000 to 70,000 lb. per sq. in.
Intermediate grade . . . . . 70,000 to 80,000 lb. per sq. in.
Hard grade . . . . over 80,000 lb. per sq. in.

In the **bending test** the bar to be tested is bent about a pin, either to a 180° turn, as in Fig. 29, or to a 90° turn, as in Fig. 30; no fracture must appear on the outside surface as the result of the bending. The severity of the test depends upon the angle of bending and also upon the diameter of the pin about which the specimen is bent. All bars of less than $\frac{3}{4}$-inch size are bent to 180°; intermediate and hard-grade bars larger than $\frac{3}{4}$-inch size are bent to 90°. The diameter of the pin used in tests of plain bars of all sizes is equal to the thickness of the test piece, for structural-steel grade; twice the thickness, for intermediate grade; and three times the thickness, for hard grade. For deformed bars, larger pins are to be used.

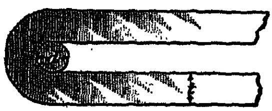


Fig. 29

Of the three grades of steel referred to, the Final Report of the Joint Committee recommends the use of the structural-grade steel because it combines a sufficiently high ultimate strength with such softness that it can be bent cold on the job.

45. Because of the higher ultimate strength of intermediate and hard grades, it has been proposed to use less reinforcement of these grades than of the structural grade, which of course would entail a saving by the use of the intermediate and hard grades. There is, however, according to the Joint Committee, no real advantage in using steel of high strength, because the safety of a reinforced-concrete structure does not depend upon the ultimate strength of the reinforcement but upon the modulus of elasticity of the steel, and this coefficient is the same for all grades of steel. On the contrary, there is a real disadvantage in the use of these steels, because of their brittleness, which makes it impossible to bend the bars cold, especially in freezing weather. They are also considered unsafe for use in structures subjected to shock or vibration, because of their inclination to snap under sudden stress.

46. Wrought Iron.—As a rule, wrought iron is not used as reinforcement, because it costs more than steel, is harder to obtain, and has less strength. It may find occasional employment in an emergency or in localities near a mill where wrought iron is made, and where it is therefore cheaper than steel. In other places iron has no advantages over steel.

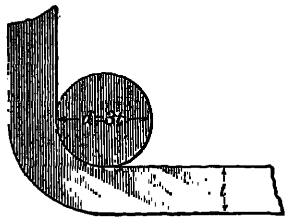


FIG. 30

Commercial wrought iron has an ultimate tensile strength of from 48,000 to 50,000 pounds per square inch, an elastic limit of from 22,000 to 24,000 pounds per square inch, and an elongation of from 8 to 12 per cent. in a test piece 8 inches long. A test piece should bend when cold, without showing failure, through 180° around a bar with a diameter twice that of the test piece.

47. Rerolled Bars.—Bars for reinforcement are sometimes made from old steel rails by cutting apart the flange, web, and head of the rail and rerolling these into square and round sections. The square sections are sometimes twisted while hot.

Much of this material has a high elastic limit and tensile strength, but this is in itself no especial advantage. This kind

of steel has the disadvantage of being brittle and lacking in uniformity, and the lapping or folding of the original material incidental to its manufacture is likely to develop laminations in the bar, which diminish its strength.

Rerolled bars should be employed only where it is possible to have rigid inspection, and every lot of this material should be carefully tested before being used in important reinforced-concrete members.

48. Wire.—Wire is used extensively in reinforced-concrete work. Thin wire is used for tying reinforcing rods together, thicker wires are used in hoops for columns and in the reinforcement of tanks and pipes, and wire woven into netting is extensively used for reinforcement of slabs and walls.

Wire is manufactured from billets about 4 inches square in cross-section and weighing between 100 and 400 pounds. These are heated and rolled hot into so-called *wire rods* by means of grooved rolls. A portion of the rolled product finds its way into the market under the name *rolled wire,* but most of the wire rods are made into thinner wire by *cold drawing,* whereby the wire rods, in a cold state, are pulled through dies by means of revolving drums upon which the wire is wound. This process is repeated with dies having smaller openings until a wire of the desired diameter is obtained. The physical properties of steel are greatly changed by cold drawing; the strength increases and with it the brittleness. In order to restore the softness of the material, so as to prevent breakage, it is necessary to *anneal* the wire repeatedly during its manufacture. By **annealing** is meant the heating of the metal in hermetically sealed receptacles to a certain definite temperature followed by slow cooling. Annealed wire is soft instead of brittle, but usually some of the strength is lost in the annealing process. The ultimate strength of drawn-steel wire is from 45,000 pounds per square inch for soft annealed to over 400,000 pounds per square inch for hard wire.

The finished wire may be *galvanized*. **Galvanized wire** is wire coated with zinc by running the wire through a bath of molten zinc. The purpose of the coating is to prevent rusting

of the steel; ordinarily, steel embedded in concrete is immune from rust, but much of the wire used in the form of netting is nevertheless galvanized to protect it against the weather until it is installed. Certain kinds of wire netting may be bought in which the longitudinal strands are of plain uncoated wire and the other wires are galvanized.

Wire and wire products are usually obtainable in single lengths up to 500 feet and in many different sizes. The size of wire is designated by its *gauge number,* which is a number indicating the diameter of the wire. There are many different gauges in use, some referring to diameters of wires and others to thicknesses of plates, and different gauges are used for wires made from different materials, so that the greatest confusion arises unless the gauge used is mentioned by name. The two most commonly used wire gauges are the American Steel and Wire Company gauge*, and the Birmingham gauge. These are entirely different, the same number of gauge indicating entirely different diameters in the two systems. The thickest wire made is indicated by 0000000, or 7-0, in the American Steel and Wire Company gauge. Smaller diameters are denoted by 6-0, 5-0, 4-0, 3-0, 2-0, 0, 1, 2, 3, and so on down to No. 40, which is a very fine wire. To give an idea of the relative sizes of wires of different gauge numbers, the American Steel and Wire Company gauge numbers that correspond approximately to various fractions of an inch are here given, as follows: $\frac{1}{2}$ inch=7-0; $\frac{3}{8}$ inch=3-0; $\frac{1}{4}$ inch=No. 3; $\frac{1}{8}$ inch =No. 11; $\frac{1}{16}$ inch=No. 16.

*The United States Bureau of Standards recommends that this gauge be used as standard for steel wire, under the name "The Steel Wire Gauge."

TYPES OF STEEL REINFORCEMENT

PLAIN BARS

49. The steel bars used for reinforcement of concrete may be *plain* or *deformed*. Deformed bars are manufactured by cold twisting of plain square bars or by passing the plain bars in a hot state through special rolls that form projections or depressions on the bars. The selection of the reinforcement for any particular case is made according to conditions of loading of the structure, considerations of economy, and the personal preference of the engineer. When the structure is subjected to shocks, or to loads causing vibration, there is a tendency to destroy the bond between the concrete and the steel rods. In such cases, deformed bars should be used.

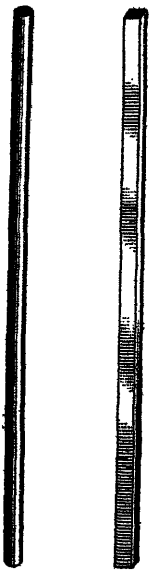


FIG. 31 FIG. 32

Plain round or square steel bars are most used, because they offer the cheapest and most available form of steel reinforcement. The price per pound is lower than for any other form of rolled steel, and the usual sizes can be obtained at short notice from the stock of mills or dealers in any part of the United States.

Plain round rods, Fig. 31, sometimes simply called **rods**, are used very extensively, and are satisfactory in structures not subjected to shock or vibration.

Plain square bars, Fig. 32, sometimes simply called **bars**, are frequently used. Rectangular bars, called *flats* or *flat bars*, are commonly employed for reinforcement of circular structures, as hoops for columns, and for web reinforcement in beams, but not as main tension steel.

In Table I are given the areas and weights of rods and bars, in sizes from $\frac{1}{16}$ inch to 2 inches. Larger sizes are seldom used in reinforced-concrete construction. This table is useful in figuring the amounts of reinforcement.

TABLE I

AREAS AND WEIGHTS OF BARS AND RODS

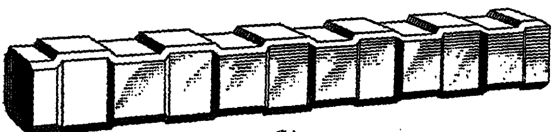
Size Inch	Square ■ Area Square Inch	Square ■ Weight per Foot Pounds	Round ● Area Square Inch	Round ● Weight per Foot Pounds	Size Inches	Square ■ Area Square Inches	Square ■ Weight per Foot Pounds	Round ● Area Square Inches	Round ● Weight per Foot Pounds
$\frac{1}{16}$	.0039	.013	.0031	.010	$1\frac{1}{16}$	1.1289	3.838	.8866	3.014
$\frac{1}{8}$	.0156	.053	.0123	.042	$1\frac{1}{8}$	1.2656	4.303	.9940	3.379
$\frac{3}{16}$	.0352	.120	.0276	.094	$1\frac{3}{16}$	1.4102	4.795	1.1075	3.766
$\frac{1}{4}$	.0625	.213	.0491	.167	$1\frac{1}{4}$	1.5625	5.312	1.2272	4.173
$\frac{5}{16}$	.0977	.332	.0767	.261	$1\frac{5}{16}$	1.7227	5.857	1.3530	4.600
$\frac{3}{8}$	.1406	.478	.1104	.376	$1\frac{3}{8}$	1.8906	6.428	1.4849	5.049
$\frac{7}{16}$	.1914	.651	.1503	.511	$1\frac{7}{16}$	2.0664	7.026	1.6230	5.518
$\frac{1}{2}$	.2500	.850	.1963	.668	$1\frac{1}{2}$	2.2500	7.650	1.7671	6.008
$\frac{9}{16}$	.3164	1.076	.2485	.845	$1\frac{9}{16}$	2.4414	8.301	1.9175	6.520
$\frac{5}{8}$	.3906	1.328	.3068	1.043	$1\frac{5}{8}$	2.6406	8.978	2.0739	7.051
$\frac{11}{16}$	.4727	1.607	.3712	1.262	$1\frac{11}{16}$	2.8477	9.682	2.2365	7.604
$\frac{3}{4}$	.5625	1.913	.4418	1.502	$1\frac{3}{4}$	3.0625	10.413	2.4053	8.178
$\frac{13}{16}$	.6602	2.245	.5185	1.763	$1\frac{13}{16}$	3.2852	11.170	2.5802	8.773
$\frac{7}{8}$	.7656	2.603	.6013	2.044	$1\frac{7}{8}$	3.5156	11.953	2.7612	9.388
$\frac{15}{16}$	.8789	2.989	.6903	2.347	$1\frac{15}{16}$	3.7539	12.763	2.9483	10.024
1	1.0000	3.400	.7854	2.670	2	4.0000	13.600	3.1416	10.681

50. Base Price.—The price per pound of rolled rods and bars is the same for all so-called *base* sizes, which in the United States include sizes from $\frac{3}{4}$ inch to 3 inches, and in Canada, sizes from $\frac{5}{8}$ inch to 3 inches. For the smaller sizes, there is an extra charge above base price; this extra charge is called the **size-differential.**

DEFORMED BARS

51. Requirements of Deformed Bars.—There are three important points to consider in the selection of deformed bars. The first is that the deformations should efficiently

(a)



(b)

FIG. 33

increase the bond; second, the deformations should not decrease the strength of the bar; third, the deformations should not prevent the proper filling of concrete into the forms.

In regard to the first point, all kinds of deformations retard sliding but not in the same manner. In some types of bars the deformations become effective before the adhesive bond is destroyed; bars of this type, of which those shown in Fig. 33 are examples, have square or nearly square shoulders on them at frequent intervals, which prevent even very minute motion of the bar; and this type of bar is preferable where great bond resistance is to be developed in short lengths, as in cantilevers used for balconies or footings.

In other types, the deformations are effective only in increasing the subsequent, or sliding, resistance. Such bars have long gradual tapers or wedges that become effective only gradually and increase their hold with increasing stress. The bar represented in Fig. 34 is of this type and is much used where the bond resistance is distributed over great lengths, as in continuous beams.

FIG. 34

In regard to the second point, deformed bars have been used in which the deformations were formed by making some parts of the bar thicker and other parts thinner. In such bars the thin portions are of course weaker than the thick portions, and as the strength of the bar is no greater than its weakest part, the metal in the thick portions is partly wasted. Well-designed deformed bars usually have the same cross-sectional area at all points along their length, as, for instance, the twisted bar shown in Fig. 34; however, for beam reinforcement, since the greatest stress occurs at the center of the beam, it is permissible to have more metal and therefore greater strength in the central portion of the rod. An example of a rod with more tensile strength at the center than at the ends is shown in Fig. 35, in which part of the metal is sheared and bent near the ends, as

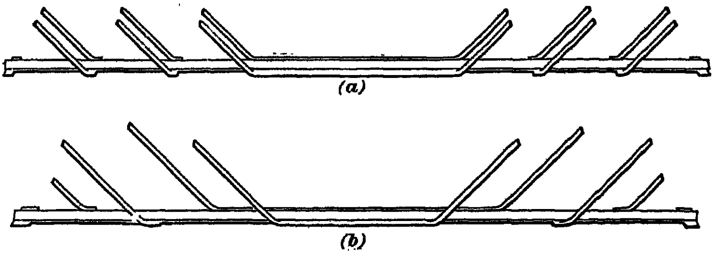


FIG. 35

will be explained hereafter, leaving the central portion with a greater sectional area than the ends.

In regard to the third point, practically all deformed bars will permit the proper filling of horizontal beams. However, types that are provided with prongs or projections, such as

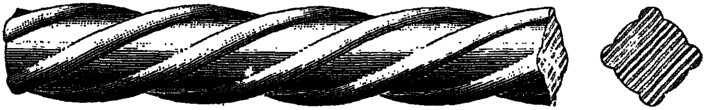
shown in Fig. 35, are not adapted for columns or other vertical members, because the projections will interfere with the placing and compacting of the concrete in the molds, by catching the stones and causing an arching of the materials which results in hollow spaces in the concrete.

52. Square Twisted Bars.—One of the most practical types of deformed bar, is the square bar, Fig. 34, twisted either while cold or while hot. Cold-twisted bars are sometimes called *Ransome bars,* as they were formerly controlled by patents issued to E. L. Ransome in 1884. The patents have expired, and cold-twisted or hot-twisted bars are now as available from either mill or dealer's stock as any rolled bar, and they cost very little more than plain steel. When the mills are not running to capacity, and the supply exceeds the demand, there is usually no charge for twisting, but when the demand is very great, an extra charge of one or two dollars a ton is made for twisting.

The hot-twisted bars are usually rerolled from old rails, and are therefore not so desirable in reinforced-concrete work as cold-twisted bars. Sometimes attempts are made to pass off hot-twisted bars as cold-twisted; but only the very inexperienced inspector is deceived by such attempts, because it is easy to tell the difference. All bars leaving the rolls are covered by a crust or film called *mill scale;* this scale always falls off the bars during the stretching of the surface accompanying cold twisting, whereas in hot-twisted bars the mill scale is usually very conspicuous, and it is thus possible to tell the difference by a mere visual examination.

The advantages of cold-twisted bars arise not only from their greater bond resistance, but also from the fact that the process of cold twisting increases the ultimate strength and raises the elastic limit of the steel, so that, by twisting, a plain square bar made of structural-grade steel adopts the properties of intermediate or hard-grade steel. As already explained, this increase in strength does not justify the use of less steel in the reinforced concrete. Hot twisting does not increase the strength of steel bars.

Twisted bars must have one complete twist in a length not greater than twelve times the thickness of the bar. When



(a) (b)

FIG. 36

twisted to the screw or auger shape shown in Fig. 34, the bar has great resistance to pulling or slipping from a mass of concrete. This is due to the fact that before the bond between the metal and concrete can be destroyed the concrete must be crushed.

53. Spiral Bar.—A bar of special section similar to the square twisted bar and called the **spiral bar** is shown in Fig. 36. This is rolled to the section shown in (b) and twisted after rolling. It is claimed that the elimination of the sharp corners that occur in the square bar avoids the tendency of the concrete to crack in setting, as it will sometimes do at a sharp edge.

Another shape of similar form, said to have the same favorable features as the spiral bar, is the **twisted lug bar** shown in Fig. 37.

54. Kahn Bars.—Fig. 35 shows two styles of the deformed bar known as the **Kahn bar,** which is used extensively. In the old style of bar, shown in (a), the prongs are opposite each other, while in the new style shown in (b) they are staggered. The bar shown in cross-section in Fig. 38 is either square or of a special section, with a fin projecting on

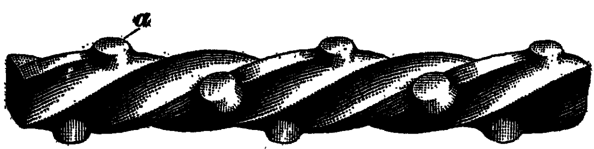


FIG. 37

each side forming extensions of the diagonals of the square. These fins are sheared partly across and also in a direction

parallel with the axis of the bar, and are bent up as shown in Fig. 35 so as to form a grip or bond with the concrete and to provide the stirrups necessary to resist diagonal stresses. The bent-up portion is called the *shear member,* and the bar is frequently called the **Kahn trussed bar,** from its resemblance to a trussed frame.

The various sizes are shown in Fig. 38, with their weights and sectional areas. Structural-grade steel is used, as this grade best stands the shearing stresses sustained in the process of manufacture.

The advantages claimed for the Kahn trussed bars are: (1) That they reinforce the concrete in a vertical plane as well

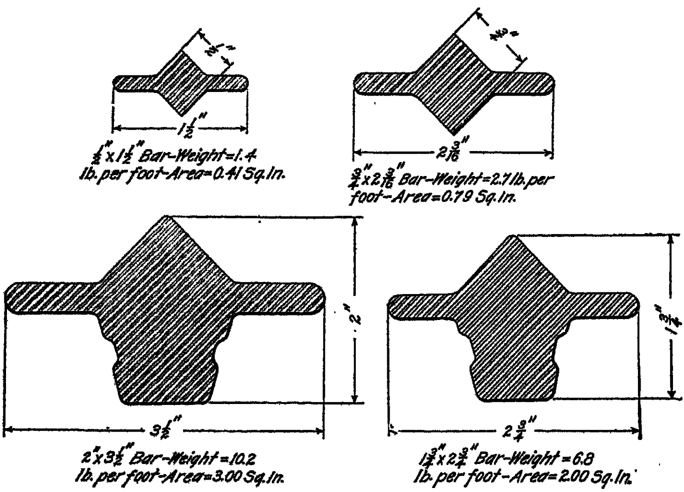


FIG. 38

as in a horizontal plane; (2) that the reinforcement is so disposed as to provide resistance against diagonal tensile stress; (3) that the metal is distributed in close proportion to the stresses existing at different points of the span, because the entire section of the bar is effective at the middle of the span where the tension stresses are greatest; (4) that the shear members are rigidly connected to the horizontal reinforcement.

The Kahn bar is used for thick slabs and for ribbed-floor construction. The fins of a bar should be cut long enough to

extend well into the top of the beam, in order to give them the required anchorage against pulling out.

55. A later type of deformed bar, called the **Kahn cup bar,** is shown in Fig. 39 (*a*) and (*b*). This is essentially a

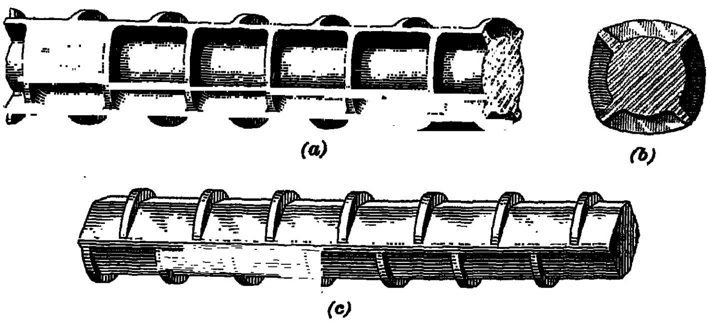


FIG. 39

round rod with longitudinal and transverse ribs and has the sectional area of a square bar of the same size.

A still later form, largely supplanting the cup bar, is called the **Kahn rib bar,** shown in Fig. 39 (*c*). The commercial sizes vary from ⅜ inch to 1¼ inches. The various Kahn bars are furnished by the Truscon Steel Company, Youngstown, Ohio.

56. Corrugated Bar.—The bar now known as the corrugated bar, but formerly as the **Johnson bar,** after A. F. Johnson, its inventor, is shown in Fig. 33, the old style in (*a*) and the new style in (*b*). This is one of the first of the patented deformed bars put on the market, and has achieved

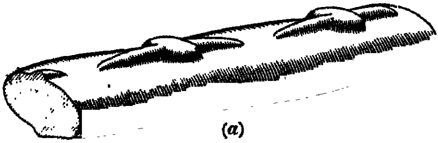
FIG. 40

very general use. It is furnished in either square or round steel by the Corrugated Bar Company, Buffalo, New York.

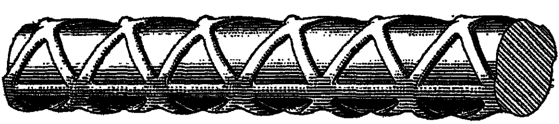
57. Havemeyer Bar.—The Havemeyer bar has the deformations extending longitudinally as shown in Fig. 40,

instead of transversely as in the corrugated bar. It is furnished by the Concrete Steel Company, of New York, in squares or rounds having a constant cross-section.

58. Thacher Bar.—The Thacher bar, shown in Fig. 41 (*a*), is one of the earliest deformed shapes contrived, but it is now rarely used. It was objected by some that this peculiar



(*a*)



(*b*)

FIG. 41

shape had a splitting tendency in the concrete, which was especially undesirable in the thin stems of **T** beams. A later shape contrived by Edwin Thacher, and known as the **Diamond bar,** is shown in Fig. 41 (*b*). This bar at present is rolled by

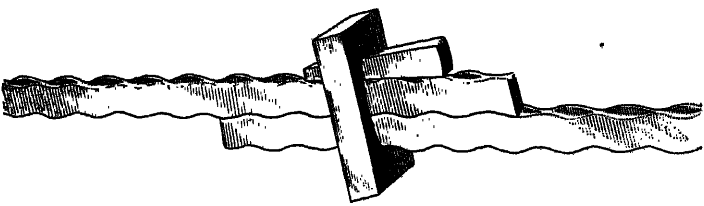


FIG. 42

the Jones & Laughlin mills at Pittsburgh. It has the section and weight of standard square bars and is of constant cross-section.

59. Slick Bar.—The bar shown in Fig. 42, called the *Slick bar,* is rolled by the Cambria Steel Company, Johnstown, Pa. and is the most recent of the deformed shapes put on the market.

The depressions and projections are staggered and so arranged that the bar has a constant cross-section. The shape is such that bars can be spliced by means of collars and wedges as shown.

60. Structural Shapes.—Rolled steel sections have sometimes been used for reinforcement. The use of structural shapes should not be encouraged, as this form of reinforcement costs more per pound than rods and bars and is not so good, because the deep indentations of shapes prevent the concrete from getting properly in contact with the steel.

MESH FORMS OF REINFORCEMENT

61. Expanded Metal.—Among the earlier forms of reinforcement for concrete is the distorted or extended steel plate known as expanded metal, illustrated in Fig. 43. This reinforcement is manufactured by partly shearing a sheet of steel in parallel lines as shown in Fig. 43 (*b*) and then expanding the sheet. This operation produces a diamond mesh of the form shown in (*a*) and increases the area of the sheet about eight times, with a corresponding decrease in weight per unit of area without waste of material. Structural-grade steel is used for expanded-metal reinforcement.

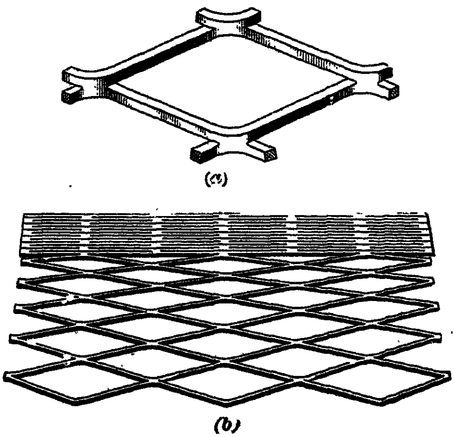


FIG. 43

62. Expanded metal is used principally for the reinforcement of slabs. In placing this reinforcement care should be taken to lay the sheets so that the long dimension of the

diamond meshes is in the same direction as the tensile stress. In Fig. 44 (*a*) is shown the incorrect way of placing the sheets, while in (*b*) is shown the correct way. It will be noticed in view (*a*) that the tensile stresses in the slab would tend to separate one mesh from another at the bridges *a*, which are already weakened by the shearing operation in manufacture. As the meshes are diamond-shaped, twice as long as they are wide, the effective section in view (*a*) is one-half of that in (*b*), since the strands are twice as far apart crosswise in (*a*) as in (*b*). Also, the strands in (*b*) run nearly in the direction of the

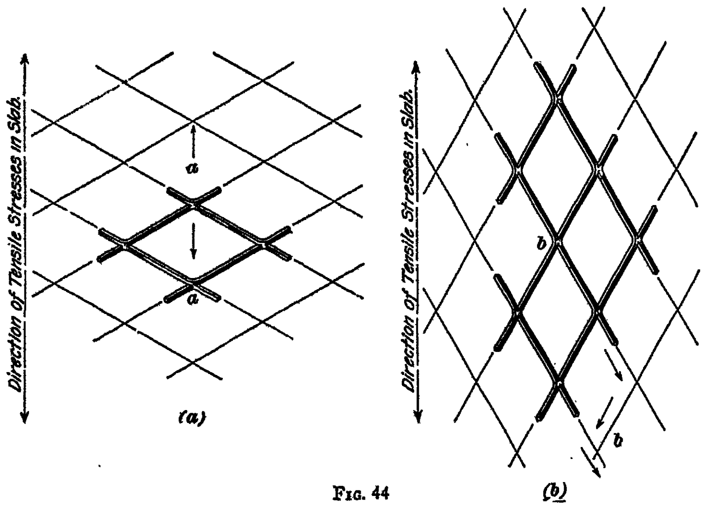


FIG. 44

stresses and are therefore better able to resist the tension than are the strands in (*a*), which are almost at right angles to the direction of the stresses. It is customary to lap the sheets lengthwise 6 inches, preferably over beams or other supports. The side lap need not be much if sheets are wired together.

63. Kahn Rib Metal.—A form of reinforcement for slabs is made from a section somewhat similar to a Kahn bar and is called *Kahn rib metal*. This expanded metal, which has the form shown in Fig. 45, is manufactured by shearing a rolled section of the form illustrated in Fig. 46 along lines

parallel witn its length, and then pulling the metal so sheared in a direction at right angles to the lines of shear. In this type of expanded metal, the main bars of the section represent the

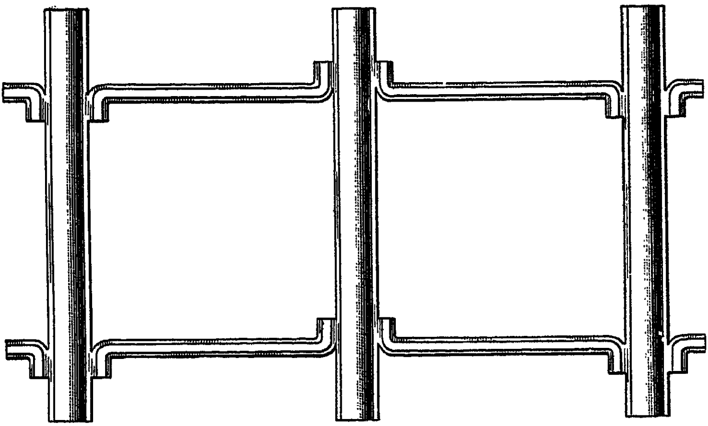


FIG. 45

material available for the reinforcement of the slab, while the light cross-bars act as spacing bars and also as shrinkage rods when embedded in concrete.

The advantage claimed for the Kahn rib metal as compared with a diamond mesh is that it transmits the load directly to the supports without any tendency to elongate or distort. Consequently, slabs reinforced with it are not liable to so much deflection as those reinforced with the diamond-mesh material. The cross-bars, or spacing bars, tend to transmit directly to the main ribs all intermediate stresses that are induced by temperature or shrinkage, and at the same time do not add materially to the weight of metal required. These secondary bars also insure the correct spacing of the main bars—a very desirable feature to be possessed by any slab reinforcement.

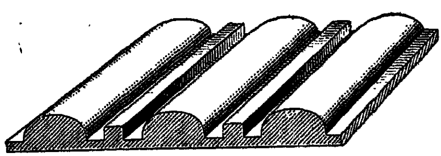


FIG. 46

The Kahn rib metal is so formed that the main reinforcing ribs always have a sectional area of .09 square inch. The

amount of metal required per foot of width of slab is controlled by making the cross-ribs longer or shorter as required and thus changing the spacing of the main ribs. Considerable range in the sectional area of reinforcement may thus be had, for the meshes vary from a 2-inch mesh, in which the main reinforcing ribs are placed 2 inches from center to center, to an 8-inch mesh, in which bars of the same size are placed 8 inches from center to center.

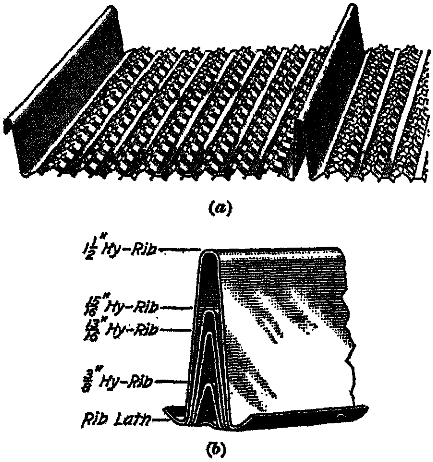


FIG. 47

64. Another form of reinforcement manufactured by the Truscon Steel Company is called **Hy-Rib.** As shown in Fig. 47 (*a*), this is a metal lath stiffened with rigid ribs made of the same sheet of steel. As the ribs are strong and the lath retains the concrete, forms are unnecessary when this type of reinforcement is used in concrete. Hy-Rib is made in several heights, as shown in Fig. 47 (*b*), and in flat or curved sheets in lengths of 6, 8, 10, and 12 feet. It is suitable for floors, partitions, ceilings, and roofs.

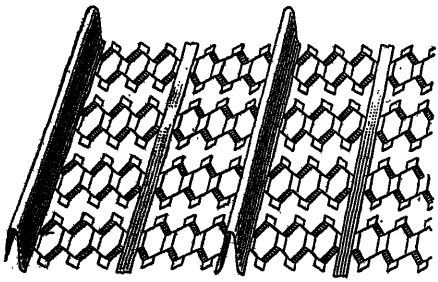


FIG. 48

In Fig. 48 is shown the **Corr-Mesh,** manufactured by the Corrugated Bar Company. This metal is of the same general character as Hy-Rib and is used for similar purposes.

65. Herring-Bone Expanded Metal.—In Fig. 49 is shown a type of expanded metal known as **herring-bone**

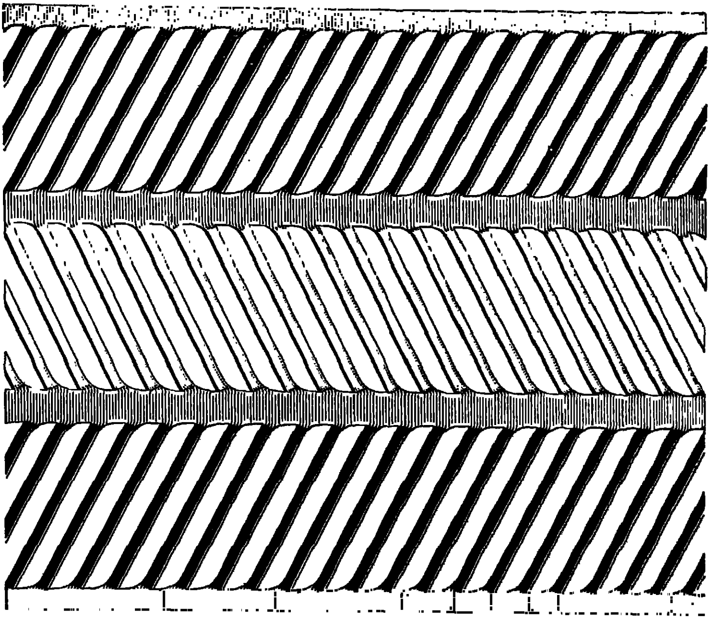


FIG. 49

metal lath. This metal when bent into the form shown in Fig. 50 is known as **Trussit.** The longitudinal ribs *a* are

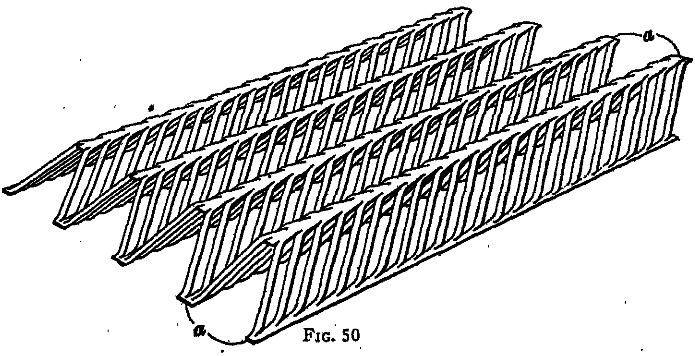


FIG. 50

intended to give it sufficient transverse resistance to allow the placing of a thin roof slab of concrete on it without other cen-

tering or support. This type of slab reinforcement is adapted for light roof construction, the expanded metal acting both as the centering and as the steel reinforcement for the slab.

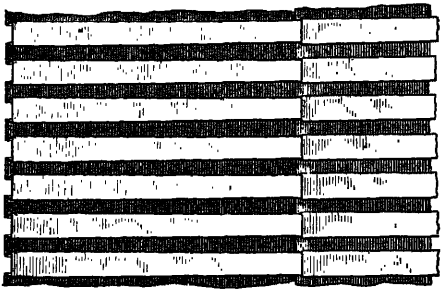


FIG. 51

66. Sheet-Metal Reinforcement.—The type of metal reinforcement known as **Ferroinclave** is used to some extent for making floor slabs and for stair and roof construction. This reinforcing material consists of sheet metal that is bent into grooves, as indicated in Fig. 51, annealed sheet steel, generally of No. 24 U. S. gauge, that is, $\frac{1}{40}$ inch in thickness, being used in its manufacture. The corrugations are $\frac{1}{2}$ inch in depth or height and are spaced 2 inches from center to center. They are also made dovetailed in section so that the end of one sheet can slip into another, as shown. The sheets are $20\frac{5}{8}$ inches in width and 10 feet in length.

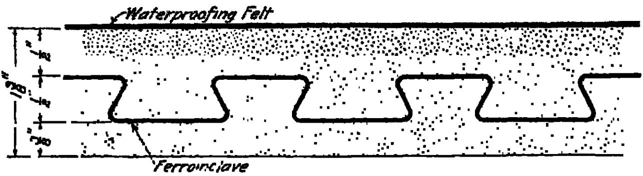


FIG. 52

Fig. 52 shows how this type of metallic reinforcement is used in constructing a slab for a light roof. The covering of the upper side of the sheet metal consists of 1 part

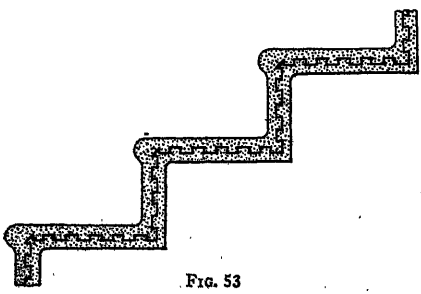


FIG. 53

of Portland cement and 2 parts of sand, and is placed first. The underside is then plastered with a mixture of 1 part of Portland cement, 2 parts of sand, and hair as required.

The Ferroinclave steel plates are also convenient for constructing fireproof steps in places where it is desired to save dead load and to obtain a fireproof flight. This type of construction is shown in Fig. 53.

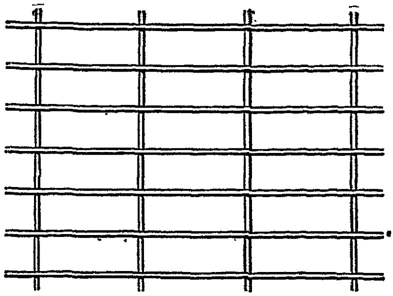


FIG. 54

67. Fabrics of Woven Wire.—There are several types of wire netting on the market that can be used in reinforced-concrete construction, both for tension reinforcement and to prevent shrinkage. Wire netting is used for reinforcement in the construction of slabs and sometimes as a web reinforcement for beams and girders. In some instances, woven-wire fabrics have been used in conjunction with small loose rods or bars as a slab reinforcement, the loose rods or bars supplying the main portion of the reinforcement and the remainder being made up by the heavy wires of the fabric. These rods, or bars, are sometimes wired to the fabric to place them accurately.

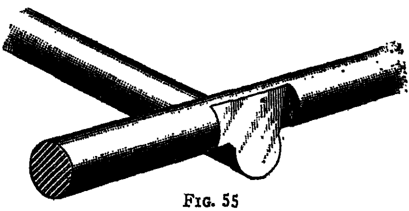
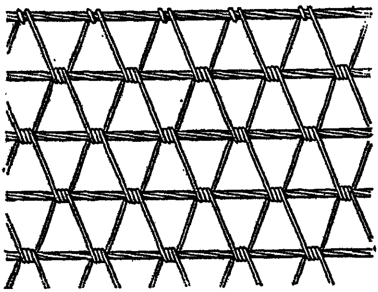


FIG. 55

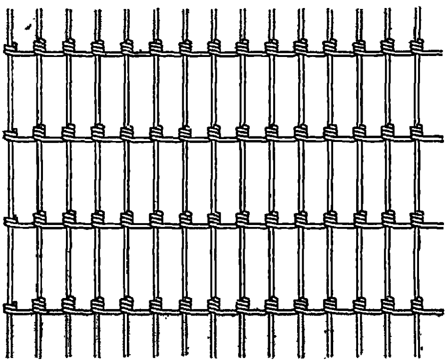
Woven-wire fabrics are made by bonding, or locking, the cross-wires mechanically or by welding them electrically. All wire fabrics are constructed with transverse, or spacing, wires, and main, or carrying, wires. The *carrying wires* are the heavy wires on which the strength of the reinforcement depends, while the *transverse wires* are those used to complete the fabric and to retain the carrying wires in their relative positions.

The wire fabrics united by means of mechanical bonds,

such as the *lock-woven wire fabric* and the *tie-locked fabric,* are ordinarily arranged with the transverse wires spaced about 6 inches apart, the space between the longitudinal, or carrying, wires varying from 4 inches upwards. The electrically welded fabric is woven with rectangular meshes of various sizes.



(a)



(b)

FIG. 56

All fabrics come in widths of 4, 6, 7, or 8 feet, and in lengths ranging from 100 to 500 feet, according to the weight of the wire. The material from which these fabrics are made is drawn wire, with a high tensile resistance.

68. Clinton Wire Cloth.—The wire mesh shown in Fig. 54 has the intersections of the wires securely welded as shown in Fig. 55. It is made by the Clinton Wire Cloth Company.

69. American Steel and Wire Company Mesh.—A mesh similar to wire fencing, made by the American Steel and Wire Company specially for reinforcement, is shown in Fig. 56. As shown in (*a*) and (*b*), it is made in triangular or square mesh and has heavy longitudinal wires, sometimes two or three wires twisted, that form the main reinforcement. These heavy wires are joined and spaced by small lateral wires. The Page Steel and Wire Company and other independent concerns also make reinforcement of similar forms.

REINFORCED-CONCRETE BUILDINGS
(PART 1)

INTRODUCTION

1. The extensive use of reinforced concrete in building construction is due: (1) to the demand for a non-combustible structure at a moderate cost; (2) to the large supplies of cement of reliable and standard qualities to be obtained at moderate prices; (3) to the growing difficulty of obtaining quickly large timbers for heavy construction; and (4) particularly to the saving in first cost of reinforced concrete over other kinds of fireproof construction for buildings having moderate spans between floor supports, reinforced concrete being even cheaper than mill construction for heavy loads.

Not only can reinforced-concrete buildings of moderate floor spans be erected at less cost than other fireproof construction, but they have superior fireproof qualities and are more rigid. They are, therefore, in particularly high favor in earthquake regions.

Probably the best examples of buildings constructed of reinforced concrete are those erected primarily for industrial and commercial purposes, such as warehouses and factories. For buildings of this kind, reinforced concrete has all but displaced the so-called slow-burning type of mill construction that was at one time extensively employed.

Although factories and warehouses furnish the majority of examples of reinforced-concrete buildings, yet many prominent and interesting concrete structures have been erected in what

may be called "civil architecture," as well as numerous hotels, hospitals, office and school buildings.

2. Kinds of Concrete Construction.—From an architectural point of view, reinforced-concrete buildings may be classified, according to the extent to which reinforced concrete is utilized in their construction, into three distinct groups; namely, (1) those in which the entire structure is of reinforced concrete; (2) those in which this material is used for the framework only, with veneer walls of other materials used especially for the architectural effect; and (3) those in which the floor construction only is of reinforced concrete. But this classification does not take into account the differences existing between various types of column construction, nor the differences existing between various types of floor construction. These differences are of vital importance in the engineering design, therefore the engineering features of the subject are here treated under the heads of: (1) Columns, and (2) Floors, which two elements enter into the construction of practically all buildings. In the highest developed type of reinforced-concrete buildings, reinforced concrete is used throughout for both columns and floors, and for the exterior as well as for the interior parts of the building. Some reinforced-concrete buildings have been built with exterior walls of solid concrete containing a small amount of reinforcement, but the majority of reinforced-concrete buildings are built with an exterior construction consisting of beams, called lintels, and columns supporting the lintels. A reinforced-concrete building in which both the interior and the exterior construction comprises isolated columns carrying beams, girders, and slabs, is said to be of reinforced-concrete skeleton construction.

3. A typical reinforced-concrete warehouse of skeleton construction is shown in Fig. 1. The most conspicuous parts of the floors are the lintels *a*. The rectangular spaces between the lintels and the piers are filled with the spandrel walls *b* and the windows. In some cases, the spaces are entirely filled with blank walls *c*, and the resulting structure is said to be **monolithic,** literally meaning *made of one stone;* but, as a matter

of fact, no building of more than one story was ever poured in one operation so as to form virtually a single stone. The

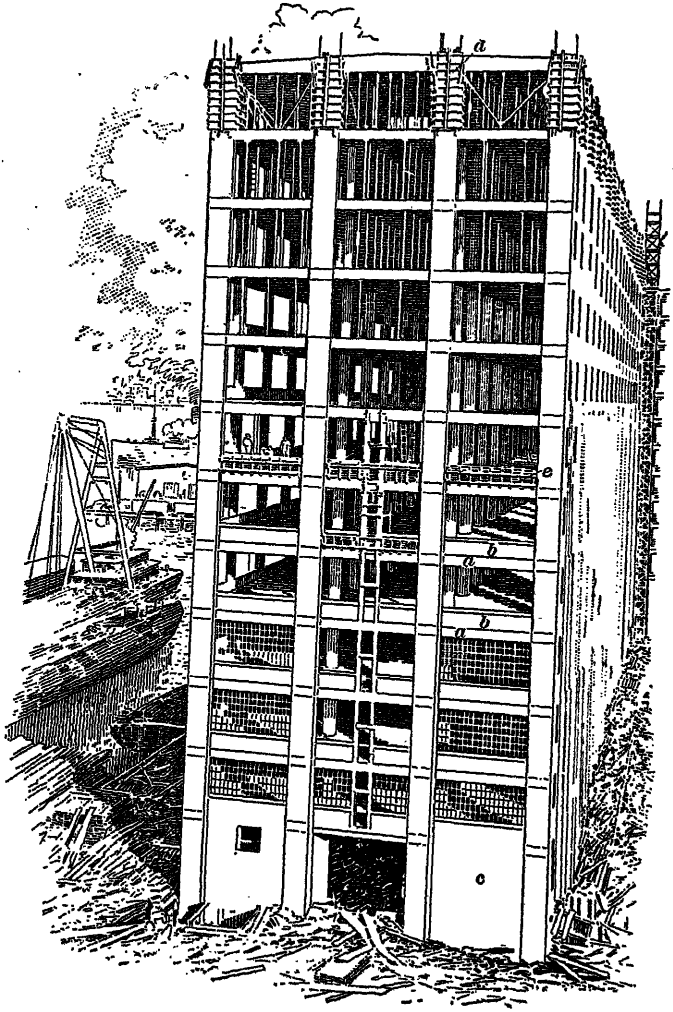


FIG. 1

impossibility of constructing a tall building in one operation will be seen by looking at Fig. 1, in which the forms or molds *d*

are shown still in place at the top story. In constructing such a building, each story in its turn goes through a similar process of evolution: First, the complete wooden false work is set up, forming a mold for the proposed structure; next, the reinforcing steel is placed within and upon these molds, and the concrete is poured into the molds; finally, after the concrete has sufficiently hardened, the forms are removed. At a later date, the molds *e* for the spandrel walls are put in place, and the reinforcement and concrete are placed within these forms, and after the building has been thus enclosed and provided with windows, the interior parts are finished.

COLUMNS

FEATURES INFLUENCING THE CONSTRUCTION OF COLUMNS IN REINFORCED-CONCRETE BUILDINGS

4. Building Regulations.—All large cities and many smaller cities have regulations or building codes prescribing the method of construction of columns. In some cities the column diameters must be a certain fraction of the spacing between columns when certain types of floors are used; a ratio between length and diameter of column which must not be exceeded is also often insisted upon. Although the methods of design and construction specified by law have become much more uniform in recent years, there is still great diversity of detail in the regulations of the various cities. Wherever, in the following, specific instructions are given relating to spacings, diameters, or thicknesses, unless otherwise stated, they are those recommended in the Final Report of the Joint Committee of the American Society of Civil Engineers and other engineering organizations. While the recommendations are not at present adopted in their entirety as law in any one community, they nevertheless represent the average professional

conception of standard practice better than any other published records.

5. Intended Use of Building.—The columns in a reinforced-concrete building must be constructed with special reference to the use for which the building is intended. Thus, if there is considerable traffic within the building, special protection must be furnished for the corners of the columns in order to prevent damage from collision. Considerations of lighting and ventilation may make narrow exterior columns desirable in order to provide a maximum of window space, or considerations of economy may, on the contrary, make it necessary to have the exterior columns wide and of thin section so as to occupy a maximum of wall space with a minimum of material. Sometimes the exterior columns are hollow in order to serve as ducts for the hot air used in certain types of heating systems. Architectural features may be added to both interior and exterior columns for the purpose of improving the appearance, and such ornamental features may dictate the selection of the column section.

6. Fire Exposure.—One important point to be considered in the construction of columns in buildings is their ability to resist fire; because experience gained in great conflagrations indicates that the columns are the most vulnerable parts of a building. Of the different kinds of fireproof columns in common use in buildings, the reinforced-concrete column is the most satisfactory. Concrete transmits heat but slowly and is not deeply injured by fire, therefore the concrete serves as a good protection for the steel imbedded in it. The action of intense heat is to dehydrate a thin outer layer of concrete; the dehydration itself and the evaporation of the water contained in the concrete absorb heat, thus keeping the concrete cooler than it would be otherwise. The portion of the concrete that loses its water in the dehydrating process loses its cohesion and cannot be depended upon for strength; therefore, in building reinforced-concrete columns it is important to keep the reinforcement so far away from the surface of the concrete that it will be protected by a layer of sound concrete although the

injured outer shell may drop off. The thickness of this protective layer is usually prescribed by local building regulations; where no such regulations govern, the thickness may be varied according to the fire exposure, greater thickness being required where highly inflammable materials are stored in large quantity. For ordinary conditions of fire exposure, the Joint Committee recommends a thickness of fire protection of 2 inches for reinforced-concrete columns, and the reinforcing steel must therefore be kept at least that far away from the surface of the concrete.

DETAILS OF COLUMN REINFORCEMENT

7. Classification of Reinforced-Concrete Columns. According to the method of reinforcing used, columns in reinforced-concrete buildings may be classified in four general types; namely, columns reinforced with (1) *vertical rods* tied together at comparatively large intervals, as shown in Fig. 2; (2) *spirally wound hoops* in conjunction with vertical reinforcement as indicated in Fig. 3; (3) *structural steel cores*, Fig. 4; and (4) *cast-iron cores*, Fig. 5. These are enumerated in the order of their importance in practical use. Although the hooped column is placed second, it is gaining in use, being commonly conceded to be the most efficient type of reinforced column both as regards strength and fire resistance.

COLUMNS WITH VERTICAL RODS

8. Vertical rods *a*, Fig. 2, are frequently used as the main reinforcement for columns, the steel area varying in amount from 1 per cent. to 4 per cent. of the net concrete area of the column. By net concrete area is meant the area left after deducting the protective layer previously referred to; so that, for example, if a column is 14 inches square and its protective layer is 2 inches thick, the net area is $10 \times 10 = 100$ square inches, and the area of reinforcing steel in a horizontal cross-section would be from 1 to 4 per cent. or from 1 to 4 square inches.

In addition to the vertical rods, horizontal ties *b*, Fig. 2, are used. These should be spaced not more than 12 inches apart if the vertical rods are not less than $\frac{3}{4}$ inch in diameter. If vertical rods of a smaller diameter than $\frac{3}{4}$ inch are used, the vertical distance between ties must not exceed 16 times the diameter of the smallest vertical rod used. The ties must not be lighter than $\frac{1}{4}$ inch in diameter.

The vertical rods are usually plain round or square rods and the ties are usually round or flat. If desired, deformed bars may be used for the verticals, but bars having rigidly attached prongs or projections are not to be recommended, because the prongs catch the concrete when the columns are filled and interfere with the spading and manipulation of the concrete.

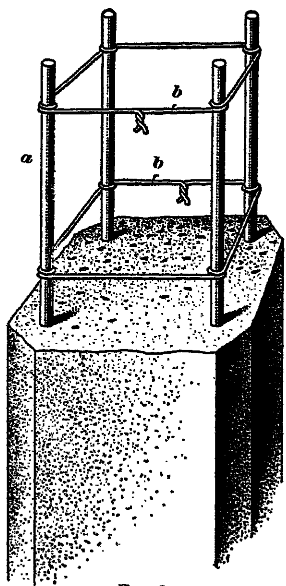


FIG. 2

9. Practical Considerations. The column with verticals and plain ties uses the reinforcement in its simplest and cheapest form. But this simplicity is obtained by sacrificing a large part of the efficiency of the steel, the vertical rods adding but little to the strength of the column, because they are likely to buckle under load unless firmly tied horizontally. Every failure of concrete columns recorded in the history of reinforced-concrete construction in this country has been with this type of column, whether failure was induced by premature removal of centering supports in cold or frosty weather or by great heat in fires. In other words, owing to the comparatively small part of the strength contributed by the steel, the strength of the column depends almost entirely on the quality of the concrete; so that one poor batch of concrete in the shaft of the column reduces the strength of the column to that of the weak portion. But this column is so adapted to meet the architectural

requirements of buildings that its structural shortcomings are endured and its use maintained, because the cross-sectional shape can be made of special forms to occupy the minimum of valuable space for exterior columns or pilasters and for special interior constructions where it is desirable to have a wide face with much smaller thickness.

COLUMNS WITH HOOPS

10. As previously stated, the vertical reinforcement adds comparatively little to the strength of a reinforced-concrete column, because the vertical rods are likely to buckle under load unless firmly tied horizontally. The most efficient way of tying the vertical rods horizontally is the one used for the so-called *hooped column* shown in Fig. 3, where continuous windings or spirals *b* encircle the vertical rods *a*. The clear distance between successive windings of the spirals must not exceed one-sixth of the diameter of the concrete area enclosed within the hoops, and must not exceed $2\frac{1}{2}$ inches; the distance between centers of successive windings, or the so-called *pitch* of the spirals, is therefore much less than for the previously described vertically reinforced columns.

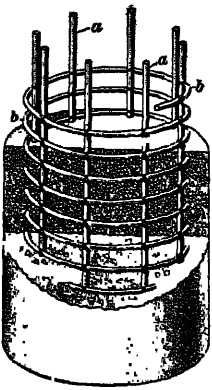


FIG. 3

Not only the efficiency of the verticals, but also the strength of the concrete, is increased by closely spaced hoops; according to the Joint Committee, the carrying capacity of a hooped column is 55 per cent. greater than that of a column with vertical rods tied at larger intervals and having the same concrete area and the same amount of vertical steel, provided the total volume of the hoops is not less than 1 per cent. of the volume of the concrete enclosed within the hoops.

Since the effect of the hoops is so much greater than the effect of the verticals, it might be thought possible to increase the strength of the column greatly by using all the steel in the

form of hoops rather than dividing the steel in hoops and verticals. This has indeed been tried, with the result that the strength of the column reached practically the anticipated higher strength, but at the same time the shortening of the column under load was too great for practical purposes. By a judicious use of both verticals and hoops, a column is obtained combining great strength with great rigidity and therefore well adapted for construction purposes.

11. The **steel used for hooped columns** is usually plain round or square rods for the verticals and round wire or flat hoop iron for the spirals. Rolled steel bars are not often used for the hoops, because they can be had only in such short lengths that there would be a great many joints in the hooping. Joints must be avoided where possible, because the steel in the hoops is in tension, and each joint is therefore a possible source of weakness. Drawn wire in coils can be obtained in great lengths and is always to be preferred where a contractor desires to make his own hooping. It is, however, in many cases more convenient to purchase the hooping ready-made from concerns specializing in the manufacture of steel reinforcement.

The wire used for hooping varies in size from No. 4 gauge to No. 7-0 gauge. The gauge is an arbitrary figure indicating the diameter of the wire, and considerable confusion arises from the fact that there are seven different wire and sheet-metal gauges in use. One commonly employed in connection with reinforced-concrete work is The American Steel & Wire Co.'s gauge. In it the heaviest wire is No. 7-0 (often written 0000000, the number of ciphers indicating the number of the wire), which has a diameter of .49 inch or a trifle less than $\frac{1}{2}$ inch. Ranged in order of their thicknesses, the wires diminish in diameter as follows: 7-0, 6-0, 5-0, 4-0, 3-0, 2-0, 0, 1, 2, 3, 4, and so on. No. 4 is the lightest wire used in reinforced-concrete columns; it has a diameter of .2253 inch, or between $\frac{14}{64}$ and $\frac{15}{64}$ inch. The flat material sometimes used for hoops is usually 1 inch or $1\frac{1}{4}$ inches wide and from $\frac{1}{4}$ to $\frac{3}{8}$ inch thick.

12. The hooped column is preferred by many engineers on account of its greater safety. This greater safety is due to the

peculiar arrangement of the reinforcement, because, if accidentally any weak spots should exist in the concrete, the hooping will nevertheless hold the weak concrete together and prevent its entire disintegration until remedial action can be taken. Where weakness develops in a hooped column, this will always be manifested by scaling off of the concrete outside the hoops and by a great shortening in the column. In practice such phenomena should of course not be manifested, because the loads actually placed on the columns are far less than those required to crush the column; nevertheless, the warning signals displayed by hooped columns make them more desirable than the ordinary reinforced-concrete column which fails suddenly and without warning.

Although the hooped column is a very safe column and is able to carry great loads, it should not be subjected to high stresses. It has sometimes been proposed to use very much higher unit stresses on hooped columns than on plain columns; but this is not good practice, on account of the great shortening of a hooped column under high compression stresses. There is great diversity of opinion among engineers as to how much greater stresses can be allowed on the concrete of hooped columns than on plain columns; the Joint Committee recommends an increase of 50 per cent. in the allowable compression stress on the concrete, but the building regulation of many cities do not permit any increase. In such cities the use of hooped columns is discouraged because of their higher cost, and they are there used only by those who are willing to pay a higher price for the greater safety. Where a suitable allowance is made for the increased strength caused by the hooping, hooped columns can compete commercially on an even basis with columns reinforced with vertical rods only.

COLUMNS WITH STRUCTURAL-STEEL CORES

13. Structural-steel shapes are used in columns in concrete buildings solely to reduce the size of the columns. These columns cost more than the previously described types of reinforced-concrete columns and they have the additional

disadvantage that it is difficult to attach reinforced-concrete floors to them. The greater cost of these columns is due to the fact that not much dependence is placed on the concrete for carrying the load, the steel alone being required to carry the greater part of the weight on the columns. Much more steel is therefore required in these columns than in reinforced-concrete columns.

In Fig. 4 are shown in section three different kinds of concrete-encased steel columns. That shown in (*a*) is made up of four angles back to back, joined together by plates. With this type, four vertical rods *a* with ties *b* are generally employed. The other forms shown in (*b*) and (*c*) do not have vertical rods, but the **H** section of (*b*) should be wrapped with wire netting or other ties to bind the concrete fireproofing, because the concrete will otherwise not adhere to the large steel flange.

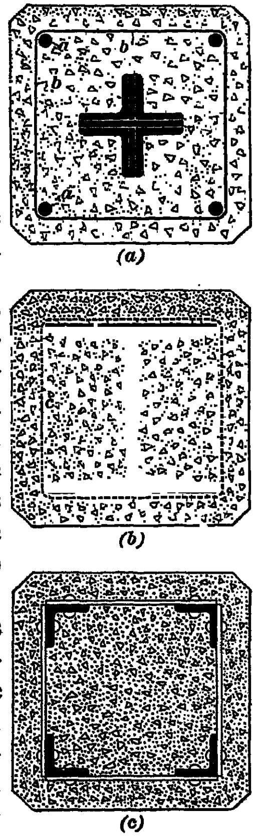


Fig. 4

The construction of the steel core is not usually undertaken by the reinforced-concrete contractor, but the cores are manufactured in structural-steel shops according to plans and specifications furnished by the engineer or architect. The best practice is to have the angle irons, or other structural shapes constituting the core, tied together with tie-plates or lattice bars; and all details of the structural steel, including the splices between successive lengths of columns, must be constructed in accordance with standard practice for structural steel.

COLUMNS WITH CAST-IRON CORES

14. Concrete columns reinforced with cast-iron cores, as in Fig. 5, have not been used extensively in this country, because it is difficult to obtain satisfactory and dependable castings. Most of the concrete columns with cast-iron cores used so far have been placed in buildings only one or two stories high where the loads to be carried and the risks assumed are not so great as in taller buildings. But tests made in 1916 at the Pittsburgh laboratory of the U. S. Bureau of Standards on a series of columns constructed as shown in Fig. 5, with cast-iron cores *a* and spiral hooping *b*, showed that cast-iron and hooped concrete, work very well together in columns; so that, where fair workmanship is obtained in the cast-iron cores this type of column can be used with safety.

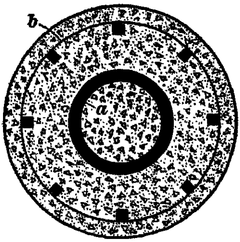
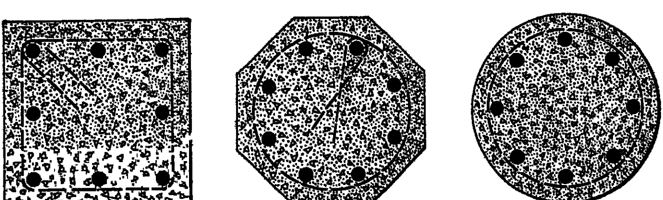


FIG. 5

COLUMN SHAFTS AND CONNECTIONS

COLUMN SHAFTS

15. The **columns** may have circular, polygonal (usually octagonal) or rectangular shafts, as shown in cross-section in



(*a*) (*b*) (*c*)

FIG. 6

Fig. 6 (*a*), (*b*), and (*c*). The *circular* section is particularly well adapted for hooped columns. The base and the top may be finished off squarely or the top may be flared into a capital,

as will be explained later in connection with flat-slab floors. The round column resists fire and shock better than one of any other shape and it looks better. The *octagonal* column, shown in Fig. 6 (*b*), is sometimes finished with a square base and top, but more often, in manufacturing buildings, the octagon shape is carried all the way from floor to ceiling. This, too, is a practical and adequate section for hooped columns. The *rectangular* section is best adapted for columns reinforced with vertical rods.

A variety of rectangular sections is in use. For interior columns, a square section with chamfered corners is used; for exterior side columns, a rectangular section, Fig. 7, with rab-

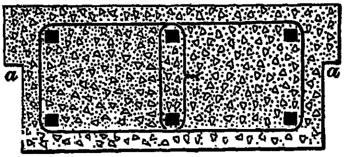


FIG. 7

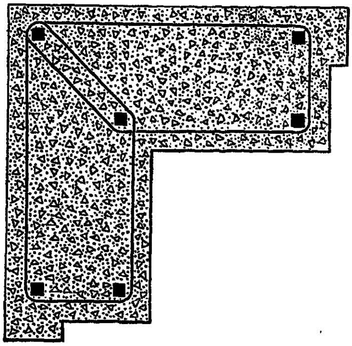


FIG. 8

bets *a* in the sides to receive the windows is employed; for corner columns, an L-shaped section, Fig. 8, composed of two rectangular sections, is usually adopted. Sometimes the rectangular columns are made hollow to carry heat ducts, flues, or conduits.

16. A special type of **wall column**, such as shown in Fig. 9, is occasionally used to advantage in one-story buildings. In (*a*) is shown a horizontal cross-section on the line *A B* of view (*b*); in (*b*) is shown a vertical section on the line *C D* of view (*c*), while in (*c*) is shown the elevation of the side toward the inside of the building. The general shape is rectangular in cross-section with a portion *a* cored out toward the inside of the building, in order to save concrete. At the top, where heavy

roof girders or steel trusses frame into the column, the full rectangular section is retained, as at *b*. Owing to the two wings *c*, this column is both stiff and strong, and it can be made to occupy a large space of wall with comparatively little concrete.

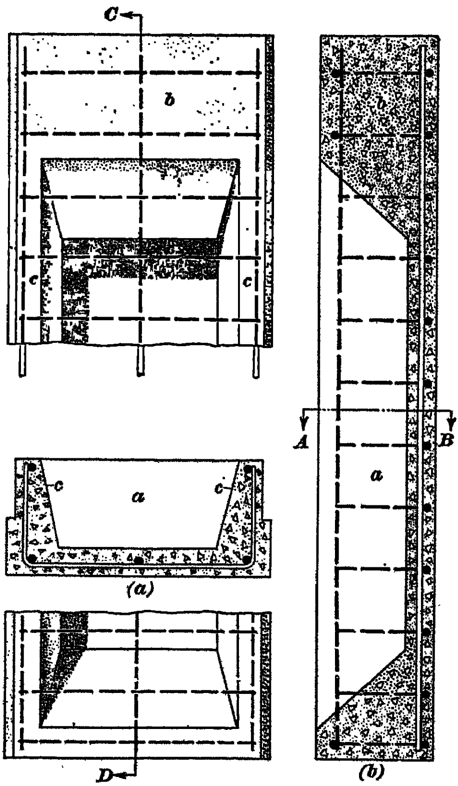


FIG. 9

COLUMN CONNECTIONS AT FLOORS

17. If the height of the building is not more than two stories, the column rods may extend in single lengths from foundation to roof. In taller buildings it would not be practical to have the rods in one single length, and in the majority of buildings means must therefore be introduced for splicing the column reinforcement of any given story to that of the column in the story immediately below. The commonly used means are: (1) lapping the rods; (2) butting the ends of the rods inside close-fitting sleeves; (3) drawing the rods together by means of threaded sleeves.

18. Lapping the rods is accomplished in the manner shown in Fig. 10. The rods are allowed to project sufficiently above the floor level so that their top ends will become

embedded in the concrete of the column of the story above. These projecting rods extend along the rods of the upper columns for a distance of 40 diameters in order to transfer the stresses from the one set to the other. The rods are sometimes wired together but more often not. The construction indicated in Fig. 10 is adapted only for columns with light rods; that is, rods up to $\frac{7}{8}$-inch square or 1-inch round.

19. Butting the ends of the rods in the manner shown in Fig. 11 is a very satisfactory detail in work not subject to

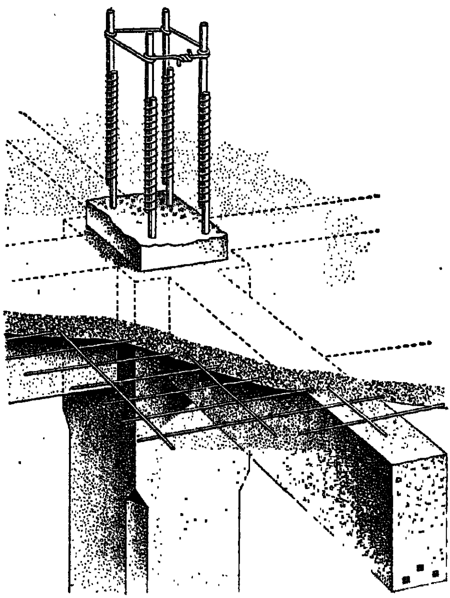


FIG. 10

tensile stresses, and in reinforced-concrete columns this is almost invariably the case. The ends of the rods are milled in order to obtain perfect bearing, and the diameter of the sleeves is made enough larger than that of the rods so that everything will come together easily.

20. Threaded sleeves, as at *a*, Fig. 12, are sometimes insisted upon in building regulations. It will be recognized that

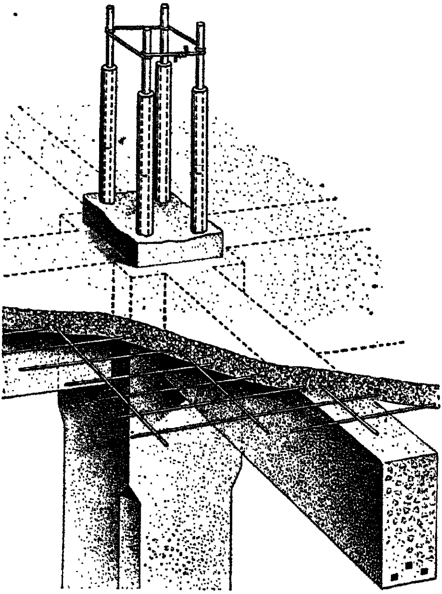


Fig. 11

in order not to lose in strength by cutting threads on the ends of the rods, these ends must be upset. By upsetting is meant enlarging the ends of the rods as at *b*, so that enough area is left at the root of the thread to carry the load without exceeding the allowable stresses. Otherwise larger rods must be used throughout. Whether upset or larger rods are employed in either case, this construction is expensive and clumsy, as in order to be of any service the sleeves must be quite large

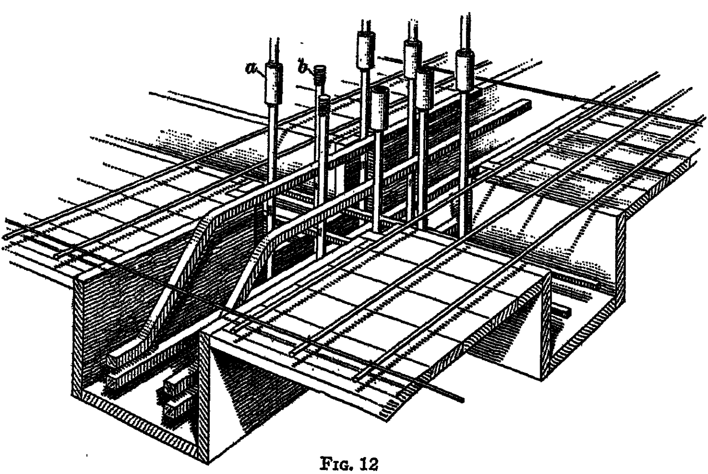


Fig. 12

and heavy. This construction can be used for round rods only.

COLUMN CONNECTIONS AT FOOTINGS

21. Connection of column to footing requires particular attention. Sometimes a steel plate, as at *a*, Fig. 13, is used to distribute the load over the footing; but where the area involved is considerable, a cast-iron chair as shown in Fig. 14 at *a* is preferable. For very great loads, the **structural-steel** detail shown in Fig. 15 is sometimes used. In either case, the steel work must be embedded in concrete of ample thickness, 3 or 4 inches at least, so that the steel will be protected against corrosion.

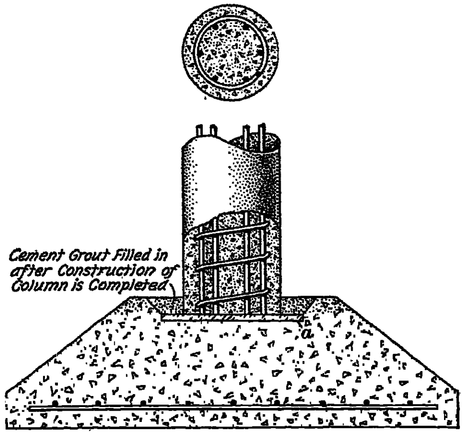


FIG. 13

Where the floor space is comparatively cheap, the basement columns may be enlarged sufficiently above ground to transmit the load to the footing without any plates. Where conditions permit, a simpler and cheaper connection is had by simply spreading the concrete column gradually below ground, as in Fig. 16. This requires considerable depth from the bottom of the lowest floor to the top of the footing and is therefore not economical where the costs of excavation and forms are high.

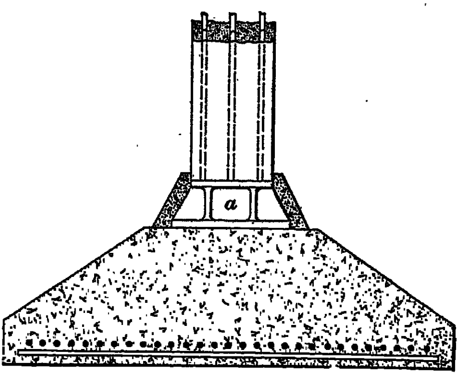


FIG. 14

22. A simple and efficient method of transmitting the column load to the footing is shown in Fig. 17, where the column reinforcement rests on two layers of steel plates *a* that cross each other at right angles and transfer the load to the footing.

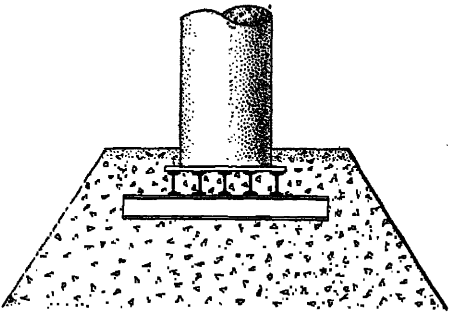


FIG. 15

The only objection to this method arises from the impossibility of correcting any slight inaccuracy in the location of the footing. When the column is independent of the footing, it is possible to shift the column mold an inch or two from the center line of the footing in order to keep the columns properly lined up even if the footings happen to be a trifle out of true line. Since in this method the column rein-

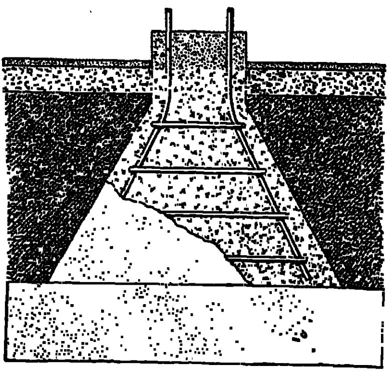


FIG. 16

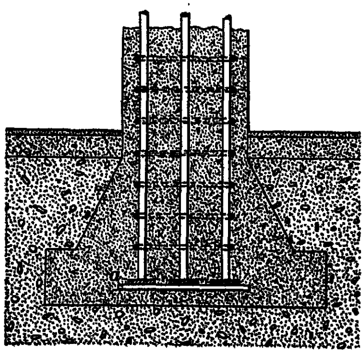


FIG. 17

forcement extends into the footing, this adjustment cannot take place, because the column reinforcement would then not be properly centered in the mold.

23. A method that is quite extensively used for connecting the column to the footing where the column load does not call

for special means of distribution over the footing, is that snown in Fig. 18. The connection is accomplished by means of dowels *a* extending from the footing into the column. These dowels guard also against a possible side motion, or lateral

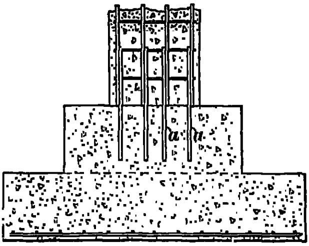


Fig. 18

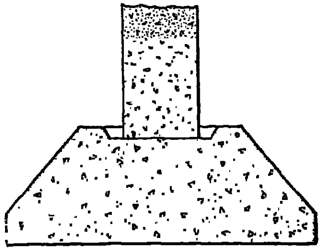


Fig. 19

shifting of the column. In some cases where no dowels are used, provision against lateral shifting is made by means of a depression in the top of the footing, forming a step for the columns, as shown in Fig. 19.

FLOOR CONNECTIONS TO COLUMNS

24. Connections With Reinforced-Concrete Columns.—The connection between floor and column is effected by extending the concrete and the reinforcement of the floor construction over the top of the column. Sometimes the concrete of the column is filled into the mold as high as to the under side of the floor construction, and afterwards the concrete of the floor is deposited in one continuous operation over the top

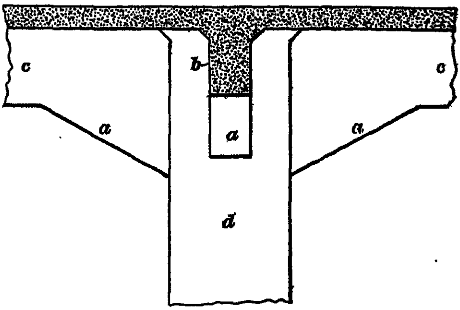
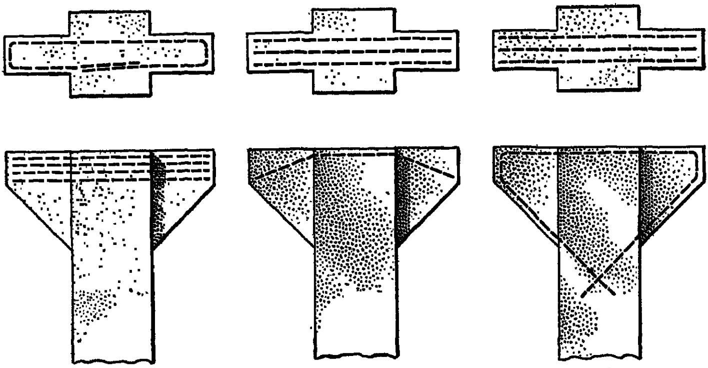


Fig. 20

of the column; in other cases floor and column are poured in one continuous operation, thus making the concrete of the floor and column integral. In either case, the reinforce-

ments of beams and girders extend continuously over the column. It is frequently desirable to have additional strength in the beam and girder over the column, because the beam and



(a) (b) (c)

FIG. 21

girder are otherwise liable to crack at that point. This strength is sometimes obtained as shown in Fig. 20, by the use of brackets *a* which increase the strength of the beam *b* and the girder *c* where they join the column *d*.

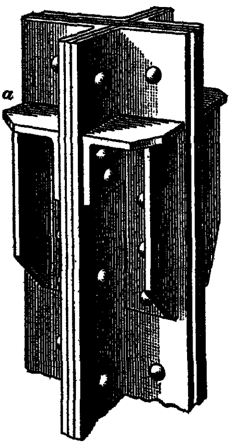


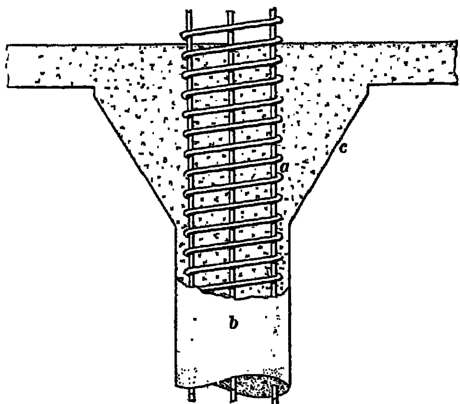
FIG. 22

Sometimes brackets are introduced for the purpose of stiffening the building and to act as wind braces. Such brackets are not looked upon with favor in manufacturing plants using overhead shafting, as they interfere with the line shafts.

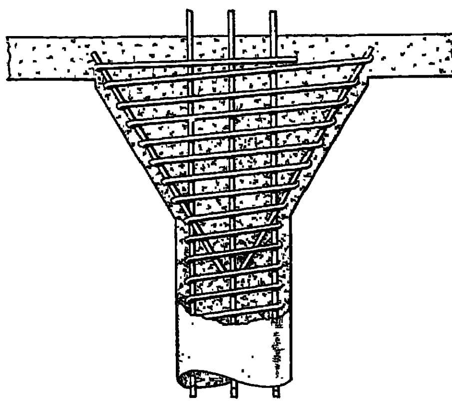
Brackets are often cast integrally with the columns to receive a crane track or a balcony floor, or some other additional element to be installed later. For instance, where it is intended to enlarge the building at some future date, brackets are provided to serve as supports for the floors of the new building.

25. Reinforcement of Brackets.—It appears from tests described by John E. Conzelman in a paper read before

the National Association of Cement Users, that the most efficient type of bracket reinforcement is that resembling a closed rectangular loop, as shown in Fig. 21 (*a*). Other types illustrated in Fig. 21 (*b*) and (*c*) gave less satisfactory results, while brackets without reinforcement failed suddenly at low loads.



(*a*)



(*b*)

FIG. 23

26. Floor Connections With Columns Having Structural - Steel Cores.—It is difficult to obtain a satisfactory connection for the support of the floor upon the structural-steel core of a column.

The connection of floor beams and slabs may be accomplished by running the reinforcement through the latticed angle columns, but bracket angles *a*, as in Fig. 22, must be provided on cores of star-shaped or H-shaped sections to transfer the computed panel load at each floor to the steel section. This detail is never very satisfactory and the use of steel cores is therefore not to be encouraged. However, it may be necessary to use a steel girder or truss in some part of a concrete building, and in that case it is desirable to employ steel

columns to support the girder or truss because no satisfactory means is available for supporting such a truss or girder on concrete columns. Nevertheless, steel trusses are sometimes supported on brackets on concrete columns; and when the span is long, roller bearings or similar devices must be used to take care of the expansion.

27. Column capitals are commonly used only in flat-slab construction. They serve the purpose of lessening the clear span between columns. As shown in Fig. 23 (*a*) the reinforcement *a* of the column *b* is carried straight up through the capitals. Enlarging the hoops at the capital to the basket shape shown in Fig. 23 (*b*) is effective but expensive.

FLOORS

28. Classification.—Reinforced-concrete floors may be classified into two general groups; namely, (1) *ribbed floors,* in which the load is carried on slabs resting upon reinforced-concrete beams and girders, and (2) *flat-slab floors,* in which the load is carried on slabs resting directly upon columns, without the intervention of beams and girders. In addition to these types of construction, reinforced-concrete slabs are often carried upon structural-steel beams and girders.

CONSTRUCTION OF RIBBED FLOORS

29. Ribbed floors may be divided into four groups according to the method of construction; namely, (1) *beam-and-slab floors,* (2) *two-way slab floors,* (3) *tile-concrete floors,* and (4) *unit-construction floors.* In each of these types of construction, part of the slab assists in taking the compression stresses in the beams and girders, both of which thus act as T beams.

30. In the practical execution of the work it is common practice to make the beams and slab in one piece, or *monolithic,* because a firm union is thus secured between the stem of the

beam and the slab, which may then be considered as acting as compression flange to the beam. This method of construction causes a great expense for form work, and the *unit-construction* methods were developed chiefly in an effort to reduce this expense, because in unit construction the same molds can be used many times with short interval, while in other types of construction, the molds can be used only a few times with comparatively long interval. The expense of the molds is a large proportion of the total cost of the building, and it is therefore advisable to keep the cost of the molds as low as possible by standardizing the sizes of the structural members. For this reason, if the loading changes from floor to floor, the dimensions of the concrete beams or girders should not be changed, but the different loads can be provided for by changes in thickness of the slab and in the steel reinforcement. Where the slab and beam or girder are cast in one piece, the depth below the slab should not be changed, thus the same molds may be used.

31. The construction of the floors, in common with other parts of a building, is influenced by building laws, which vary in different cities and must always be complied with. The use for which the building is intended influences the floor construction mainly in regard to the arrangement of accessories, such as devices for fastening machinery and the like, which will be described in another Section.

32. Thickness of Protective Coating.—In order to protect the reinforcing steel of floors against injury by fire and rust, the steel must be well embedded in the concrete. The depth of embedment depends upon the anticipated exposure. For ordinary conditions of fire exposure, the Joint Committee recommends that the steel be protected by a minimum of 2 inches on girders, $1\frac{1}{2}$ inches on beams, and 1 inch on floor slabs.

The Joint Committee also recommends that the corners of beams and girders be beveled or rounded because a sharp corner is more seriously affected by fire than a round corner.

TYPES OF RIBBED FLOORS

BEAM-AND-SLAB FLOORS

33. A widely used form of floor design is the slab supported on parallel beams. For light loads, the slab span may be 12 to 16 feet, and sometimes more, although the prevailing practice is to limit the spacing of the beams to between 5 and 10 feet in order to avoid thick and heavy slabs. An arrangement of this kind is shown in Fig. 24, where (*a*) is a plan view, (*b*) a cross-sectional elevation, and (*c*) a section on the line *A B*. The slabs *a* rest on beams *b*, which in turn are carried upon girders *c* or are framed directly into the columns *d*. The girders *c* are generally made somewhat deeper than the beams *b*, so that, as shown in (*c*), the reinforcement *e* of the beam may extend over the reinforcement *f* of the girder *c* without interference.

34. The reinforcement used in ribbed floors may be classified in two groups as *principal* and *secondary* reinforcement. The principal reinforcement serves the purpose of carrying, in conjunction with the concrete, the loads for which the floor is designed. The strength of the structure depends largely upon the skill and care with which the principal reinforcement is placed, and in practical construction every precaution must therefore be taken to place the steel exactly as shown on the drawings. The purpose of the secondary reinforcement will be explained later on.

35. The **principal reinforcement of the slab** extends parallel with the span, as indicated by the dotted lines *g* in Fig. 24; the rods *g* are uniformly spaced over the entire floor, but only a few of them are indicated in the plan (*a*). The rods *g* must be at least a few inches longer than the clear distance between the beams *b*; commonly they are made long enough to cover several spans in one length, the rods being usually bought in lengths of 20 to 30 feet. The rods are placed in the bottom part of the slab and most of the slab rods continue

straight in that location, but some of them are usually bent up, as at *h* in (*b*), over the tops of the beams, in order to take care of tension stresses existing there. The number of slab rods

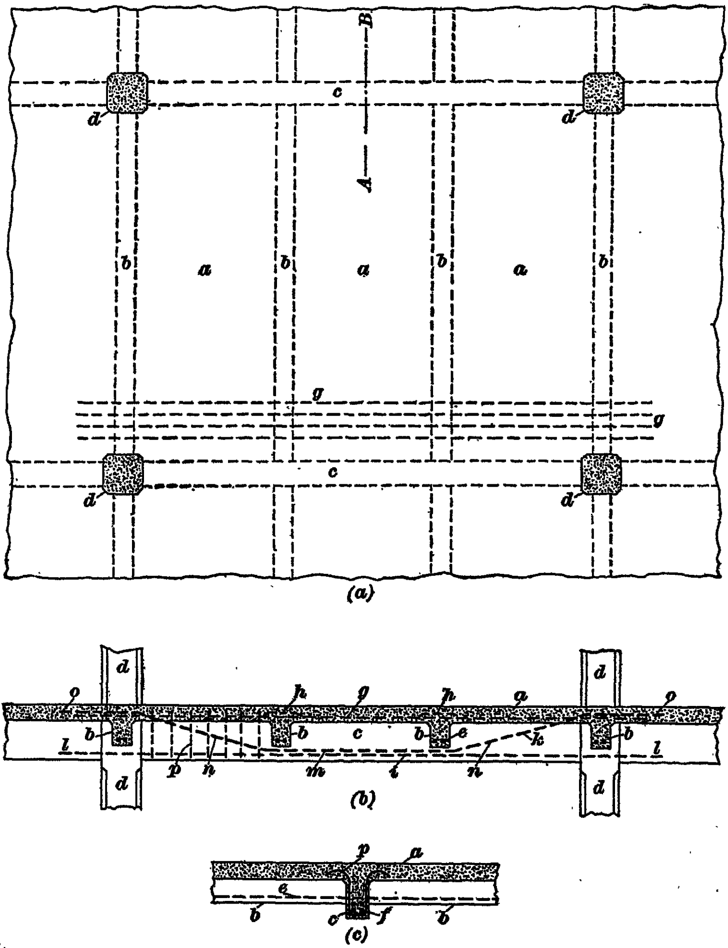


Fig. 24

bent rarely exceeds one-half of the total number. Sometimes the rods are bent to template before being laid in the slab, but they can also be laid straight and be bent after laying by means

of special rod benders of which a number of different designs are in use.

The size of rods used in slabs varies with the depth of the slab, more steel being required for a thick slab than for a thin one. Light, short spans may be reinforced with rods as light as $\frac{1}{4}$ inch in diameter, while $\frac{7}{8}$-inch to 1-inch rods are the heaviest rods used for slabs. Rods with a diameter of $\frac{3}{8}$ in., $\frac{1}{2}$ in., $\frac{5}{8}$ in., and $\frac{3}{4}$ in. are those most commonly used.

36. The **principal reinforcement of beams and girders** is of two kinds; namely, *tension reinforcement* and *shear reinforcement.*

The **tension rods** *f*, shown in Fig. 24 (*c*), are usually divided into straight rods *i* and bent or trussed rods *k*, as shown in (*b*). The straight rods may extend from center of support to center of support or, better, they may extend a distance beyond the center of support into the adjoining span as at *l*. The trussed rods have a horizontal portion *m* in the center of the span, inclined portions *n* at the ends of the span, and horizontal ends *o*, overhanging into the adjoining span. It is upon this overhang or lap that the strength of the floor depends to a great extent, partly because the length of embedment of the ends of the rods in the adjacent spans furnishes anchorage against the pulling out of the rods, but mainly because horizontal reinforcement in the top of the beam or girder over the support is required to withstand the tension stresses existing at that point.

The **shear members** *p* are vertical or inclined rods bent usually to a **U** shape and circling, at the bottom of the **U**, the main tension rods. In (*c*) is shown how the shear members *p* circle the main tension rods *f*. The shear members are often referred to as **U** bars or stirrups on account of their shape. They are always most numerous near the ends of the span, being either uniformly spaced over the end portions of the girders as in (*b*) or else arranged with closer spacing at the supports and with spacing increasing toward the center of the span. At the middle of the span shear rods are seldom required and are usually omitted.

The size of the tension rods and the shear rods depends upon span, load, and arrangement; and nothing definite can be said about the size and number of rods required without careful analysis of each particular case. A short light beam may contain one or two $\frac{3}{4}$-inch tension rods, while a long span, heavily loaded girder may have as many as ten or twelve 2-inch square bars. The **U** bars are mostly made of round rods varying in diameter between $\frac{5}{16}$-inch and $\frac{5}{8}$-inch, the $\frac{3}{8}$-inch and $\frac{1}{2}$-inch sizes being commonly used.

37. Secondary Reinforcement in Slabs.—In a reinforced-concrete building the tying of the floors into the walls prevents the contraction of the concrete of the floor slab, which takes place while hardening. As a consequence, the concrete of the slab is subjected to tension stresses. A lowering of the temperature also causes the concrete to contract, thus creating additional tension stresses in the slab. These stresses are not of sufficient magnitude to influence the design of the main tension reinforcement of the slab, nor do they have appreciable effect upon the concrete in the direction parallel with the main tension rods. It is, however, customary to introduce in the slab a light reinforcement at right angles to the main tension rods of the slab. These additional rods are commonly referred to as *shrinkage rods*. The shrinkage rods should be securely fastened to the main tension rods in order to prevent displacement of the latter during the depositing of the concrete on the floor.

Practice in regard to the amount and disposition of the shrinkage rods varies greatly. In a short span of slab, such as 5 or 6 feet, one rod at the center of the span and one along each support will suffice. For larger spans, two additional rods should be used, making five in all for each span. The diameter of the rods may conveniently be made the same as the diameter of the main tension rods of the slab.

38. Secondary reinforcement is used also for the purpose of tying new work to old. When the day's work has been completed, short rods, called *stubbs*, are bedded half way in the concrete so that the projecting half of the rods will be sur-

rounded by the concrete to be deposited when work is again resumed, thus assisting in creating a solid connection between the two days' work.

Secondary reinforcement is also frequently embedded in the concrete of a slab where a slab joins to a beam or girder. Experience has shown that shrinkage cracks often occur at the place where a slab joins a beam, and especially where a slab joins a girder. One form of reinforcement intended to counteract such cracks consists of rods, several feet long and ½ inch in diameter, placed about 6 inches apart, embedded in the slab at right angles to the girder. Since the slab is usually assumed to act as the flange of a T-shaped girder these cross-rods are also of advantage in tying the slab to the girder.

TWO-WAY SLAB FLOORS

39. Slabs that are square or nearly square and supported along all four sides are constructed with reinforcement placed in two directions at right angles, as shown in Fig. 25. Floors consisting of slabs *a* resting in this manner upon girders *b* framing directly into the columns *c* are called *two-way slab floors*. In a construction of this kind, the beams are built as already explained for beam-and-slab floors, with straight and bent tension rods and vertical U bars, the main difference between this and the other type being in the reinforcement of the slab. As indicated in the plan, Fig. 25, the slab reinforcement is placed closer together toward the center of the panel, two-thirds of all the steel rods being placed in the central half of the span. Some of the slab rods are bent or trussed over the beams as at *m* in the section *A B*; the beams *b* shown in this section are of course reinforced in the usual manner, although no reinforcement is indicated for them in order not to obscure the drawing.

Two-way slab floors are strong and rigid and the forms are not so expensive as those required for beam-and-slab floors, so that floors of this type are well adapted for manufacturing plants and similar structures. They are, however, no cheaper than beam-and-slab floors, because they require more steel and

concrete if designed in accordance with current building regulations, and they are therefore not much used, especially since

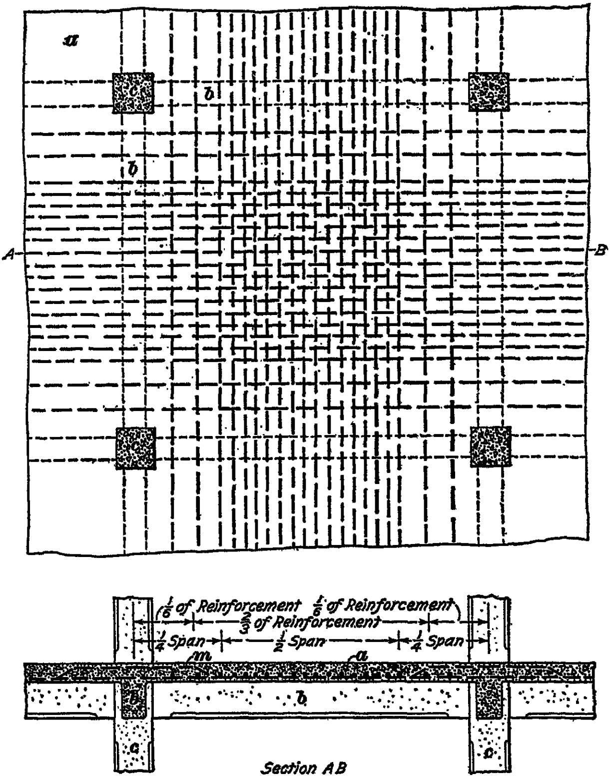


FIG. 25

it is possible to dispense altogether with the beams, by using the flat-slab floor construction, which will be explained later.

TILE-CONCRETE FLOORS

40. For light loads and long spans, the dead weight of the floor construction becomes out of proportion to the live load, making the construction uneconomical. The weight of the floor is therefore often lightened, and the reinforcement reduced

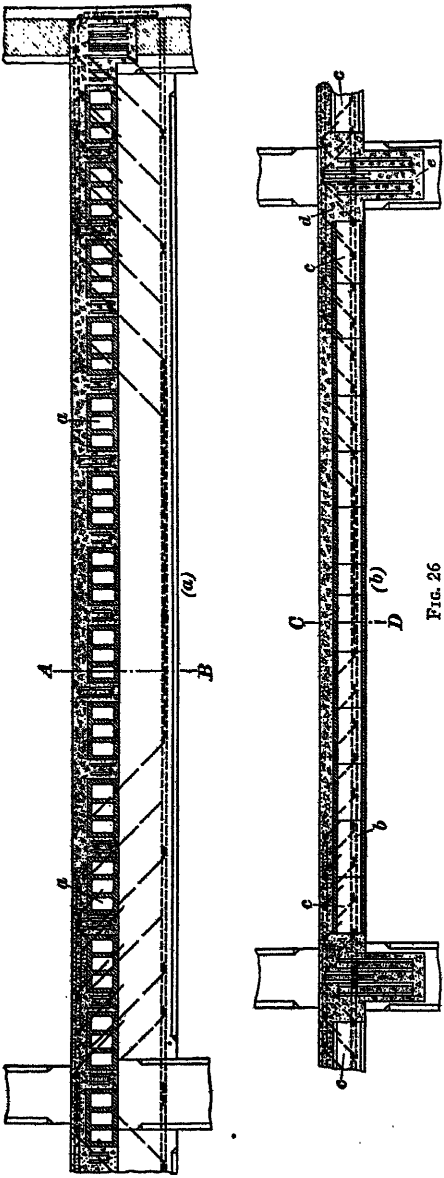


FIG. 26

accordingly by eliminating part of the concrete in the bottom of the slab and using instead rows of terra-cotta hollow tile which are placed between reinforced-concrete ribs. This construction is illustrated in Fig. 26, where the tiles are shown in section at *a* in the cross-section (*a*), which is taken on the line *C D*. A section extending longitudinally on the line *A B* through a row of tiles is shown in (*b*), where the tiles are marked *c* and the reinforcing bars of the ribs are marked *b*.

The hollow tile serves merely as a filler, and in designing the tile-concrete floors, *no dependence whatever is placed upon the strength of the filler, or tile;* in other words, the ribs and slabs are assumed to act in the usual manner as a series of **T** beams side by side. In (*b*) the portion *d*

of the slab next to the girder *e* is made solid in order to furnish the concrete area required for the compression flange in the T-shaped girder.

Tiles may be used not only in connection with parallel girders to form a construction similar to a beam-and-slab floor, but they may also be used in connection with girders running both ways in the building to form a two-way slab construction.

Many building regulations call for a thickness of concrete of not less than 2 inches over the tops of the tiles and a width of ribs, between tiles, of not less than 4 inches. A temperature and shrinkage reinforcement of ⅜-inch round or square rods is often used in the concrete over the tile.

41. In the most common form, the type of floor just described is constructed with **terra-cotta tiles,** which may be obtained in sizes 6 in.$\times$12 in. and 12 in$\times$12 in. in plan, with a depth ranging from 4 inches to 16 inches. **Gypsum hollow blocks** are sometimes used instead of terra cotta; the gypsum blocks are usually from 16 inches to 24 inches wide. Terra-cotta and gypsum blocks are always left in the floor and become a permanent part of the structure; but **steel forms** of shape similar to the tiles may be obtained which serve merely the purpose of molds and are removed after the concrete has properly hardened. Steel forms or "tiles" of another type are designed to be left permanently in the concrete; these forms are usually of very light metal and are provided with metal lath along the bottom for plastering.

Tile-concrete floors are particularly adapted to long-span floor slabs as in school buildings; also to any ceiling requiring plastering, as the plaster adheres well to the tile.

UNIT CONSTRUCTION

42. Separately molded members are sometimes used in constructing reinforced-concrete buildings. The individual members are called *units;* hence the name, *unit construction.*

43. The **advantages** claimed for unit construction are: (1) The cost of elaborate form and false work is saved;

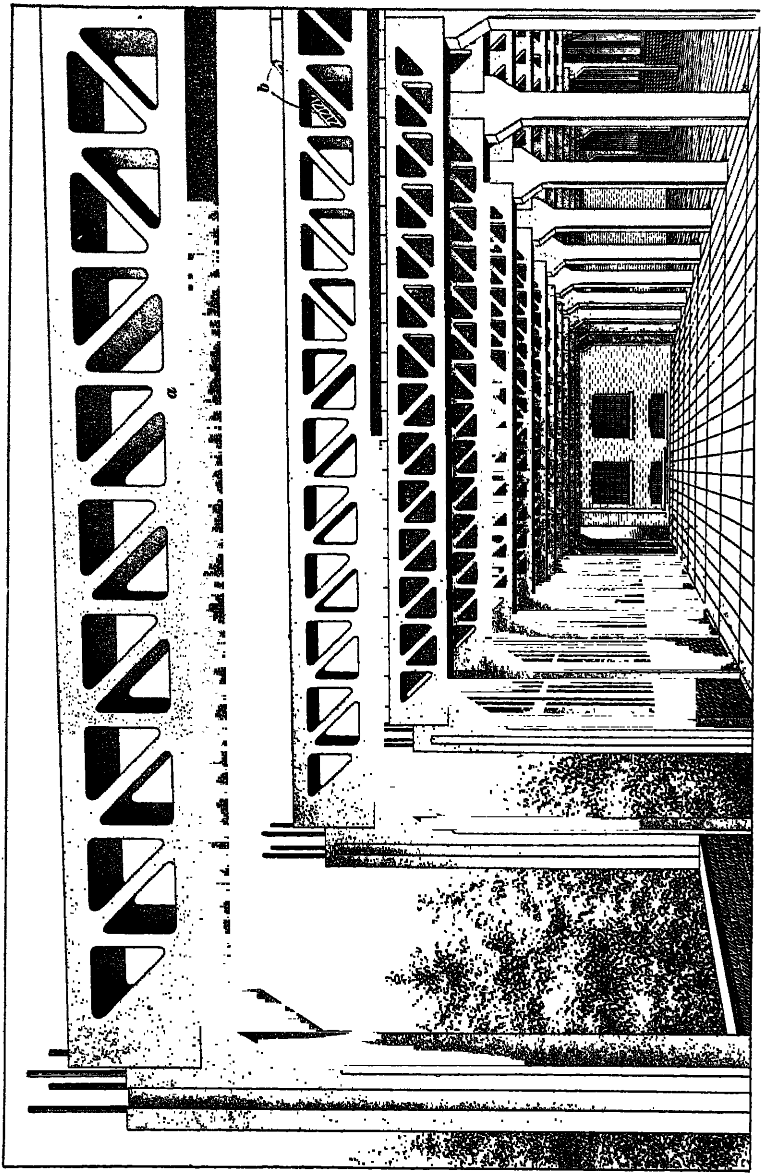


FIG. 27

(2) by building the members in advance of the operation and allowing the necessary time for them to harden properly, the erection may be carried on without interruption; (3) buildings of this type may be erected in winter without danger of frozen concrete, provided the members are previously molded or molded in a shed that is heated; (4) the molded members may be tested, and if found deficient they may be discarded; (5) the forms may be used over and over again with slight alterations, thus saving timber and labor.

The **disadvantages** arise from the large space required for the proper and economical manufacture of the units, from the large investment necessary for machinery used for handling the units, and from the practical difficulty encountered in sealing the joints between pieces.

Unit construction is principally adapted for long low buildings in which there is much repetition of details; and the majority of buildings so far erected in this manner have been of that kind, although some buildings 6 and 8 stories high have been successfully constructed.

44. One of the first attempts at unit construction in this country was a machine shop built on the Visintini system shown in Fig. 27. In this system, the beams and girders are molded separately and then erected in place, the columns being arranged with brackets to receive the beams and girders. Both the beams and the girders consist of reinforced-concrete truss construction with accurately defined top and bottom chords and oblique web members. The girders *a* are necessarily of large section and are heavily reinforced. The beams *b* are placed close together side by side, so as to form the floor slab spanning over the top of the girder on which they rest, cement being poured between the abutting ends in order to seal the joint.

This method is a radical departure from the common practice in reinforced concrete, owing to the use of trusses instead of beams, and although widely used in Europe, has not found extensive use in United States. Other and later unit systems have adopted beam-and-girder types of construction of appearance similar to that of monolithic construction.

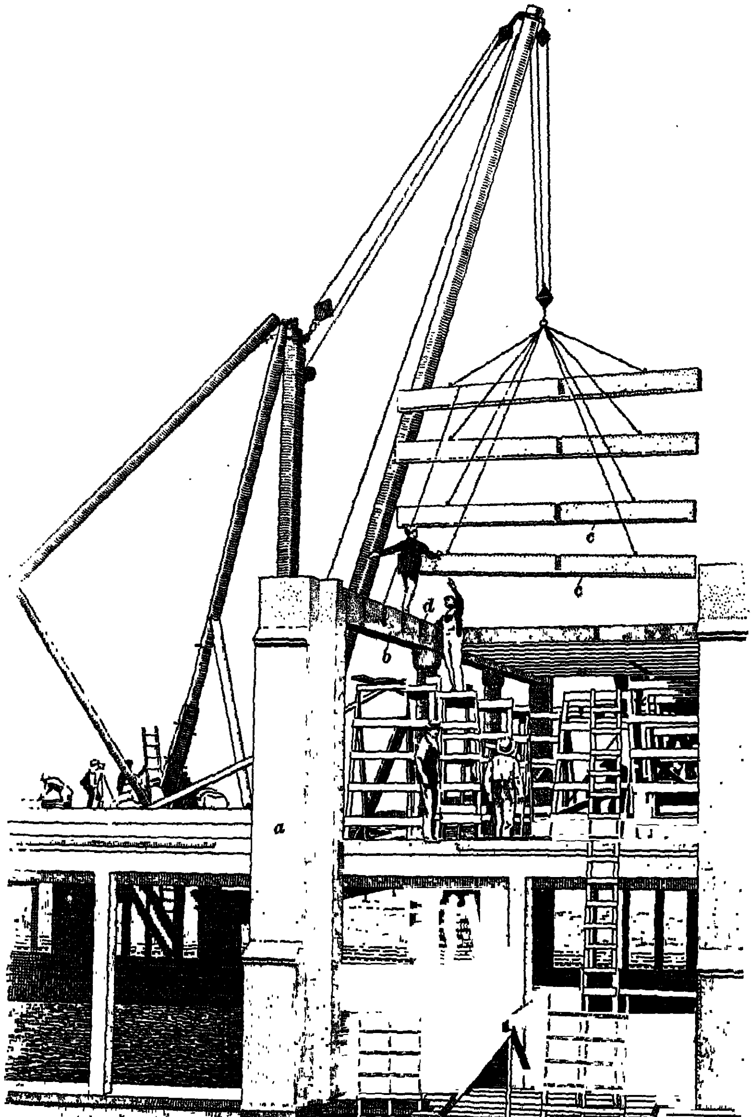


Fig. 28

45. In Fig. 28 is illustrated a method devised by the late Ernest L. Ransome. The large flue columns *a*, which carry

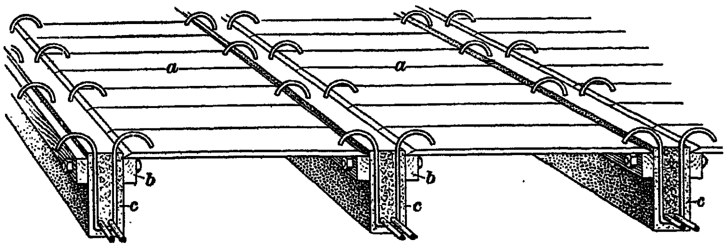


Fig. 29

the hot air ducts to the several floors, and the main girders *b*, are already in place. A number of beams *c* are being lifted by the derrick, all ready to find their seats *d* on the girder *b*. The slab is subsequently cast in place on forms *a*, Fig. 29, which rest on temporary wooden stringers *b*, bolted onto and through the beams *c*.

46. In all unit construction, **tying the units** together is the real problem. In the Ransome method, the tying is easily accomplished because the slab is cast right in place and thus an opportunity is had for embedding additional tie-bars. Thus, in

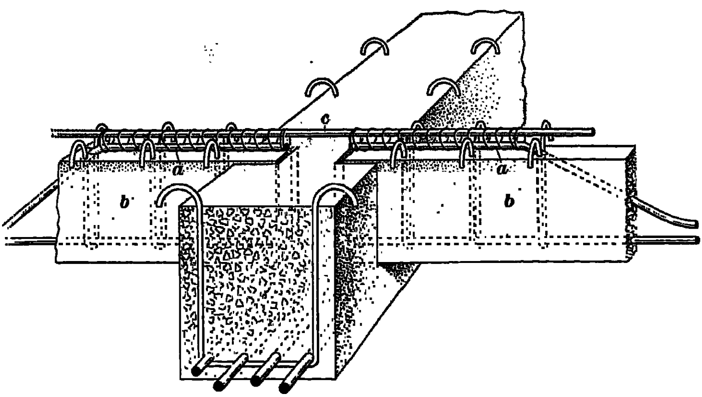


Fig. 30

Fig. 30 the trussed rods *a* of the beams *b* project over the tops of the beams sufficiently to be engaged in the concrete of the

subsequently molded slab, and the connection is effected by additional parallel tie-bars *c* laid loose in the concrete or wired to the main tension rods.

47. An attractive feature of unit construction is the great ease with which **walls** may be built, so as to dispense with practically the entire cost of form work. First, a space of ground is carefully leveled, and the slab which is to form the

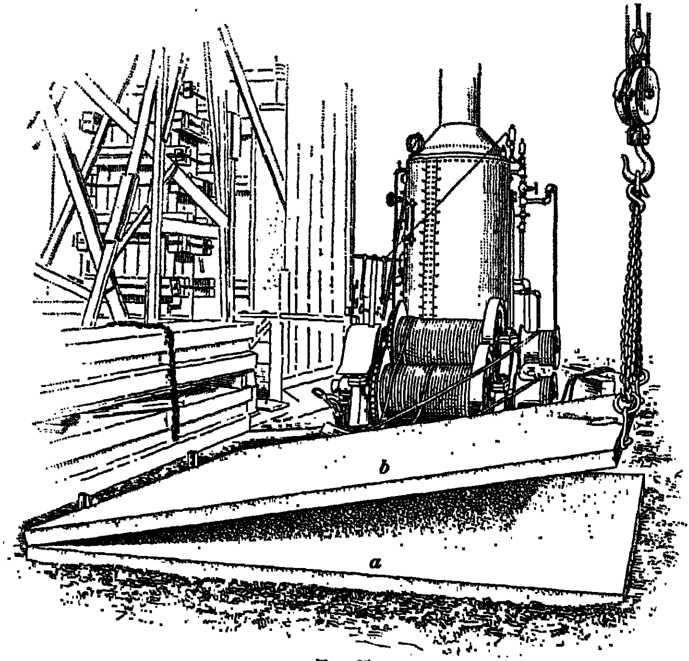


FIG. 31

wall, is cast there in a horizontal position on a bed of sand. The lower slab *a* in Fig. 31 was cast in this manner, and it will be understood that the only form or mold needed is a low box, without bottom, surrounding the concrete, the height of the box being equal to the thickness of the slab, or 4 inches. This slab after hardening is heavily coated with lime whitewash, the surrounding mold or box is moved up 4 inches, and the concrete of the next slab *b* is then deposited right on top of the

first, the whitewash separating the two slabs. This is carried on until a stack of slabs, a dozen slabs deep, has been completed,

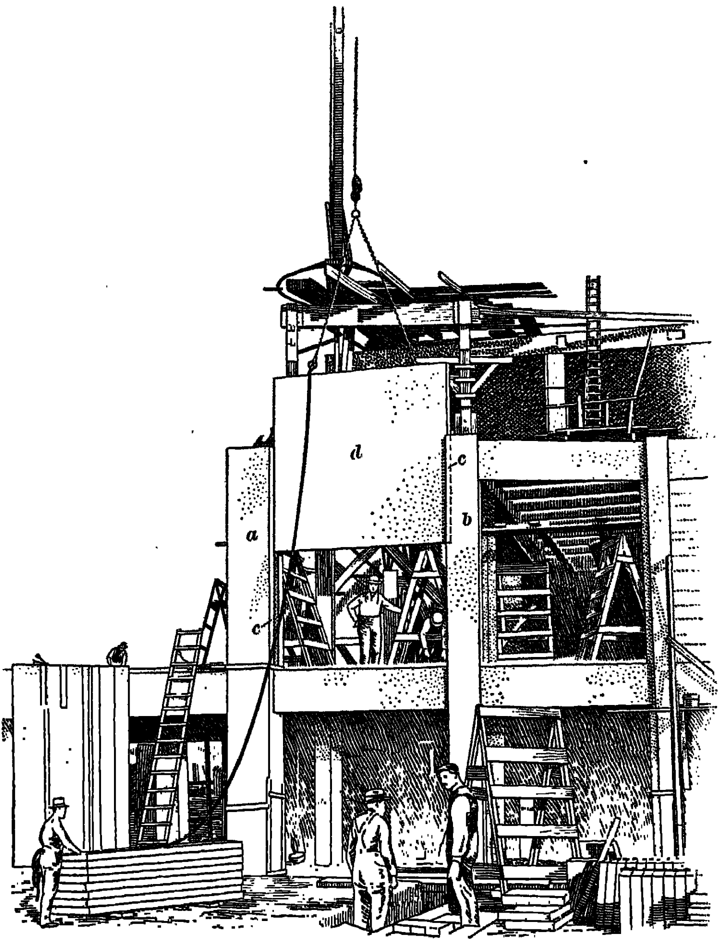


FIG. 32

whereupon the operation may be repeated in some other convenient place.

In Fig. 31, at *b* is shown the last slab but one being removed from a pile of slabs. Fig. 32 shows a slab arriving at its final

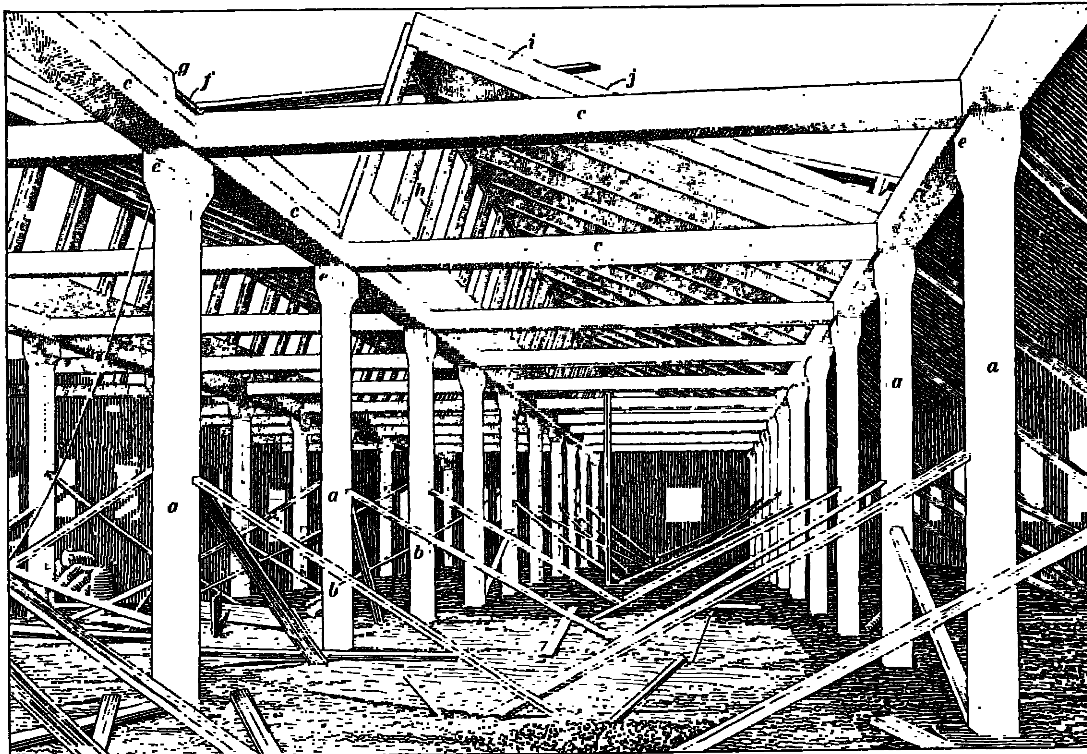


Fig. 33

destination in the wall. In the sides of the two columns *a* and *b* are rabbets *c*, into which the slab *d* is sliding. These rabbets must of necessity be somewhat wider than the thickness of the slab, so that the slab will *slide easily;* the surplus width is pointed up with cement mortar after the slab is in place. In the preparation of plans for unit construction, the following principle must be carefully followed throughout. Rabbets and seats must be plenty large enough, with ample excess clearance; and the units themselves, whether beams or wall panels, must be made of proper length and width, to fit, not snugly, but loosely, into their places. Hence, in unit construction, the office design involves a more thorough knowledge of the field operation than for other methods, and in fact, the two go hand in hand. Every seat, rabbet and tie-rod must be indicated in the detail drawings.

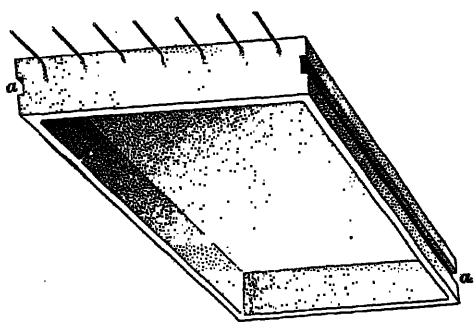


FIG. 34

48. Unit construction can be used to advantage in **saw-tooth roof construction.** Fig. 33 shows an example built by the Unit Construction Company of St. Louis. In this construction, the separately molded columns *a* are first set up and temporarily braced with scantlings *b*, while the roof beams *c* are being placed upon the enlarged column heads *e*. At the ends of the beams, steel ties *f* project so that the several beams *c* can be joined together by pouring cement mortar or concrete into the openings *g*, and this cement when hardened around the rods will prevent the units from tearing apart. Next, the skylight window-frame work *h* is set, and upon it rest the beams *i* of the inclined roof slab *j*. The slabs for roofs and floors are usually cast in advance and set in place by means of derricks in the same way as the beams and girders are erected.

49. The Unit Construction Company has also developed a method of floor construction embodying a **floor unit of box shape,** the sides of the box serving as stiffening ribs. One of these floor units is shown in Fig. 34. Notches *a* extend along the sides of these panels so that, when two of the panels are placed side by side and the interval between them is filled with cement mortar, the mortar in the notches *a* will form keys preventing displacement.

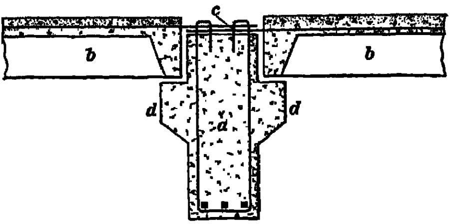


FIG. 35

When erected, these panels *b*, Fig. 35, rest upon ledges *d* projecting from both sides of the girders *a*. The union between the parts *a* and *b* is effected by the projecting steel rods, which meet in the space *c* and are then embedded in cement mortar placed at a later time.

In Fig. 36 is shown the connection over the column. The main girders *a* rest upon the heads *b* of the column *c*, so as to leave an open space *e* between the girder ends. Into this space project steel rods from the girders *a* and column *c*, as well as the rods extending downward from column *d* of the story above. The space *e* is filled with mortar or cement and a firm joint is obtained by the hardening of the concrete around the projecting rods.

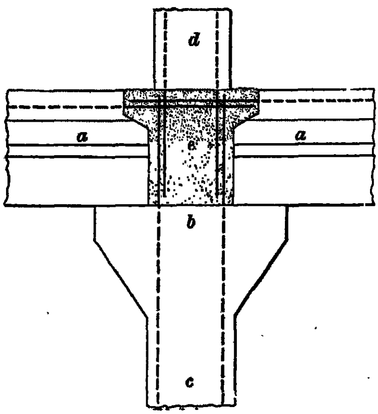


FIG. 36

50. In the methods mentioned in the foregoing articles, the separately molded members have all been designed especially for the building in which they were erected. How-

ever, for buildings having light loads and simple construction, such as school houses and apartments with brick walls, it is possible to standardize the design of the units, and such units of **standardized design** are now frequently manufactured in permanent stationary plants. The advantages in producing concrete units by factory methods are several, the chief advantage being that the many contingencies of reinforced-concrete construction as usually practiced are eliminated by factory work. Many engineers believe that the future of unit construction will involve the construction of permanent plants

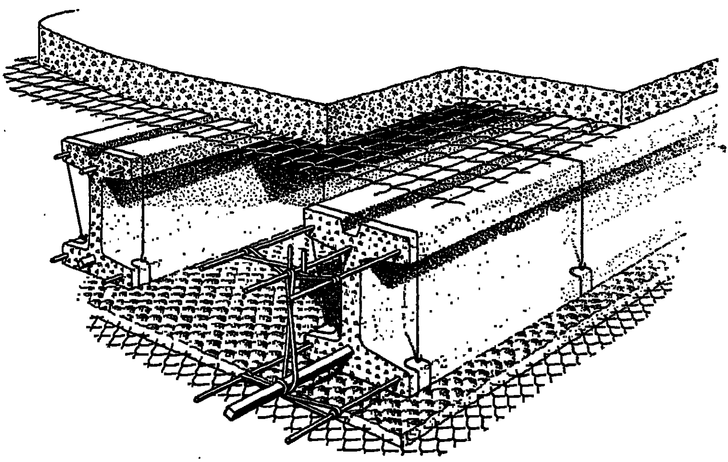


Fig. 37

of large capacity in the industrial centers of the country, where a reinforced-concrete building may be bought ready made except for assembling. There exists already a number of smaller plants engaged in the manufacture of beams.

51. The I beams shown in Fig. 37 are manufactured in a stationary plant by the R-C Products Trust, in Cleveland, Ohio. Usually steel ties project from the top of the beams so that a subsequently molded slab will adhere to the beam. The I beams may be set closely together, in which case the bottom flanges of the I beams form a flat ceiling, or they may have an open interval between the flanges, in which case a light form work is required for the floor slab. The ceiling formed by the

beams is often covered with plaster on metal lath as indicated in Fig. 37. The beams are usually supported on brick walls or on steel beams; less frequently on concrete girders.

52. Common to all unit systems is the method of **setting the units** on prepared beds of fresh mortar. Other methods

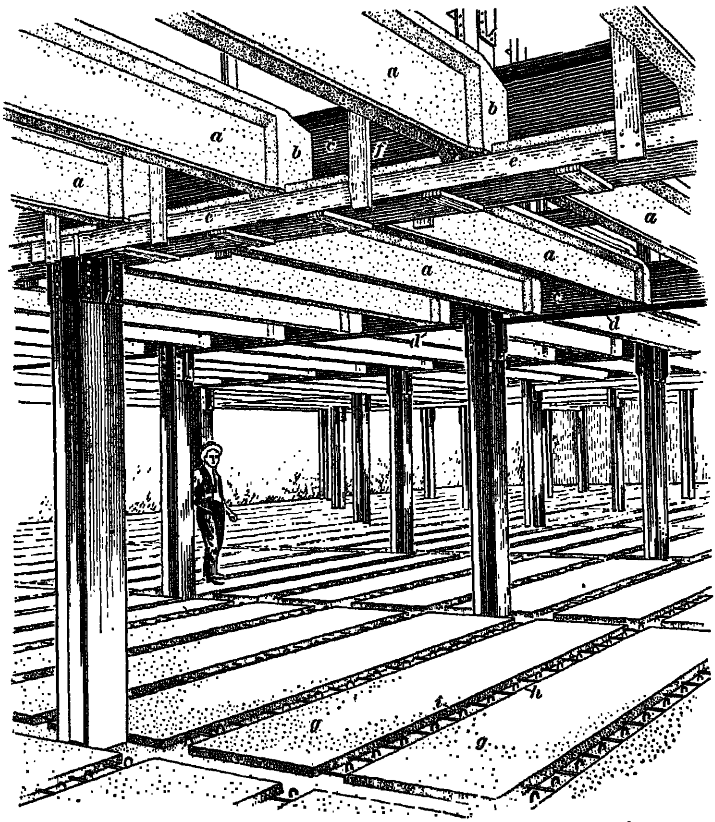


Fig. 38

of setting the units occasionally used for special places involve the use of ducts or openings in the units through which grout is poured into cavities designed for the purpose. But this method is open to the serious objection that there is no certainty that the cavity has been filled entirely.

53. Where the skeleton of the building is constructed of structural-steel members, the floors may be built of reinforced-concrete units according to the design developed by Chas. D. Watson.

In this construction, shown in Fig. 38, the unit beams *a* have enlarged ends *b* that rest upon the steel girders *c*. The lower flanges *d* of the steel girders are fireproofed with concrete cast in place, and the molds *e* used for this purpose may be seen surrounding the lower part of one of the girders. The remaining exposed steel areas are to be fireproofed with precast concrete blocks, but these are not indicated in the illustration.

Concrete slabs *g* are shown in the floor construction covering the spaces between the concrete beams that support the floor. Directly over the tops of the beams, spaces *h* between the slabs *g* are purposely left open to receive the cement grout which is intended to flow around the rods *i* projecting from slab and beams; and the grout when hardened will tie the slabs to the beams and each slab to its neighbor.

CONCRETE SLABS ON STEEL BEAMS

54. Structural Details.—In tall buildings the frame work supporting the loads is commonly built of structural-

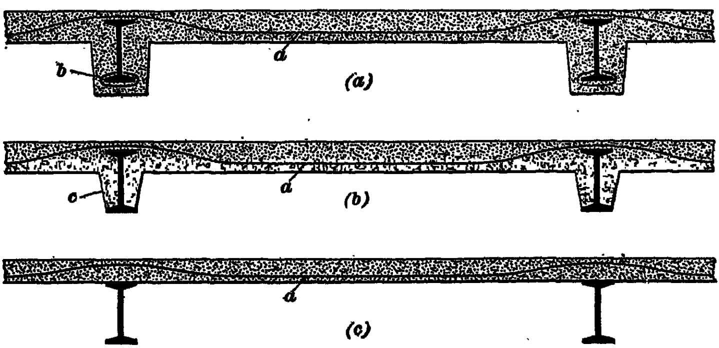
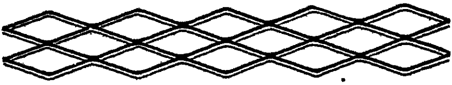


FIG. 39

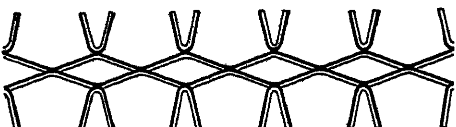
steel shapes with concrete slabs serving for floors. These slabs are supported on I beams of structural steel spaced usually

from 5 to 10 feet apart as shown in Fig. 39. According to the purpose for which the building is intended, the structural-steel work may be entirely encased in a fireproofing of concrete, as in (*a*), or partly encased as in (*b*), or entirely exposed as in (*c*).

The three types, (*a*), (*b*), and (*c*), are equally strong, but that shown in (*a*) is the only one that can be considered fireproof. In the construction indicated in (*b*), the web of the I beam is fire-proofed with concrete *c*, called the haunch, but the lower flange is left exposed. In order fully to protect the steel work against fire, it is necessary not only to cover the steel entirely, but also to surround the steel shapes with wire in order to bind the concrete to the steel shape. This may be accomplished by wrapping the beams or columns with wire or wire netting or expanded metal; or merely the lower flange of the I beam may be wrapped, as at *b* in (*a*). Special strips of wire mesh or expanded metal may be obtained for wrapping the lower flange. The so-called *Steelcrete* beam wrapper is made from expanded metal strips two diamonds wide and 6 feet long. In Fig. 40 (*a*) is shown the original strip. This strip is first cut at each diamond, as shown in (*b*), and the strands are then pulled out, as shown in (*c*). In (*d*) is shown the strip as it is in place wrapped around the lower flange of the steel beam.


(a)


(b)


(c)

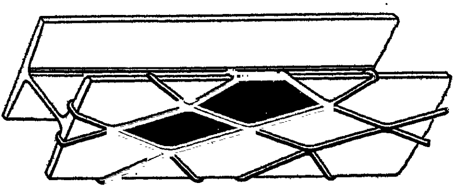

(d)

FIG. 40

The reinforcement of the slabs of Fig. 39 consists of wire mesh *a* raised to near the top of the slab over the I beams, and dropped to near the bottom of the slab midway between beams.

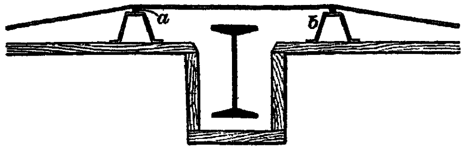


FIG. 41

Where the spans are long and the loads heavy, wire mesh may not be sufficient to resist the stresses. In this case round or square rods are then used in addition to the mesh in order to increase the cross-sectional area of steel. Light rods as shown at *a* in Fig. 41 are sometimes used to support the wire mesh in its proper place over the I beams, and the rods *a* are in turn supported on the chairs *b*.

55. Where **head room is limited,** the slab may be lowered by bringing the top of the slab to the same level as

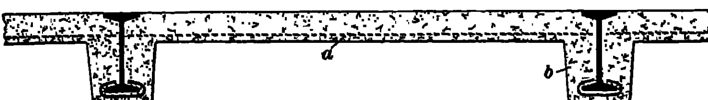


FIG. 42

the top of the I beams, as in Fig. 42. In this construction the I beams separate adjacent spans of slabs and the reinforcement *a* is therefore not continuous, but extends merely from support to support; the slab is usually supported upon the haunches *b*, but sometimes the haunches are dispensed with, and the slab is supported upon angle irons *a*, Fig. 43, riveted to the webs of the I beams. This construction, because of its light weight, is quite common for roofs of manufacturing

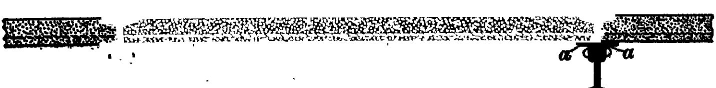


FIG. 43

buildings where there is no fire danger; where there is danger from fire the entire I beam must be encased in the concrete as previously explained.

56. A type of construction presenting a **flat ceiling** is shown in Fig. 44. The reinforced-concrete slab *a* is supported on the bottom flanges of the I beams, and the reinforcement *b* of the slab is curved up into the concrete *c* surrounding the web of the I beam, in order to insure a sufficient anchorage for the reinforcement. Where a perfectly fireproof floor is wanted, beam wrappers *d* are used for the lower flange of the I beams, as previously explained, and the top flange of the I beam is encased in concrete as at *e*. The structural concrete work upon completion presents the appearance of long inverted troughs; in order to obtain a level foundation upon which the floor finish may be placed, the troughs are filled with cinders or cinder concrete *f*. The cinder concrete also serves as a sound insulation.

57. As explained in *Elements of Steel Reinforcement,* **self-centering types of reinforcement** are sometimes

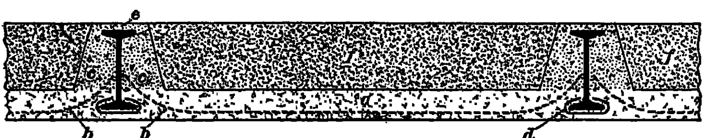


FIG. 44

used. Examples of their application are shown in Fig. 45 (*a*) and (*b*). In (*a*) is illustrated an arched floor, in (*b*) a flat slab curved at the haunches. The self-centering reinforcement *a* is placed with its ends *b* resting upon the lower flange of the I beams, and concrete is deposited, first at the crown *c* of the arch and then simultaneously toward both sides, as otherwise the concrete might squeeze the ends of the sheets away from the I beams and precipitate the concrete and forms into the story below. After the concrete has hardened, the plates *a* must be plastered on the under side, for which purpose a Portland-cement plaster should be used, composed of one part Portland cement to two parts of coarse, sharp sand. It is very difficult to plaster with a mixture of this kind, and the practical plasterer prefers to handle a mixture containing lime. The addition to the 1 : 2 cement mortar of an amount of hydrated lime equal to 10 per cent. by weight of the cement will make the plaster

flow more easily without detracting from the strength of the mortar.

58. Floor arches of the type shown in Fig. 45 (*a*) and (*b*) are often constructed without any reinforcement in the top of the slab, although both practical and theoretical considerations

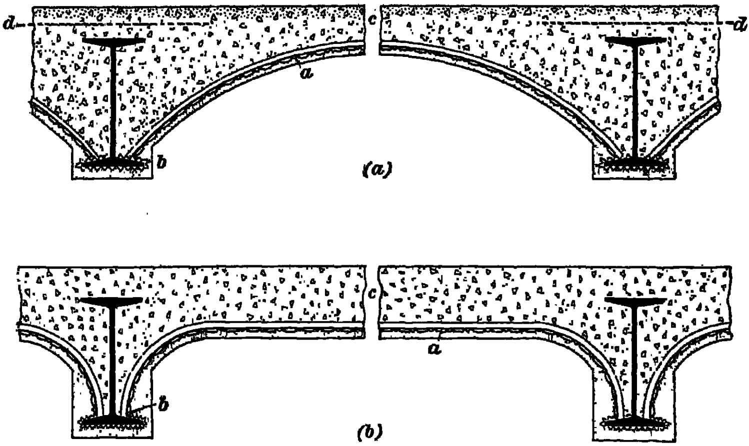


FIG. 45

dictate the use of a top reinforcement over the supporting I beams, as shown at *d* in (*a*). The practical experience with such slabs is that they are likely to crack over the tops of the I beams, and the theory of these slabs indicates the presence of tensional stress over the I beams. The reinforcement *d* should therefore not be omitted.

FLAT-SLAB FLOORS

59. In Fig. 46 is shown the under side of a flat-slab floor, which consists of a slab *a* extending continuously over columns *b*. It is distinguished from the previously described ribbed floors by having no beams or girders. The concrete slab, reinforced by a net work of steel rods, transfers its load directly to each column through the medium of the column capital *c*. The absence of ribs or other obstructions underneath the slab has made this type of floor equally popular with owners and contractors. The owners are pleased with the

ideal lighting and ventilation obtainable, and with the high unobstructed storage space available. The contractor finds this construction desirable because of the simplicity of the form work, the forms being repeated practically unchanged from floor to floor and even from building to building.

There are many different types of flat-slab floors in use, differing mainly in the arrangement of the reinforcement.

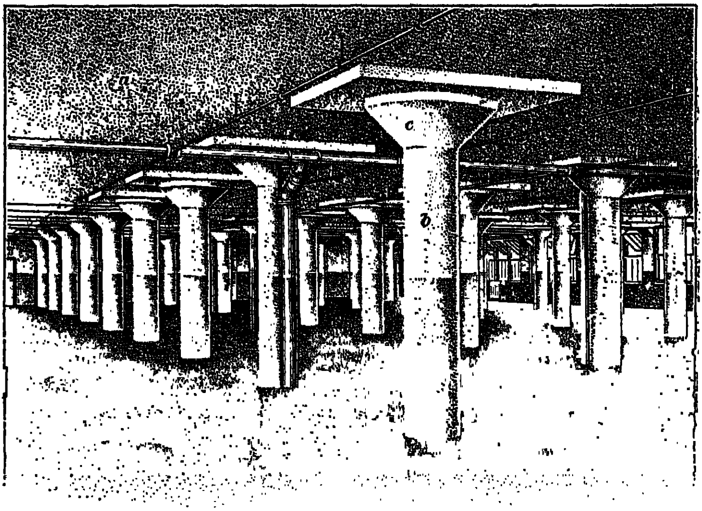


FIG. 46

According to the arrangement of the reinforcement, the flat-slab floors may be classified as *two-way, four-way, circular,* and *combined belt and circular* types. This classification is, however, not rigidly adhered to in the following, where a description is first given of the so-called mushroom system, because this, although a combination type, was the first system introduced commercially.

FLAT-SLAB FLOOR SYSTEMS

60. The introduction of the flat slab on a commercial scale was made by C. A. P. Turner with his **mushroom system** shown in Fig. 47. In this system, radial rods *a* extending from the columns, and concentric rings *b* circling the columns, com-

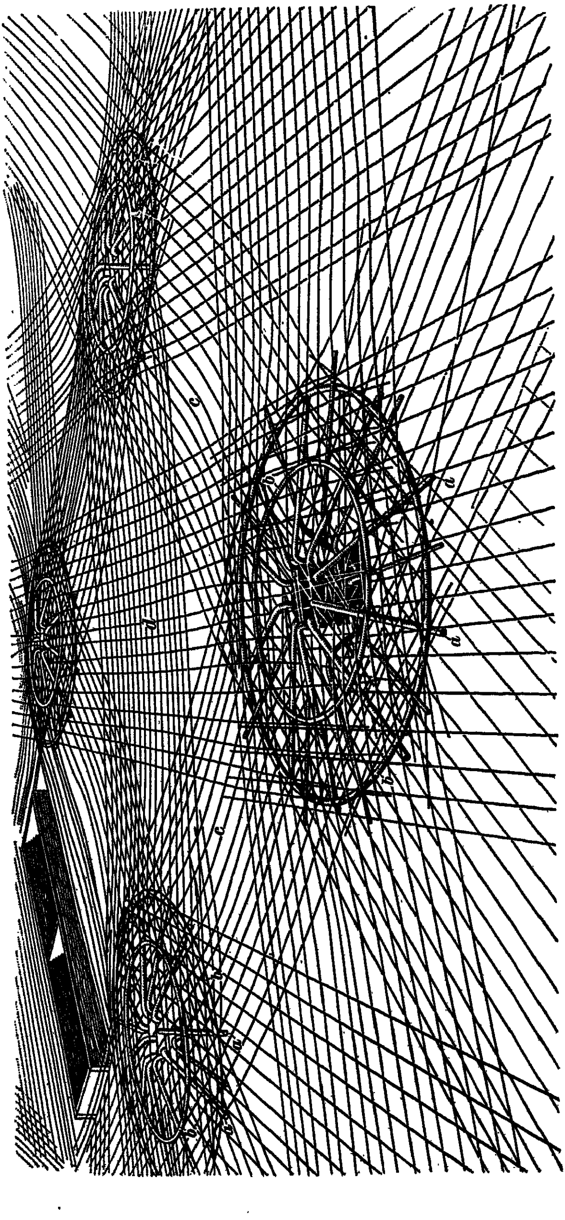


Fig. 47

bine to form a column head having the shape of a mushroom, to which circumstances the system owes its name. From column head to column head, extend belts of lighter steel rods; some of these belts, called *direct belts*, span the short way from column to column, as shown at *c*; other belts *d*, called *diagonal belts*, span diagonally from column to column. The width of

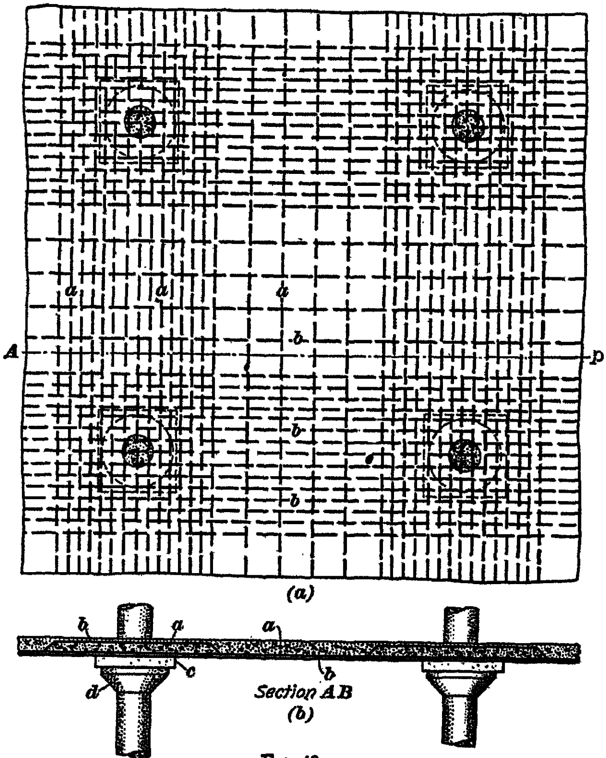


FIG. 48

the belts is so arranged that together, the belts cover the entire floor surface. Midway between columns, the belt reinforcement is near the bottom of the slab while over the columns, the belt reinforcement reaches nearly to the top of the slab.

61. The **two-way type**, shown in Fig. 48, is represented by the *Corrplate* floor of the Corrugated Bar Company, and the

Akme system of the Condron Company. In these floors the reinforcement extends in only two directions, namely, lengthwise in the building as at *a*, and crosswise as at *b*, without diagonal belts. Tension rods thus disposed in two belts at right angles can exert resistance in any direction and in reality reinforce the slab as effectively as would a thin plate of the same weight as the total of all the rods used in the slab.

62. The two main belts of reinforcing rods *a* and *b* in Fig. 48 extend over the column head as shown in the plan

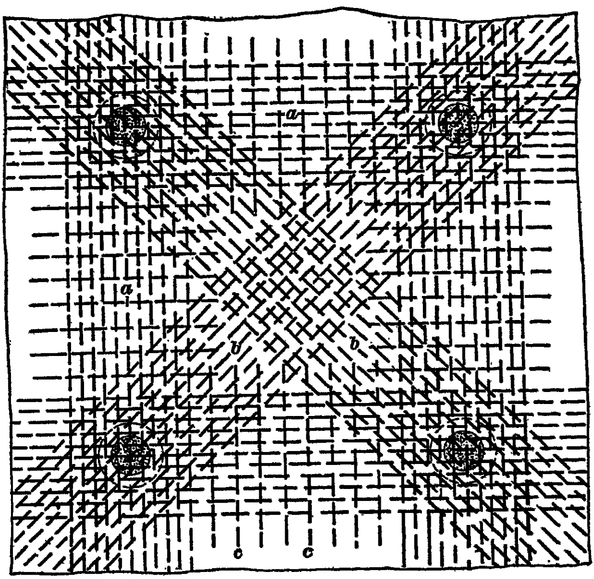


FIG. 49

view (*a*). Some of these rods are bent up at about one-quarter of the span from the column to extend into the top of the slab over the column as shown at *b* in the cross-sectional elevation (*b*), while other rods are continued straight into the adjacent panel. At the central portion of the slab the reinforcing rods *a b* are spaced farther apart.

The slab may or may not have a thicker portion *c* called a *drop*, over each column; in either case the top of the column is flared into a column cap *d*, as shown in Fig. 48 (*b*).

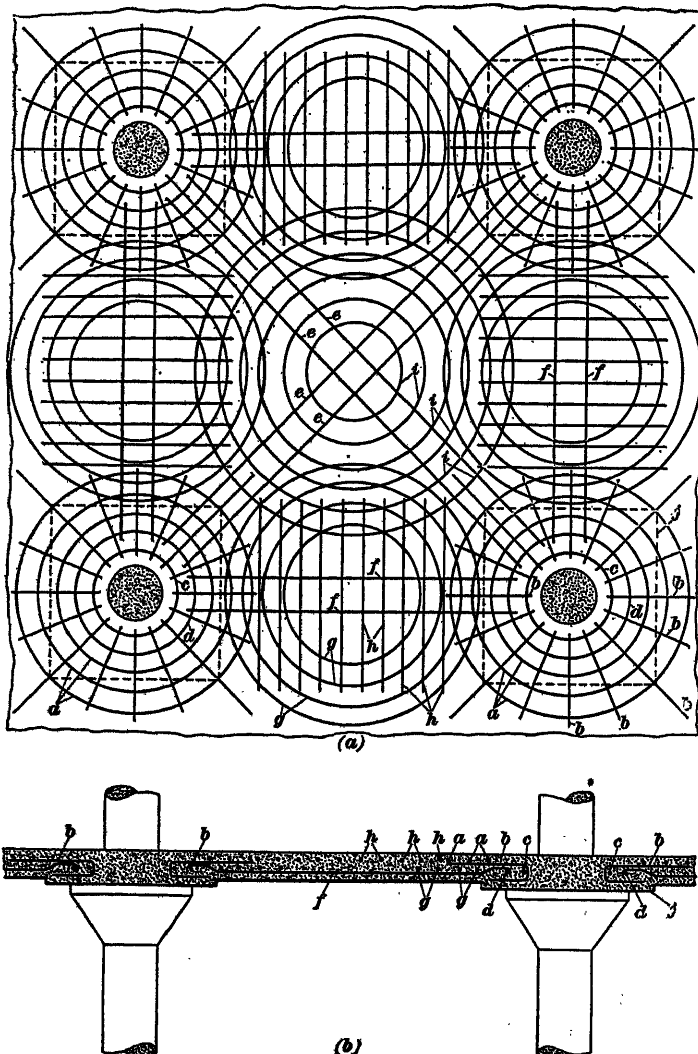


FIG. 50

The drop may be used with any system of reinforcement when a greater thickness of slab is desired near the column head than at the center of the panel.

63. The **four-way type,** shown in Fig. 49, is similar to the mushroom type of floor, except that no radial rods or concentric rings are used. Both the direct and the diagonal

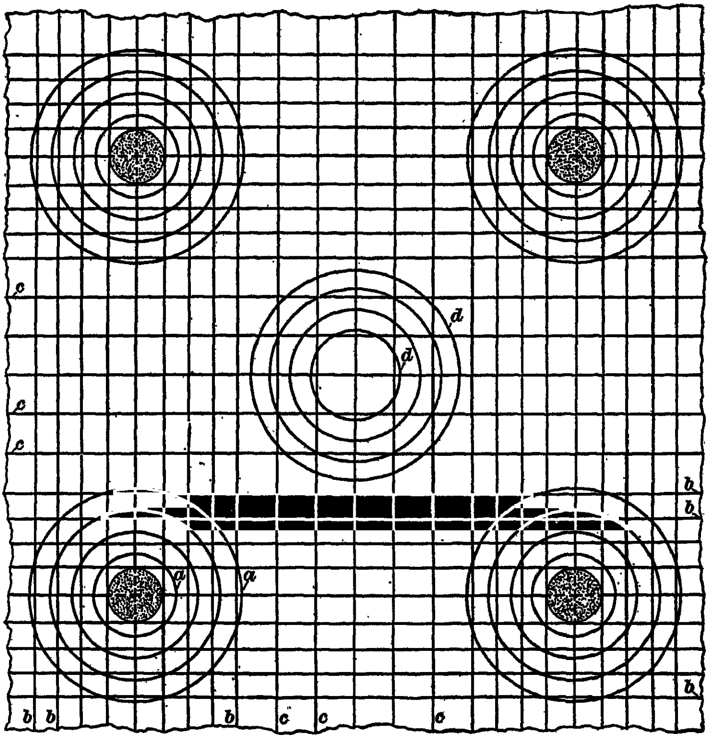


FIG. 51

belts are usually bent up over the column head, but some authorities maintain that the better construction is to have the direct belts *a* straight and near the bottom of the slab for their entire length, and to bend merely the rods of the diagonal belts *b*. Rods *c* are sometimes used near the top of the slab midway between columns; these rods should cross the direct belts at right angles.

64. Circular reinforcement is the invention of Edward Smulski. As shown in the plan view, Fig. 50 (*a*), and in the sectional elevation (*b*), the columns are surrounded by concentric rings *a* resting upon radial rods *b*. In addition, there are two anchor rings *c* and *d*. The radial rods *b* are looped around the anchor ring *c*, and the anchor ring *d* connects the hooked ends of the truss rods *e* and *f*. Upon the direct truss rods *f* rest four concentric reinforcing rings *g*, which serve as reinforcement for the bottom of the slab, while straight rods *h* reinforce the top of the slab. The central portion of the panel is reinforced by means of concentric rings *i* disposed near the bottom of the slab.

In the illustration is also indicated a square drop *j*, shown in section in (*b*) and represented by dotted lines in (*a*).

65. Combined belt and circular reinforcement, shown in Fig. 51, is a development by C. A. P. Turner. In

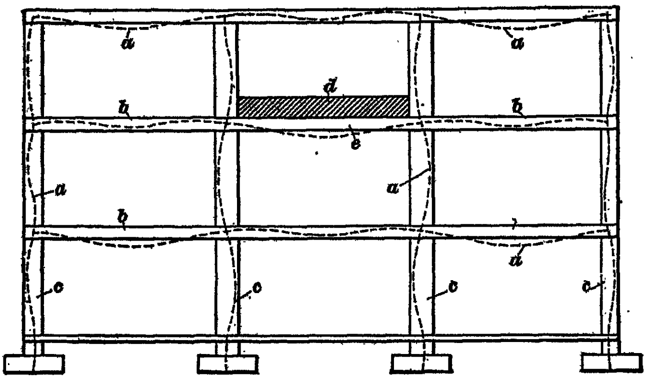


FIG. 52

this construction, concentric rings *a* surround the columns and furnish the necessary tensile resistance over the column heads in connection with the raised ends of the two-way belt rods *b*. Rods *c*, parallel with the side belts, span the central portion of each panel and in conjunction with concentric rings *d* serve as tension reinforcement at the center of the panel.

66. Diameter of Columns.—The various tests on flat-slab floors have shown that the columns take part in the gen-

eral deflection of the floor slab with which they are united, and that under certain loading the columns bend slightly sidewise midway between floor and ceiling. This condition is illustrated in Fig. 52, where the dotted lines *a* show, greatly exaggerated, the deflection of the adjoining floor panels *b* and the columns *c* due to the test load *d* in the panel *e*. The columns should therefore have a diameter large enough to resist the bending tendency in addition to the direct load which they are required to carry.

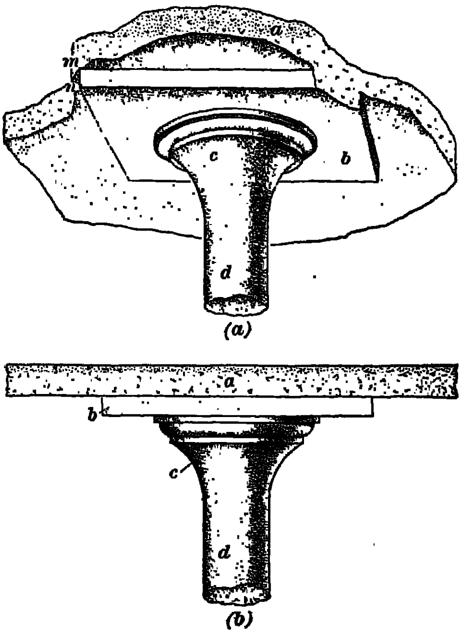


FIG. 53

The Chicago building code, for example, prescribes a minimum column diameter of one-twelfth of the floor span, or one-twelfth of the clear height of the column. Whichever of these is the largest is to be adopted. Thus, for 20-foot spans the columns must be at least 20 inches in diameter, regardless of how small the load may be, provided the clear height of the column does not exceed 20 feet. But if for the same 20-foot spans the clear height was 24 feet then a column diameter of 24 inches would have to be used.

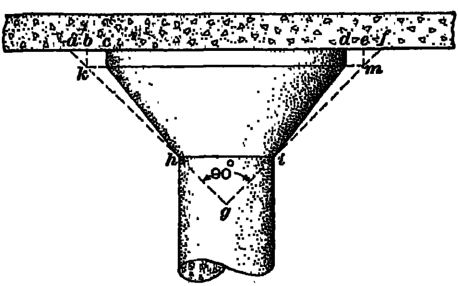


FIG. 54

67. Column Capital.—In all flat floor systems, the columns are generally flared out at the top as illustrated in Fig. 53, where *a* is the floor slab, *d* the column shaft, and *c* the flaring capital. In many cases, the capital is surrounded by the **drop** *b*.

According to the Chicago building code, the column capital must not spread out at too flat an angle. The slope of the column capital, such as *a g*, Fig. 54, must nowhere make an angle of more than 45° with the vertical. In other words, the column capital *c h i d* must always be within the cone *a g f* with angle $agf = 90°$. The diameter *c d* of a column capital is usually between one-fourth and one-fifth of the distance between centers of columns. According to the Chicago building code this diameter must not be less than 0.225 of the distance between centers of columns, and the thickness *e m* must be at least 1½ inches.

When used the drop, which is sometimes called the *drop panel,* is made either square or circular for square panels of the main slab, and rectangular or elliptical for oblong panels. The length of the drop, according to the Chicago building code, must not be less than one-third the distance between centers of columns.

REINFORCED-CONCRETE BUILDINGS

(PART 2)

DETAILS

LINTEL CONSTRUCTION

1. Influence of Framing Plans.—In the planning of reinforced-concrete building construction, one of the most important features is the design of the *lintel* construction. A **lintel,** shown at *a* in Fig. 1, is a cap or beam over an opening for a door or window, and it supports the *spandrel wall b,* which is the part of the wall between the lintel and the sill of the next story above. If the bays are narrow and the distance between the jambs of the window openings is small, so that each beam can have a bearing upon a wall pier, as indicated in Fig. 1, then the floor slab can span from beam to beam, and the only load that comes upon the lintel *a* is that due to the weight of the spandrel wall *b*.

Ordinarily, however, in order to obtain the large amount of window space desired in modern commercial buildings, it is necessary to have twin or triple windows and to make the distance between piers from 16 to 20 feet or even greater. If this plan of construction is followed, two methods can be employed, as indicated in Fig. 2. In the construction shown in (*a*), one-half of the floor load from the beams *a* and *b* is concentrated at two points upon the lintel, and the lintel has to carry this load in addition to the weight of the spandrel wall. Also, as the

beams *a* and *b* extend into the lintel, the depth of lintel should be at least equal to that of the beams, although a smaller lintel would actually carry the load. Thus the height of the window opening is materially reduced and part of its lighting and ventilating efficiency is consequently destroyed.

2. The second method of framing the floor to obtain the greatest amount of window surface with reference to piers and lintel construction, illustrated in Fig. 2 (*b*), is frequently

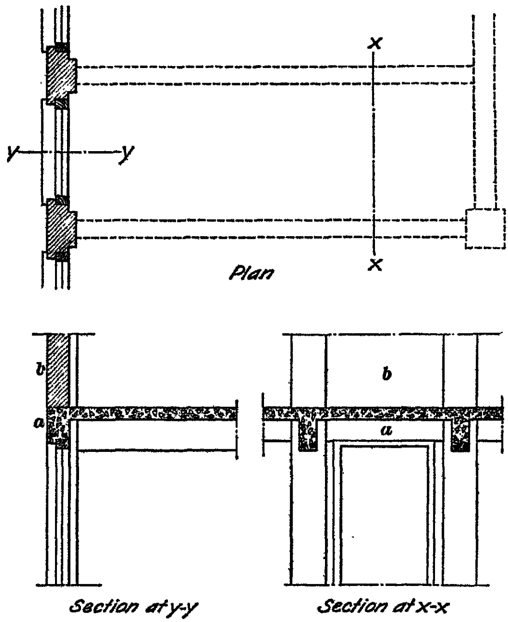


Fig. 1

employed. The girders, instead of extending from column to column as shown at *c* in view (*a*), extend from column to wall pier; consequently the beams extend in a direction parallel with the lintel.

A study of Fig. 2 (*b*) will show that the slab spans from the beam *a* to the lintel *b* and that as a result the lintel carries one-half the slab load between these structural members as well as

the weight of the spandrel wall. The advantage gained by this method of construction arises from the fact that there are no beams abutting the lintel. Thus, this member may be reduced in depth to the minimum required to support the small floor load and the weight of the spandrel wall. Some of the light entering the window will, however, be intercepted by the first

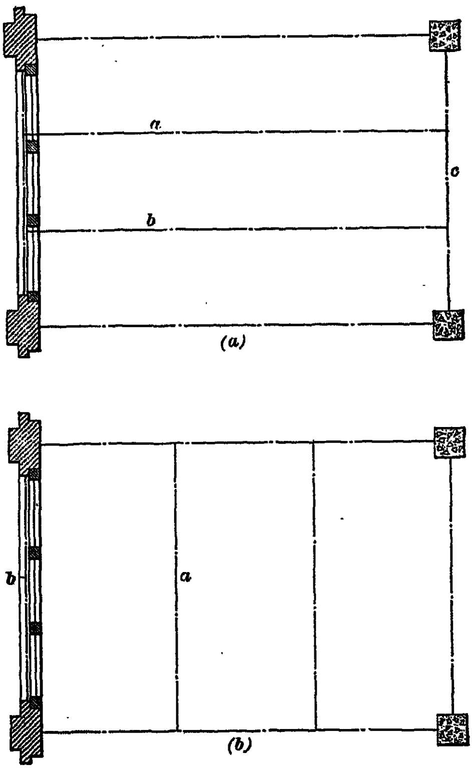


Fig. 2

cross-beam *a*, which will also interfere with free ventilation from the interior. One of the advantageous features of the flat slab floors described in Part 1 is that there are no cross-beams to intercept the light, and also that a shallow lintel can be used. In some cases the lintels have been dispensed with altogether.

3. Construction of Arched Openings in Brickwork. Frequently, with brick walls and piers, the opening in the brickwork over the window head is arched, as shown in Fig. 3. In this case, the reinforced-concrete slab is made to span from girder to girder, with its reinforcement parallel to the arch span. It therefore needs no support over the arch head, and for this reason the soffit of the arch, that is, the under side of the arch, can come flush with the under side of the slab. In Fig. 3 the slab construction is shown at *a* and the brick arch at *b*. Although the soffit of the arch is flush at its highest point with the under surface of

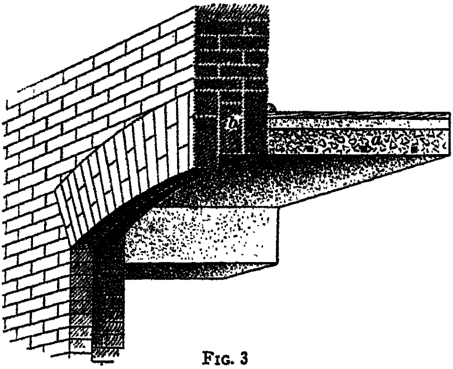


FIG. 3

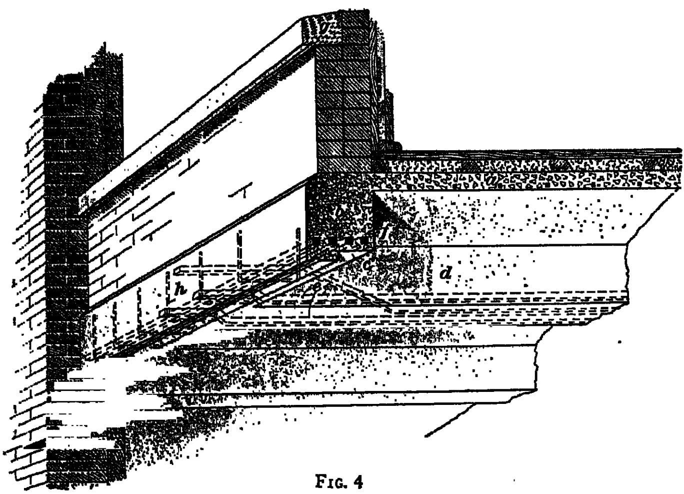


FIG. 4

the floor slab, it will be observed that at either side of this point some light is cut off by the spandrels of the arch, thus reducing the available daylight opening.

4. Concrete Lintels in Brickwork.—In Fig. 4 is shown a method of forming a concrete lintel over a side window opening where the beams extend from side to side of the building, as in Fig. 2 (*a*). Here the slab spans in a direction parallel with the wall in which the windows occur, but the beams concentrate part of the slab loads on the lintel. As shown in Fig. 4, the reinforced - concrete slab *a* is monolithic with the lintel *b*, which is rabbeted in the soffit (that is, a notch, or **rabbet,** is formed on the under side) at *c* to receive the window frame. The floorbeams, or secondary members of the construction, as at *d*, enter the lintel so that the soffits of the beams are flush with the soffit of the lintel. Where the beams are framed into the lintel in this manner, it is necessary to raise the reinforcing rods *e* of the beams upwards, so that they will have a bearing above the reinforcing rods *f* of the lintel.

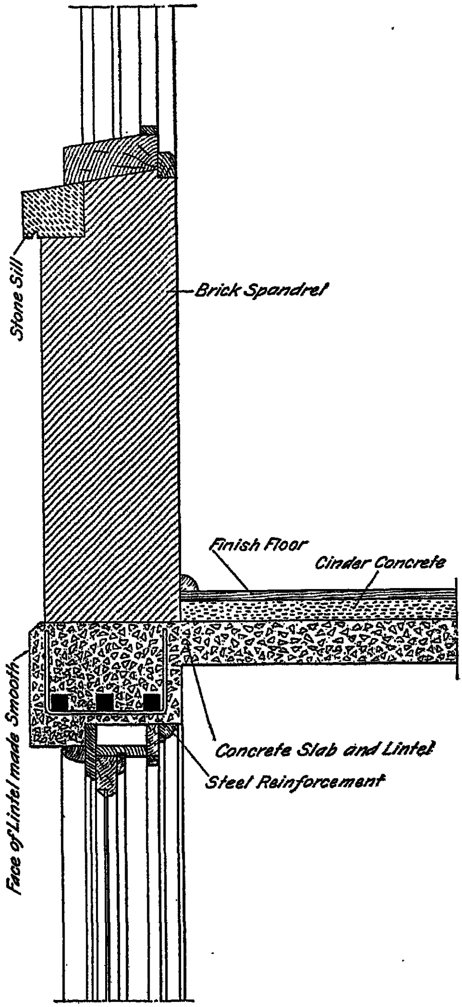


FIG. 5

The spandrel wall in constructions of this kind may be built up of brick, capped with a stone sill *g*, and arranged to receive the window frame. Where the beam rods overlap the lintel rods, it is also advisable to provide additional stirrups *h* in the lintel.

5. When the framing plan shown in Fig. 2 (*b*) is used, the lintel and slab construction shown in Fig. 5 is usually employed, in which the window head is as near the bottom of the slab as possible. In this case, also, the slab is monolithic

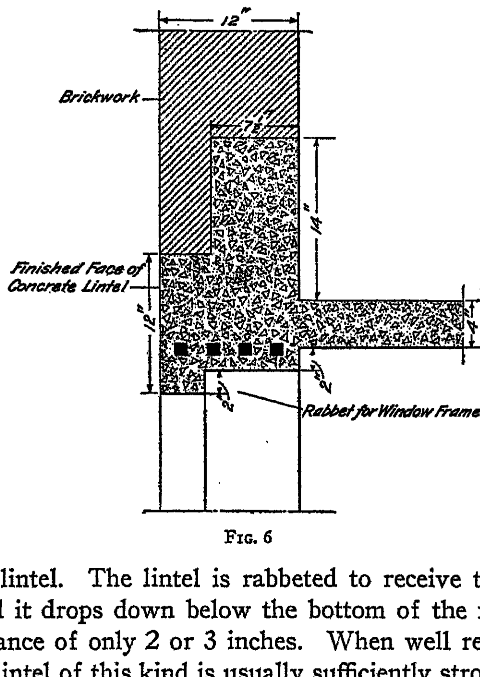


FIG. 6

with the lintel. The lintel is rabbeted to receive the window head, and it drops down below the bottom of the floor slab a total distance of only 2 or 3 inches. When well reinforced, a concrete lintel of this kind is usually sufficiently strong to carry the load of a portion of the floor slab and the weight of the spandrel. In this construction, part of the floor slab is assisting in taking the compressional stresses in the lintel.

The construction shown in Fig. 5 will be found to be too weak where the distance between piers is greater than 8 or 9 feet, and where the floor is designed to carry heavy live loads.

In order, therefore, to obtain the necessary strength, the lintel beam is increased in depth, being extended above the floor slab, as shown in Fig. 6. A lintel constructed in this manner loses the advantage gained by having the adjoining part of the slab act in compression at the upper section of the lintel, as the slab section occupies a position near the neutral axis of the lintel, and the resistance to compression of the slab is lost. The lintel must therefore have its section extended upwards to an extent sufficient to give it the needed compressive strength.

Some difficulty is encountered in forming a lintel of this kind if the material is very plastic, because such material will ooze out from underneath the form board that forms the projection of the lintel above the slab. Such difficulty, however, can be avoided by using a semidry mixture.

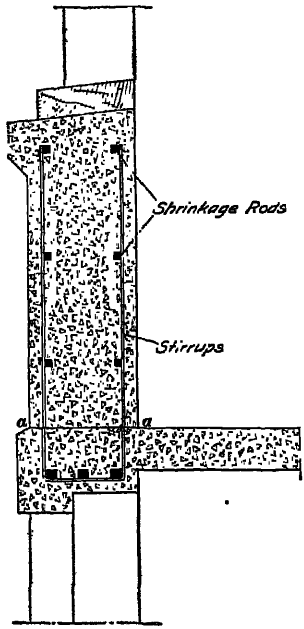


FIG. 7

6. Concrete Lintel and Spandrel.—If the span between piers is great and the floor loads heavy, as is usually the case in warehouse construction, it may be necessary, where the soffit of the lintel is brought close to the under side of the slab, to form the entire spandrel of reinforced concrete, as shown in Fig. 7.

In constructions of this kind, it will be very difficult to form the spandrel monolithic with the floor slab and lintel. The lintel shown in Fig. 7 is designed so that its junction with the panel of the spandrel may be made on the line *a a*, and the spandrel constructed afterwards. The junction of the work is hidden by the projection of the lintel beyond the face of the spandrel, and the projection finishes on top with a bevel.

When this construction is used, the lintel may be designed in two different ways. It may be assumed that only the part

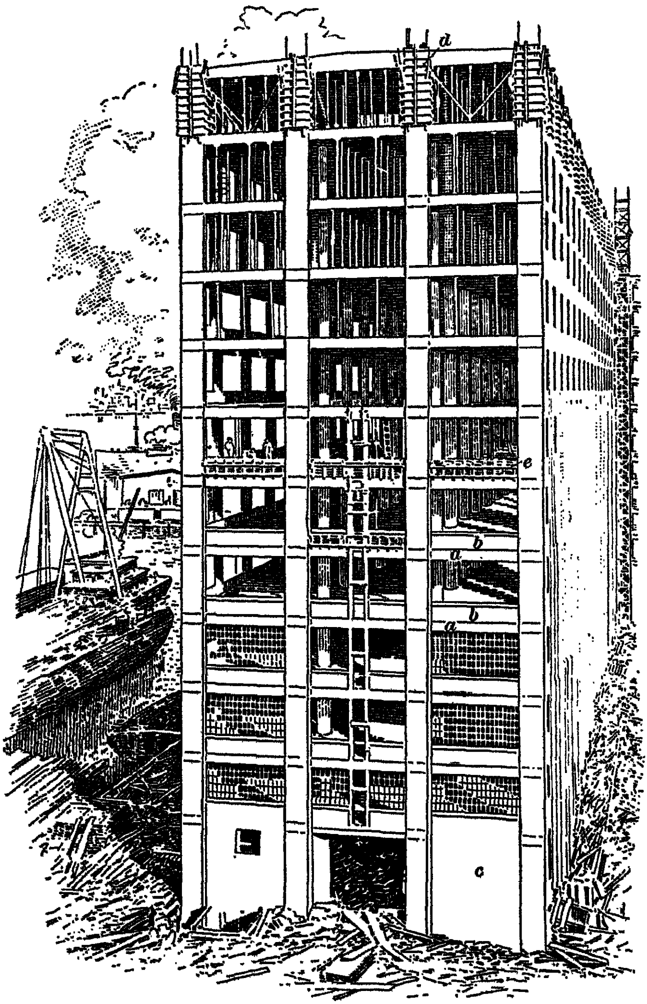


FIG. 8

below *a a* acts as the lintel, in which case part of the slab assists in taking the compressional stress in the lintel. The spandrel

wall will then be designed to carry only its own weight. Otherwise it may be assumed that the entire lintel and spandrel act together as a rectangular beam. In this case, the spandrel should be thoroughly bonded to the lintel by means of long stirrups projecting from the lintel into the spandrel as indicated in Fig. 7.

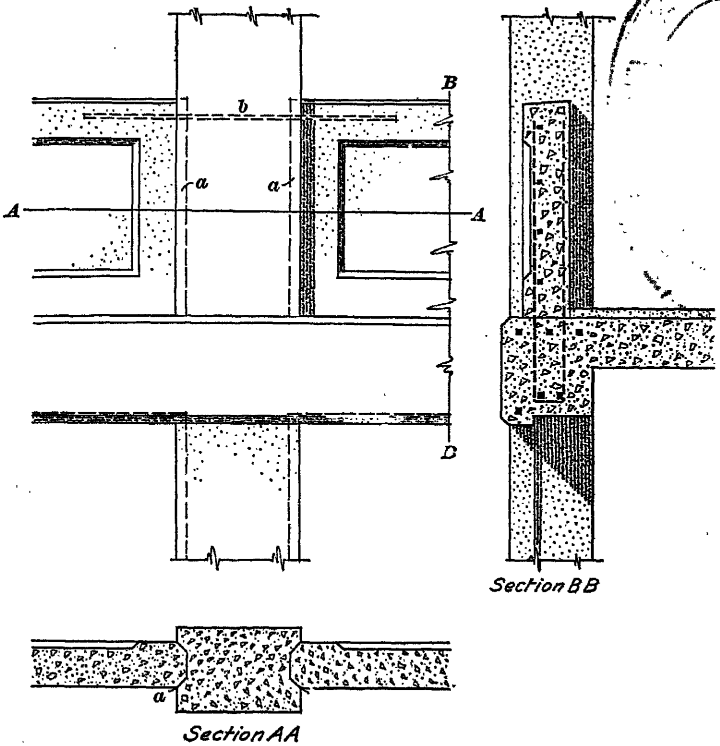


Fig. 9

7. In the construction shown in Fig. 7, a great depth of beam is obtained, thus lessening the amount of steel reinforcement required, while the concrete is needed in any case to form the wall. In some cases the spandrel wall is built in one continuous operation all the way around the building, and where the exterior columns come the wall is made thicker in order to furnish a sufficiently large base for the column. The

more common method of procedure is to build the entire frame of the building first, as shown in Fig. 8, leaving the spandrel walls to be put in at any convenient time, for which purpose but a few molds are needed as they can be used a great many times. Recesses are left in the sides of the piers to receive the spandrel walls, as shown in Fig. 9 at *a*. In some cases, the walls have been tied into the columns by means of the reinforcing rods indicated at *b*, but these are better dispensed with, so that the spandrel wall is free to shrink in the recess while it hardens.

The spandrel wall is reinforced with light horizontal rods in addition to the stirrups. The stirrups extend vertically from the lintel, and extra shrinkage rods are introduced in the top of the lintel. These rods continue all the way around the building to create a strong circumferential tie at each floor level.

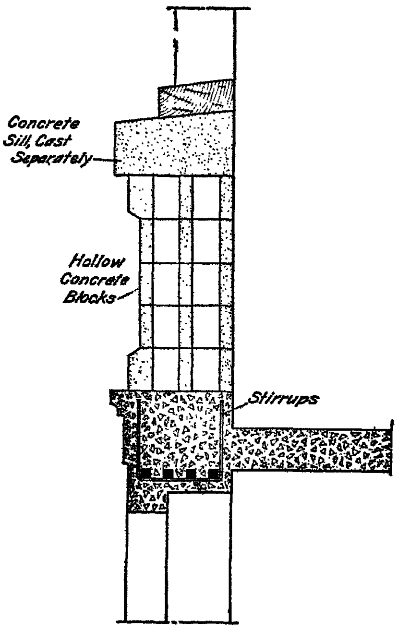


FIG. 10

8. Spandrel of Hollow Concrete Blocks.—In Fig. 10 is shown a reinforced-concrete lintel and spandrel construction that can be used on a building that is more or less ornamental. In this construction, the lintel is formed of carefully molded reinforced concrete that is hammer-dressed on the face after it has set. The spandrel wall is formed of molded hollow concrete blocks, and the window sill of either solid molded concrete blocks or artificial stone.

9. Surface Finish for Concrete Lintel.—Owing to the difficulty of obtaining smooth work in placing concrete, exposed

lintels should be finished with a bush hammer or by brushing the partly set concrete with wire or rattan brushes. The concrete may be smoothed by troweling or finished with cement mortar, but such a finish is liable to peel in time and thus disfigure the work. If a neat finish is desired on a building that is not too ornate and is constructed with brick piers and spandrels, the lintel and spandrel construction shown in Fig. 11 may be employed. The concrete

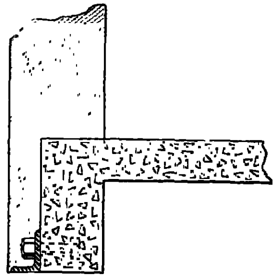


FIG. 11

lintel is formed with its face about 5 inches back of the finished lintel surface and is provided with a steel angle at the soffit; this angle iron supports a brick facing extending over the surface of the lintel, thus concealing the concrete work. Usually bolts are cast in the concrete lintel 3 to 4 feet apart for the fastening of the angle, which is placed after the forms have been removed.

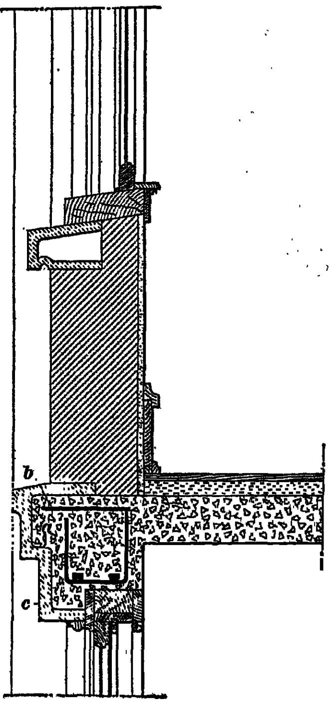


FIG. 12

10. Concrete Lintels With Terra-Cotta Faces. Terra-cotta facings are used in conjunction with reinforced-concrete lintels where a building is to have an architectural finish. One of the best methods to employ in such work is to arrange the concrete lintel as shown in Fig. 12, with a projection *a* for the support of the terra-cotta facing. In order to strengthen the projection and prevent it from shearing off, the

rods *b* are introduced. When the terra-cotta facing *c* is properly bedded upon the projection, the brickwork of the spandrel will hold it securely in place and the back of the facing will finish against the window frame. One objection to this construction is that the transverse webs of the terra cotta are cut away, thus tending to weaken the work.

Another method of supporting terra-cotta lintel facing in conjunction with reinforced-concrete work is shown in Fig. 13. Here, instead of a projection on the lintel, a steel angle *a*

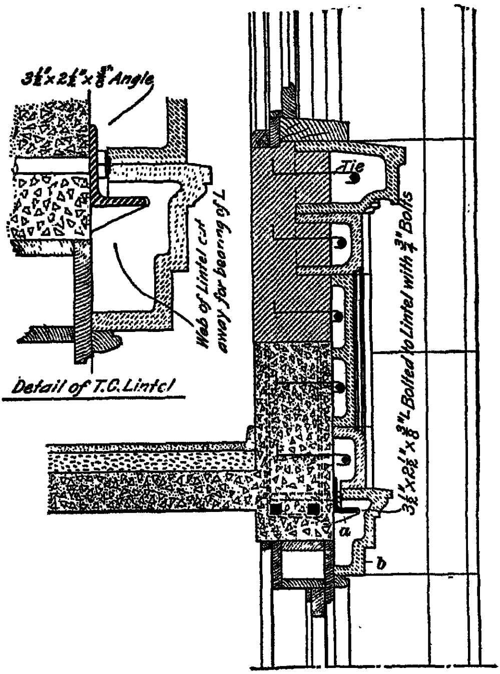


FIG. 13

extends along the face of this member. This angle is secured at intervals of 2 or 3 feet by means of bolts that are properly embedded in the concrete work. By using the angle a more nearly true and level bed is likely to be obtained for the bearing of the lintel facing *b*, and, besides, more of the transverse webs of the terra-cotta work are retained.

11. Lintel Construction With Composite Floor Construction.—If the floor construction consists of reinforced-concrete joists with hollow terra-cotta blocks between, it is necessary to use a deep reinforced-concrete lintel over the window openings, as shown in Fig. 14. The construction illustrated shows a composite floor system in which the concrete is reinforced with Kahn bars. The wall piers *a* support girders that run crosswise, the tile *c* and reinforced-concrete joists *b* extending between the girders. The tile is placed next

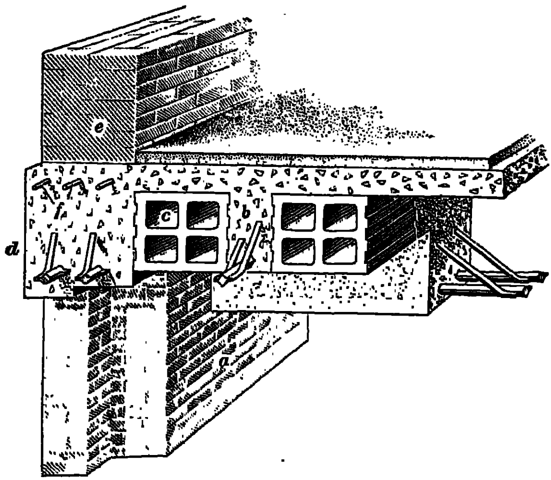


Fig. 14

to the concrete lintel *d*, which, owing to the depth of the floor construction and the fact that the piers are usually about 18 to 20 feet from center to center, is made of considerable depth. As will be observed, the lintel beam is reinforced with two main bars at the bottom and is provided with rods *f* in the upper part at the pier. The lintel projects about 2 inches beyond the face of the pier and supports a brick spandrel wall *e*; it is also rabbeted in the soffit to receive the sash frame.

CONSTRUCTION OF EAVES AND CORNICES

12. Construction of Overhanging Eaves in Concrete.—There are several ways of arranging the construction of monolithic buildings along the eaves of the roof, and there is no material that lends itself so readily to good construction at this point as concrete.

If a factory or warehouse is to be erected in a country or a suburban district at a minimum cost, the simplest way to arrange the roof at the eave line so as to take care of the rainwater will be to follow the method shown in Fig. 15 (*a*). Here, the roof slab is projected beyond the wall line, and a copper or galvanized-iron gutter supported by hangers conducts the rainwater from the edge of the roof to outside conductor pipes. This means of catching the drip from the roof is often omitted, as shown at (*b*), and it is not unusual, especially where there is no basement and where the floor level is somewhat above grade, to allow the drip from the roof to drain directly upon the ground.

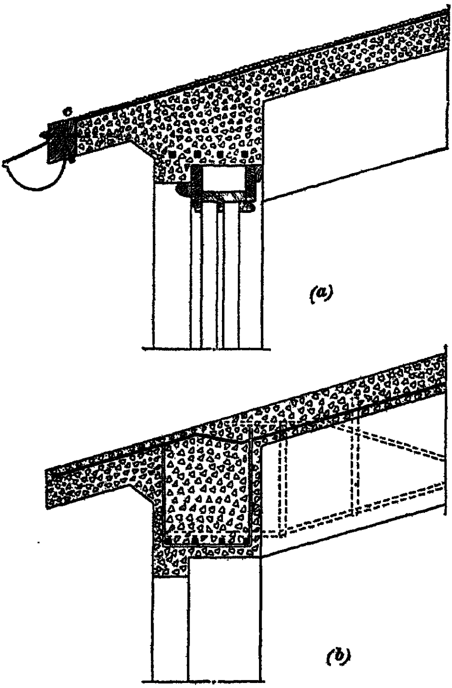


Fig. 15

Where the slab spans between beams or girders that extend from the wall piers across the building, the lintel can be made of minimum depth, as in (*a*), and the window frame brought

close up underneath the roof slab. The reinforcing rods of the slab should be bent up as indicated in (*b*), in order to provide the necessary resistance to the cantilever effect of the overhang of the slab.

In this construction, the custom is to employ a slag or gravel roof covering and to secure this covering under a cleat *c*, Fig. 15 (*a*), bolted against the edge of the concrete slab by means of bolts that are embedded in the concrete work as it is placed. As slag will adhere to steeper slopes than will gravel, a slag covering should always be used on roofs whose slopes exceed a rise of 3 inches to the running foot.

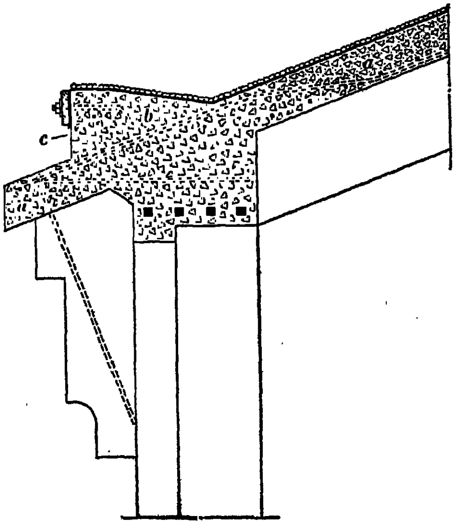


Fig. 16

13. Formation of Gutters in Concrete Roof Slab. A more permanent and durable construction than the foregoing for the eaves of a reinforced-concrete building is shown in Fig. 16. Here, the roof slab *a* is extended, being supported in some instances by brackets to provide an ornamental feature. A gusset, and consequently a gutter, is formed by raising the concrete work above the roof, as shown at *b*. The rainwater is thus drained to inside cast-iron conductors, an eaves box being formed in the concrete roof slab or provided by embedding in the concrete work a cast-iron box in the nature of a cesspool. The roofing material is carried down the slope of the roof, across the gutter, and turned down the vertical side *c*. At this point it is secured to a wooden strip covered with galvanized iron. This strip is held in place by bolts previously embedded in the concrete work.

14. Another method of forming a gutter is to keep all the work adjacent to the gutter below the roof slab by depressing the eaves in the manner shown in Fig. 17. This construction has no particular advantage over the one just described. It is more difficult to place and secure the roof covering, though this can be done, by using a cleat as shown, or by flashing the gutter with metal and running the flashing up under the felt. This last method is not good, because there is no secure way of fastening the felt other than cementing it to the flashing.

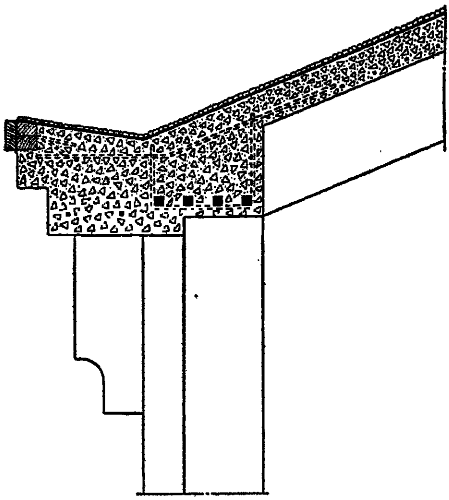


FIG. 17

15. Construction at Eaves, and Parapet Walls.—In monolithic buildings of a more finished nature, where there is pretension of a cornice formed in the concrete, it is customary to finish the structure at the eaves line by running up a monolithic parapet wall, as shown in Fig. 18. The work should be so arranged that the portion below the dotted line *a a*, which is practically a continuation of the roof slab, can be put in at one time and the parapet wall constructed subsequently. This type of construction provides a guard around the roof, supposedly adding security from fire, and is frequently required by city and fire-insurance laws, though in monolithic buildings such a parapet wall offers no additional protection.

In forming the gutter in conjunction with this type of construction, the gusset at *b* is made of a rather lean mixture of cinder concrete, such as 1 : 3 : 6 or 1 : 4 : 8. This gutter should be properly graded to drain to eaves boxes connected to inside conductor pipes.

16. The best way to secure the roofing material in conjunction with a parapet wall of this kind is to place upon the top of the wall a concrete coping *f*. This coping should either be formed monolithic or of artificial stone or concrete blocks previously cast. The metal flashing is bent over the top of the parapet wall and caught underneath the coping; it is also carried down and under the felt, which forms the gutter of the roof. In some instances where it is desired to save the cost

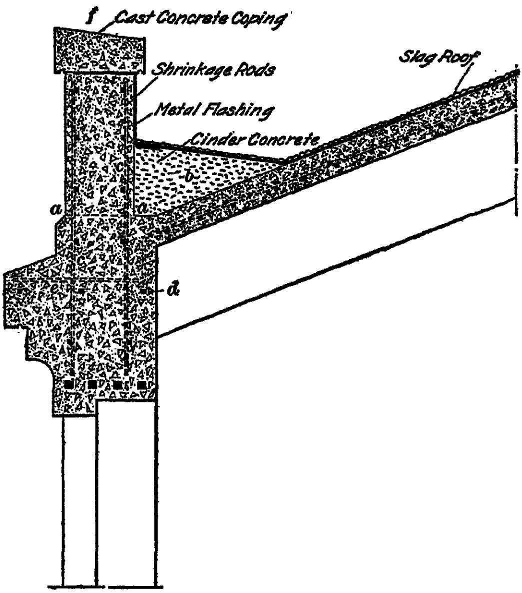


FIG. 18

of the flashing, which should, especially in all concrete buildings, be of copper, the felt is carried up the vertical face of the wall and turned in under the coping. Gravel or slag, however, will not adhere to the vertical surface of the felt; consequently, the felt, being deprived of this protection, is liable to be torn, damaged, broken, or destroyed by the elements. However, if the cinder filling that forms the gusset is carried to the top of the parapet wall on an easy slope, as shown in Fig. 19, metal flashings may be dispensed with and the felt

carried directly under the coping and covered with slag or gravel.

In this construction, ample shrinkage rods should extend through the parapet wall, and the slab rods should be arranged in such a manner as to reinforce any projecting corners or brackets. In Fig. 18, vertical and horizontal shrinkage rods are shown respectively at *c* and *d*, and the projecting rods for

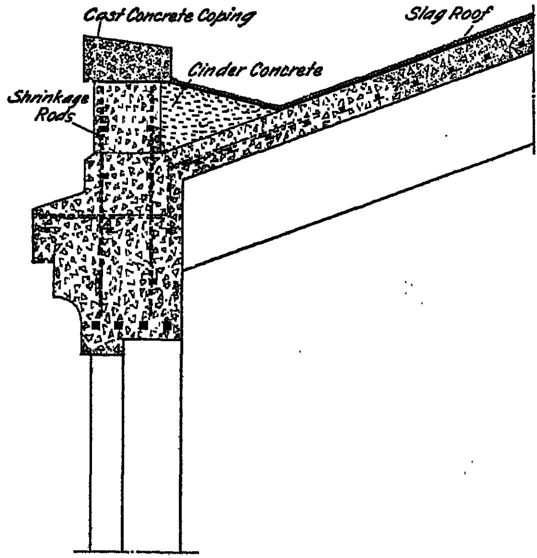


FIG. 19

the reinforcement of the overhanging portion of the eaves are shown at *e*.

17. Monolithic Cornice Construction.—Buildings constructed entirely of reinforced concrete are frequently finished with an elaborately molded cornice at the top of the main façade. A cornice of this kind sometimes extends above the roof 5 or 6 feet and has an overhang of 4 or 5 feet. Special attention must be paid to the design of such a cornice in order to provide a reinforcement that is ample and so bonded in with the other concrete that there will be no possibility of its falling by reason of the weight of the overhanging portion.

A monolithic cornice is shown in Fig. 20. The profile of this cornice consists of the crowning member *a*, the mutules *b*, and the frieze, or entablature, *c*. The lintel *d* must be designed to carry the weight of the entire cornice, with sufficient reinforcing rods, as at *e*, in the bottom of the lintel. There should also be provided numerous shrinkage rods *f*, usually of $\frac{3}{8}$-inch round or $\frac{5}{16}$-inch square twisted bars, running lengthwise of the cornice. The rods *h* projecting from the slab can be extended to the top of the projecting portion so as to furnish

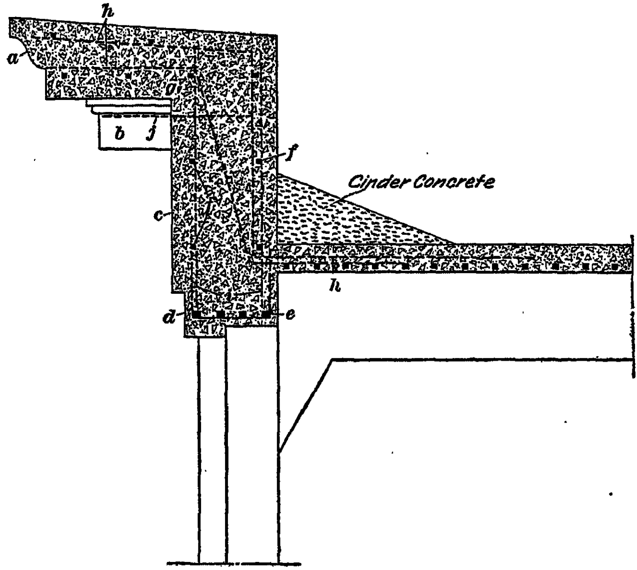


FIG. 20

the necessary resistance, and, if necessary, additional rods *i* extending down through the lintel and hooking to the reinforcing rods in this member can be introduced.

18. Where mutules, modillions, or brackets occur, it is well to place some additional reinforcing rods, as shown at *j*.

The top of the construction may be covered with copper, which should be securely fastened to cleats embedded in the concrete work and carried down so as to make water-tight junction with the roof covering.

Sometimes the mutules and the crown molding consist of separate blocks that are molded previously and then built, or set, in the concrete work. By planning the construction in this way, any defect in the finer molded members can be corrected before they are built in; whereas, if the work is cast monolithic, it is difficult to make good any faulty work caused by carelessness or accident.

19. Terra-Cotta Cornice With Reinforced-Concrete Construction.—Unless the terra-cotta work is made a por-

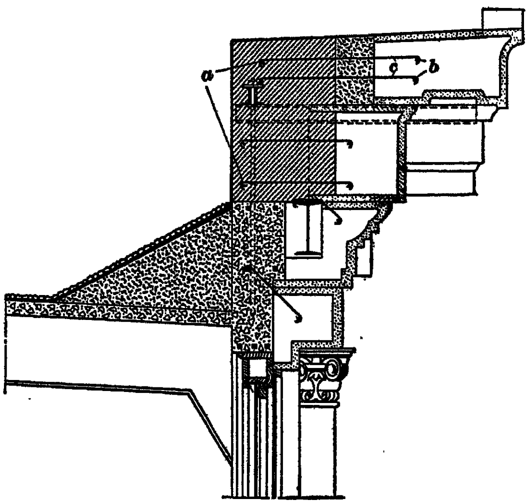


FIG. 21

tion of the form and the concrete poured in, so that the terra-cotta facing becomes securely embedded in the monolithic construction, it is only with difficulty that a terra-cotta cornice is properly designed and constructed in conjunction with the reinforced-concrete work.

A familiar type of terra-cotta cornice construction used in conjunction with reinforced-concrete work is shown in Fig. 21. The main overhang of the cornice is upheld by small I beams called outlookers. These outlookers are held down at the end by two channel irons that extend the entire length of the cornice. The channels are placed back to back, with anchor bolts

between them extending down into the concrete lintel. Rods *a* are embedded in the concrete work, and other rods *b* are inserted through holes in the webs of the terra-cotta blocks. Each pair of rods is connected by wires, or strips, *c*, bent around the rods. The method of supporting the several parts is clearly shown in the figure. The support of each separate block should be carefully studied to see whether it is properly secured in place and whether or not the work can be put together in the field.

BRICK, TERRA-COTTA, AND STONE FACINGS

20. Fastening of Brick Veneer.—One difficulty encountered in the construction of reinforced-concrete buildings whose exterior walls are to be faced with brick is to arrange the proper means of securing the brick veneer, or facing, to the wall piers and spandrels. If the bricks rest upon a solid ledge of concrete, a stone band, or a base course, and their weight is consequently securely sustained in this manner, they can be tied to the concrete wall at intervals with flat copper ties. These ties are placed through slots in the form boards, usually made by boring two small holes side by side and cutting out between, as shown in Fig. 22.

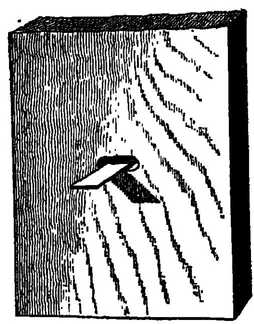


FIG. 22

The method of tying a brick facing to a concrete pier by means of copper or galvanized-iron ties is illustrated in Fig. 23. As shown, the ties, which consist of a strip of copper about $\frac{1}{16}$ inch thick, $\frac{3}{4}$ inch wide, and 7 inches long, are placed every seventh joint. These ties are spaced about 2 feet apart horizontally, and are embedded in the concrete work to a depth of about 4 inches, which gives them, when a $\frac{5}{8}$- or $\frac{3}{4}$-inch space is left between the facing and the concrete, a surface about $2\frac{1}{4}$ inches long in the bricks. It is customary in arranging the brick facing to allow about 5 inches for the width of the brick and the space back of the brick,

which is necessary on account of irregularities in the forms. This space, which is shown at *a*, Fig. 23, is usually flushed in with mortar as the bricks are laid.

21. Brick Facing With Terra-Cotta String-Courses.—In the architectural treatment of a building constructed with a reinforced-concrete frame it frequently happens that much of the brickwork bears upon terra-cotta band courses or string-courses, as illustrated in Fig. 24. If the terra cotta is adequately supported and filled with concrete so as to have great bearing resistance, it will safely sustain the load of the brickwork. Usually, however, such terra-cotta band courses possess little strength and are built and tied in with only sufficient security to support their own weight. It is inadvisable therefore to allow the entire weight of the brick facing to come upon such decorative work. The brickwork facing should therefore be supported by means of concrete ledges *a* at each story.

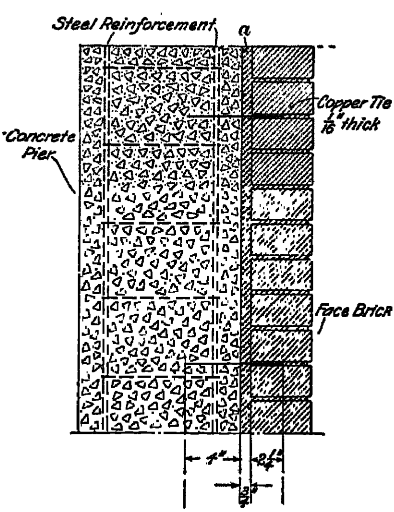


FIG. 23

22. Cut-stone facing is secured to the concrete work as described for brick and terra cotta. As cut stone is the most expensive kind of facing in common use, the buildings on which it is used require particular care in order to prevent discoloration or efflorescence from marring the appearance of the finished work. The efflorescence is a deposit of crystals of salts which the water dissolves out of the concrete and leaves upon the surface of the building when it evaporates. The remedy is simply to waterproof all surfaces where the concrete touches the masonry and especially the cut-stone work. Any good

waterproofing compound will answer, but it stands to reason that this must be applied during the progress of the work; all seats, beds, ledges, and corbels must be treated with the com-

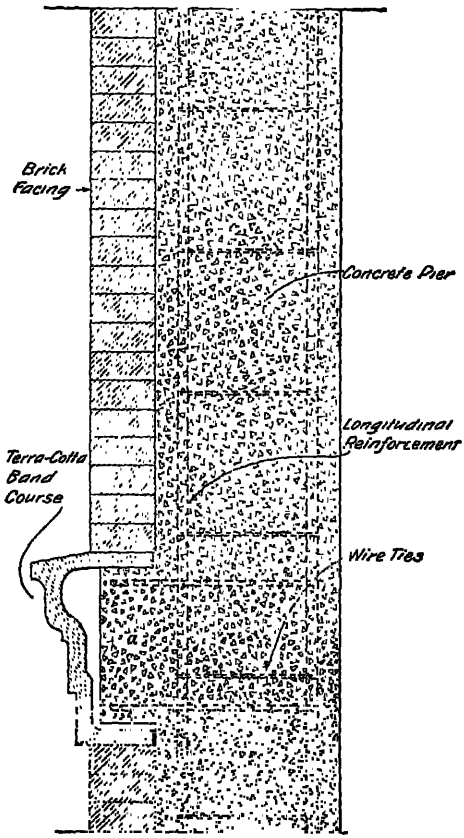


Fig. 24

pound. There is no absolutely certain way of stopping efflorescence in an *old* building, as the white film usually returns some time after being removed.

BEARINGS FOR CONCRETE BEAMS AND GIRDERS

23. Length of Bearing.—In determining the seat or bearing of a beam on a masonry wall, two conditions must be considered; sufficient length of bearing must be provided for anchoring the reinforcing bars at their ends, and the area of bearing must be sufficient to prevent the wall from being crushed by the load transmitted by the beam.

Long-span slabs have sometimes been built with only a 4-inch bearing on a brick wall, or even on a projection (called a *corbel*) from the vertical wall. Such construction has resulted in serious collapses and loss of life. The practice of supporting floor slabs on corbels is objectionable, as the reinforcement of the slab does not have sufficient anchorage. Slabs having spans of more than 10 feet should have a bearing of at least 8 inches on the wall.

The length of bearing for a beam or girder should, where possible, be not less than 12 inches, and preferably more.

24. Bearing Plates and Grillage.—The area of bearing already referred to is equal to the product of length of bearing times width of beam. In cases where this area is

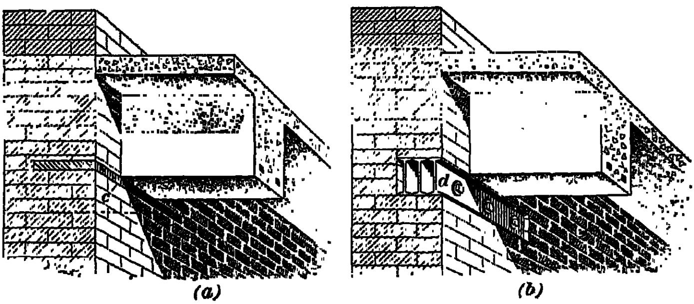


Fig. 25

insufficient to carry the loads imposed upon it, the area can be increased by increasing its width beyond that of the beam, in order to distribute the load over a greater width of wall. This

can be accomplished by using cast-iron or steel bearing plates *c* or grillage beams *d*, as shown in Fig. 25 (*a*) and (*b*). Where it is necessary to secure only a small additional bearing area, the end of the beam can be molded with flared sides, as shown in Fig. 26. This device will usually increase the bearing area as much as 50 per cent.

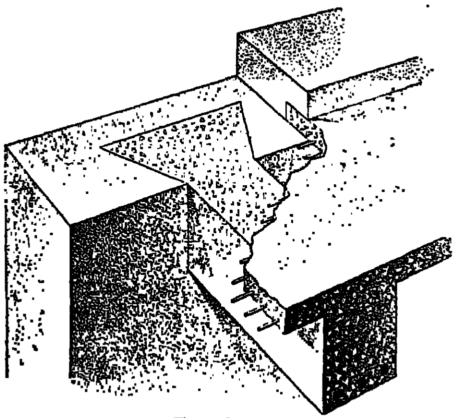


FIG. 26

25. Another method sometimes used to increase the bearing area of concrete beams and girders is shown in Fig. 27. The end of the beam is formed with projecting spurs at right angles to the axis of the beam, these spurs being of such length as to distribute the bearing over a considerable area. If the line *a b* is at an angle of 60° with the horizontal, no steel reinforcement need be used in the cross-beam or bearing lintel; but if this angle is reduced, it is well to use reinforcing rods, as shown at *c*. In the best practice, reinforcement is, however, always used since it adds much to the safety without adding much to the cost.

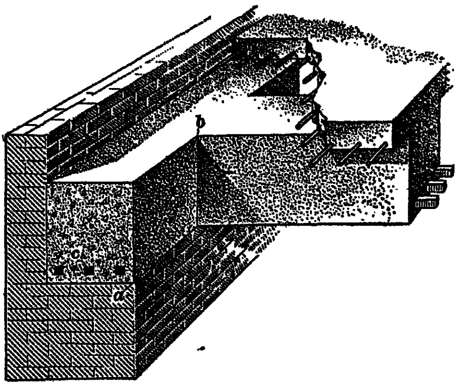


FIG. 27

REINFORCED-CONCRETE STAIR CONSTRUCTION

26. In concrete buildings it is customary to construct the stairs of reinforced concrete. Although such stairs are more costly than iron and steel, it is convenient where the operation of reinforced-concrete construction is being carried on to build the stairs of this material. Steel stairs, however, are entirely practical and quite common in concrete buildings, as they are enclosed in a fireproof shaft with brick, terra-cotta, or concrete walls with fireproof doors.

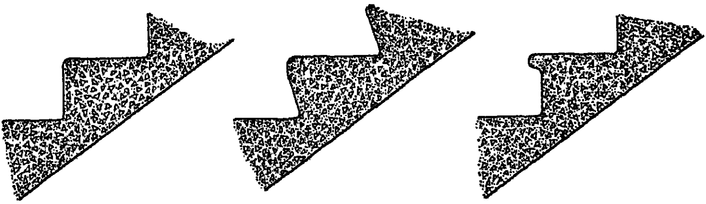
Although the general design of the reinforced-concrete stairs must comply with the requirements of the architectural plan, yet the details of construction are governed by the manner in which it is proposed to construct the building.

Practice differs with regard to the construction of stairs in a building of reinforced-concrete skeleton construction with columns, pilasters, and floors forming a complete structural monolith. In some sections of the country it is usual to arrange to build the stairs with the structural frame and to pour the stair concrete at the same time as that of the columns and floors. This method secures a strong bond between the various parts. It may also be so arranged that the enclosure walls can be built after the entire concrete work is completed.

It is also quite common to construct the stairs after the floors and columns are poured. Sometimes stair construction follows up a story or two behind the floor construction and sometimes after the structural frame is complete. Unfortunately, with this method of building, especially where the stairs are supported partially one flight by another, it frequently happens that not enough attention is paid to bonding the stairs to the floors and columns already poured. Bonding rods are absolutely necessary at the ends of the runs and around the landings. Also, rabbets should be provided in the floor and column construction for support of stairs constructed later. For the ordinary stair spans a 4-inch rabbet with bonding rods is sufficient for bearing. Ends of short beams at the head of stairs or at intermediate landings may be supported by the enclosure

walls. In this case the walls may be built first and holes left into which the beams are run when the concrete stairs are poured. If, on the other hand, the stairs or landings are to be supported along the edge by the walls, it is best to pour the stairs first and build the walls under before removing the forms. Otherwise concrete posts must be provided to support the stairs and landings from the floor level.

27. Sections of Reinforced-Concrete Steps.—Reinforced-concrete steps are formed in several ways, the usual sections for such steps being illustrated in Fig. 28. In (*a*) is shown a section of reinforced-concrete steps with a horizontal tread and a vertical riser. This type of step may be easily constructed and finished. Generally, the angle between the



(*a*) (*b*) (*c*)

FIG. 28

riser and the tread is finished with a slight fillet, and the nosing of the step is rounded off as shown in the figure.

As reinforced-concrete steps are used mostly in the industrial type of building, though they have been constructed in hotels and fine residences, the space apportioned to them is generally restricted. Therefore, any design of step that will cut down the run, or horizontal extent, of the stairs is of advantage. In such instances, the step section shown in (*b*) is sometimes employed. By arranging the step with a horizontal tread and a sloping riser, the tread is widened; or, with the same width of tread, the run of the stairs may be reduced. This section of step is nearly as easily constructed as the one shown in (*a*), though its appearance is not so good.

In (*c*) is shown the section of a concrete step that presents a better appearance than either of the sections described. This step is formed with a nosing that has the advantage of a wide

tread and gives a finish to the stair construction. The nosing, however, is sometimes worked on with the finish of the step, instead of being formed in the actual concrete construction, and is then liable to be knocked off or marred with hard usage.

28. Cement Finish for Concrete Steps.—In finishing concrete steps with cement, the mortar is placed on the steps in the manner indicated in Fig. 29. The cement finish, which consists of 1 part of cement and 1 part each of sharp coarse sand and crushed rock, is usually placed directly upon the concrete work while it is still fresh; a capping 1 inch in thickness is placed on top of the tread, and the finish is reduced to a mere skin coat of about $\frac{1}{8}$ or $\frac{1}{4}$ inch on the face of the risers. In the illustration, Fig. 29, the molding and rounding of the nosing is shown formed in the cement finish; it is, however, so difficult to form the nosing in the cement finish that this construction is but little used. Where a nosing is desired it is better to cast the nosing in one piece with the step, as in Fig. 28 (*c*).

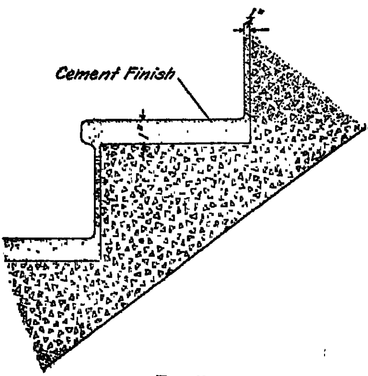


FIG. 29

29. Safety Treads.—If the steps are on a main stairway of an important building, cement finish is not satisfactory, because it is dusty and does not present the best appearance. One of the best ways of finishing concrete steps in such buildings is to use safety treads, as illustrated in Fig. 30 (*a*). These treads are of the section shown, and are made of brass or steel, with lead or carborundum secured in grooves or fastened in some other way. They are usually about 5 to $7\frac{1}{2}$ inches in width and are furnished with a nosing piece, as shown.

In placing these treads upon concrete steps, cast-iron blocks, as indicated in (*b*), are embedded in the concrete work or

cement finish, and the treads are fastened to them by countersunk machine screws. These treads form a finish at the nosing, make a good appearance on the steps, prevent the concrete steps from being worn into unsightly holes by the traffic upon them, and also overcome any danger of slipping.

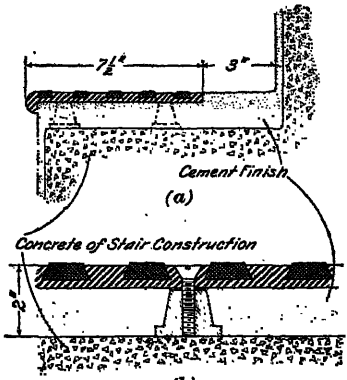


FIG. 30

30. Slate, Marble, and Wood Finish for Steps. Concrete steps are often finished with rubbed slate or marble treads with molded nosings, and if a fine finish is desired, the same material is used for facing the risers. The marble or slate should be set in cement, and is best secured by fastening it with screws to castings or blocks embedded in the concrete.

Wood may be used for finishing reinforced-concrete steps by embedding nailing pieces in the concrete and finishing the steps with hardwood treads and risers with molded nosings. This finish, however, is hardly consistent with the material of construction, the slate or marble being more suitable.

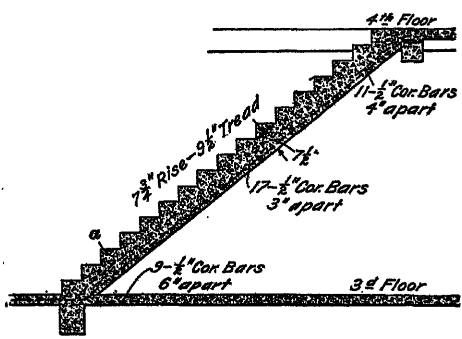


FIG. 31

31. Simple Concrete Stairs. Reinforced - concrete stairs of simple construction are illustrated in Fig. 31. Here, the stairs extend from floor to floor, and are supported upon header beams, which receive them at the foot and the top. Such stairs are reinforced with rods placed near the soffit, the rods extending the length of the

flight and lapping over the beams of the floor systems and stair landings.

Frequently, small rods *a* are placed so as to run parallel with the steps, as shown. These rods act as shrinkage rods and in no way increase the strength of the stairs; the strength depends

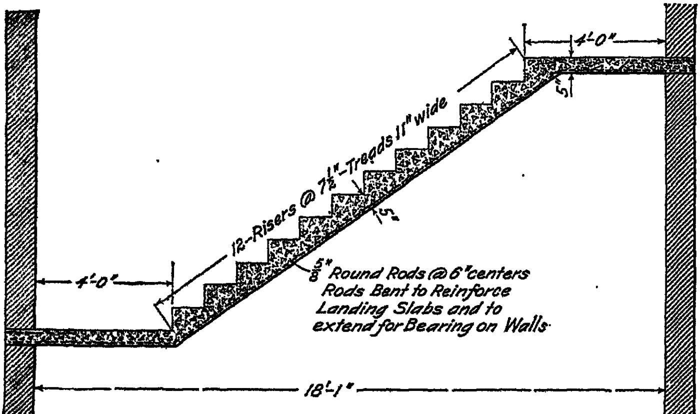


FIG. 32

upon the thickness of the concrete and the number and size of rods extending from support to support.

32. Stairs to be placed in a brick-enclosed shaft with landing at top and bottom of flight, are constructed as shown in Fig. 32. The concrete slab should extend a minimum of 8 inches into brick walls to provide a bearing for the landing, and the reinforcing rods should extend through the flight from the wall bearing of one landing to the wall bearing of the other.

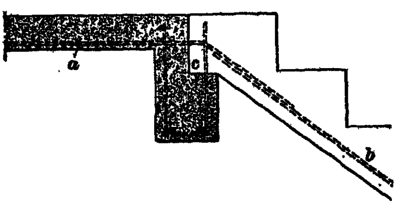


FIG. 33

The reinforcing rods are bent to the necessary shape to form this continuous reinforcement. In this construction no header beams are used.

For spans of 8 to 12 feet a 4-inch bearing in brick walls would suffice, while for spans up to 8 feet the stairs might

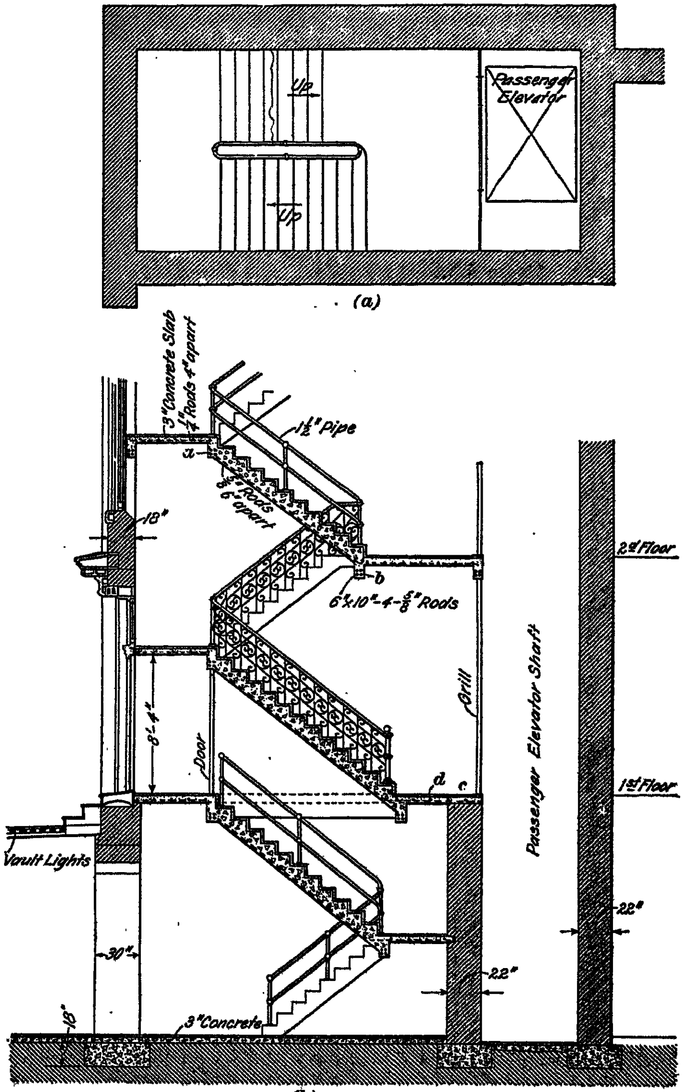

Up
Passenger Elevator
Up
(a)
3" Concrete Slab
1/4" Rods 4" apart
1 1/2" Pipe
a
3/8" Rods 6" apart
18"
b
6"x10"-4-5/8" Rods
2nd Floor
Grill
Passenger Elevator Shaft
8'-4"
Door
d
c
1st Floor
Vault Lights
22"
30"
22"
18"
3" Concrete
(b)

FIG. 34

even be supported on brick corbels built out on the walls. Such construction is positively dangerous for long spans such as that in Fig. 32.

If the stairs are to be constructed after the floor has been completed, and are to extend from floor to floor and rest on concrete header beams, the beams are made as in Fig. 33, with a rabbet *c* to form a bearing for the top of the stairs. In such cases, the reinforcing rods *a* of the floor system are left to protrude, so that they can tie in with the reinforcing rods *b* of the steps to be constructed later.

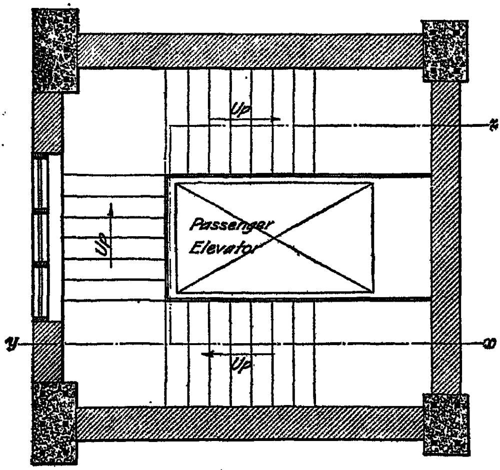


Fig. 35

33. Stairs With Half-Pace Landing.—A type of stairs much used in all buildings of commercial or industrial type is shown in Fig. 34. Stairs of this kind are necessary where the distance from floor to floor is 12 or 14 feet, and where it is required that they occupy as little room as possible in the building.

As shown in the plan (*a*), the stairway is composed of two half flights, with an intermediate landing between floors. Sometimes the stairs are arranged with an open well that is 8 or 10 inches in width, but more frequently the face of the stringer of one flight is directly over the face of the stringer of

the flight below. The stairs are usually constructed and reinforced as shown in the section (*b*). By arranging reinforced-concrete beams at *a* and *b* so as to extend across the stair shaft, the construction is greatly strengthened. The wall, as at *c*, receives the slab at the first-floor level. Where the wall is built afterwards, the slab is received upon a reinforced-concrete beam that is supported at the corners of the shaft either by temporary shoring or by concrete posts or columns.

In some cases the reinforcing rods are arranged to run through the slab and the stairs, as shown, and in others the reinforcement for the stairs extends only over the reinforcement of the beams *a* and *b*, and the slab rods are independent, extending from the wall to these beams. Shrinkage rods are placed through the slab, as shown at *d*.

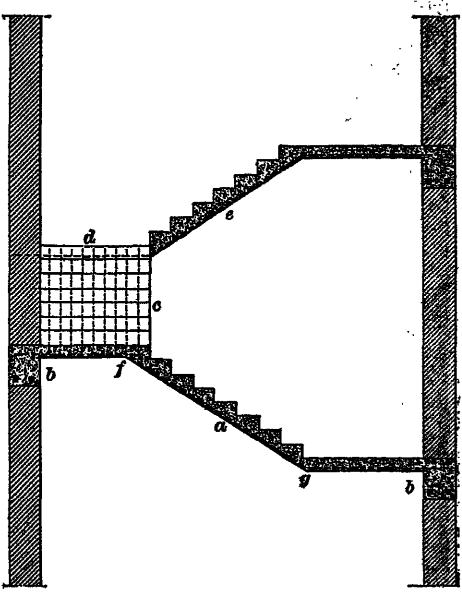


FIG. 36

34. Stairs With Center Well Hole. In many buildings the bays are arranged so as to be nearly square, and as it is convenient to arrange the stairs in a stair well built between piers and columns so as to occupy the space of a bay, the type of stairs with a central well hole, as shown in plan in Fig. 35, is extensively used. Stairs of this construction can also be conveniently used around the elevator shaft, it being customary to arrange the passenger elevator inside of a stair well in this manner.

The method of constructing stairs of this kind is illustrated in Fig. 36, the section being taken along the lines *x y z*, Fig. 35.

In constructing such stairs of reinforced concrete, the several flights between floors act as supports for one another. The flight *a*, Fig. 36, is supported at each end by the beams *b*, and acts as a support for the intermediate flight *c*. This flight is supported also by the landing *d*, which is either built into the

FIG. 37

wall or supported upon corbels. Therefore, the upper flight *e* can be considered as suspending, or tending to hold up, the intermediate flight *c*.

35. Frequently, the stairs can be greatly strengthened by the use of header beams at such places as *f* and *g*, Fig. 36,

where they do not interfere with the headroom nor the elevator shaft; or, a reinforced stringer acting as a beam can be run

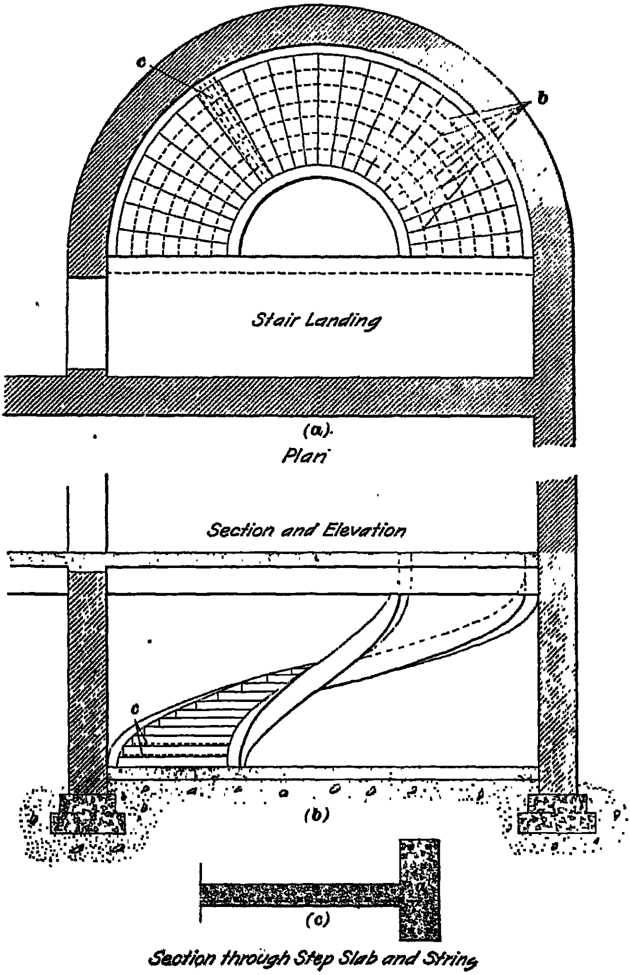


FIG. 38

along the edge of the stairs. Such a stringer molded in conjunction with the reinforced-concrete stairs is shown in Fig. 37. A reinforced-concrete stringer, or curb, provides a means for

receiving the elevator grill or stair railing, and also prevents dust or dirt from falling down the well. Such a curb should, however, not be run across the landings where the sliding doors to the elevator enclosure are located, as they interfere with the formation of the sill as well as the installation of the sliding doors. When no elevator is present, however, they may be made continuous.

36. Spiral Stairs.—Several types of spiral stairs have been constructed in reinforced concrete. In some instances the spiral has been continuous, in the form of a helix, around a circular well hole, and in others straight flights have been combined with winders, so that a portion of the run is straight and a portion spiral. As a rule, spiral stairs in reinforced concrete are too expensive in cost of form construction for use in commercial buildings, though they may be used if an imposing vestibule or stair entrance is desired.

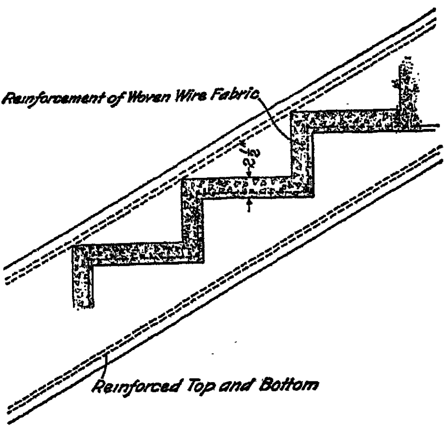
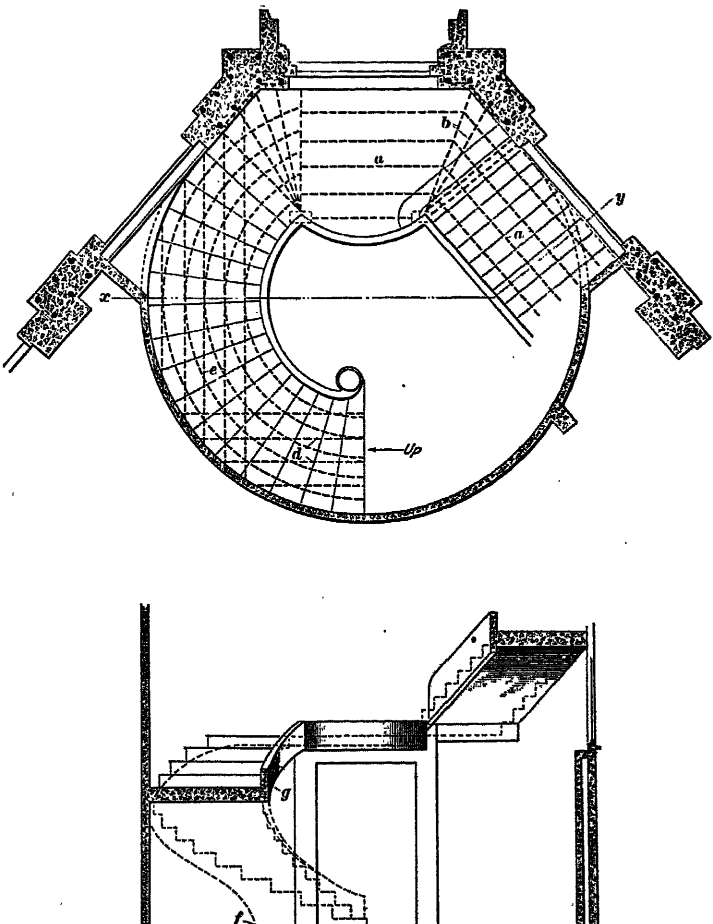


FIG. 39

37. A type of spiral stairs is illustrated in Fig. 38. From the plan view (*a*) and the sectional elevation (*b*), it will be seen that these stairs are constructed with a well and have outside stringers of reinforced concrete. The stringers are reinforced at the top and the bottom with steel rods and are usually combined with the slab portions of the stairway, as shown at *a* in the sectional detail view (*c*). Besides reinforcing the concrete stringers, it is best to run reinforcing rods *b* in the form of a helix along the stair slab and parallel with the stringer. The stairs should likewise be reinforced transversely with rods *c*, in order to avoid shrinkage, and also to give the

stair slab transverse resistance between the stringers. If the stringers can be made of a good size and are well reinforced, the



Sectional Elevation On x–y

FIG. 40

stairs may be made of the lighter section illustrated in Fig. 39, in order to minimize the dead-weight and lessen the transverse stress upon the stringers.

38. In the type of stair shown in Fig. 40, where a straight flight connects with a spiral portion, the rods *a* of the straight flight can run through the landing slab and rest upon cross-rods *b*. This forms a heavily reinforced slab, from which the spiral portion of the stairs can start. In addition to the main reinforcement, cross-rods *d* and *e* can be used in the construction of such a stair. The stairway is greatly stiffened by the use of a small spandrel *f* at the commencement of the flight. By reinforcing the run of the stair with an outside closed stringer *g* of reinforced concrete, the stairs are improved in appearance and greatly strengthened. The stringer also acts as a structural member from which to start the balustrade, which may be of ornamental ironwork, with a bronze pipe or hardwood molded rail.

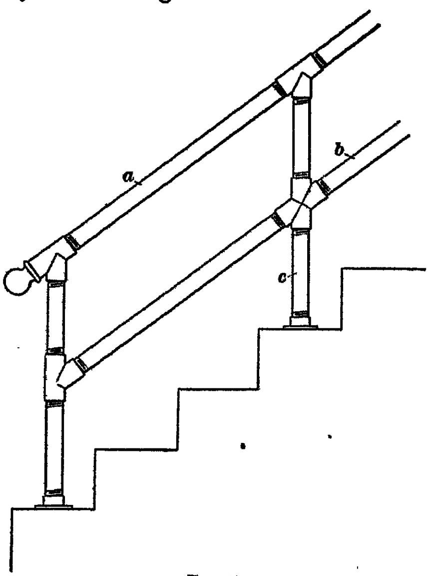


Fig. 41

Concrete spiral stairs with architectural pretensions may be finished with ornamental plaster work or covered with scagliola. The ornamental moldings, band courses, and other ornaments need not be attempted in the rough concrete work, but can be a portion of the finish.

39. Construction of Hand Rail for Concrete Steps. The hand rail generally used on reinforced-concrete steps consists of galvanized-iron pipe that is put together with galvanized-iron screw fittings. The usual construction of such a hand rail is illustrated in Fig. 41. In this figure, the hand rail *a*, the guard-rail *b*, and the stanchions *c* are made of $1\frac{1}{2}$-inch galvanized pipe. With inferior construction, the hand rails and

guard-rails are run through the fittings and are not fitted into them.

Many methods are employed to secure the stanchions in the steps. Sometimes a pipe sleeve of larger size is embedded in the concrete steps, as shown in Fig. 42 (*a*), and the stanchion is leaded into the socket, being finished with a flange, as shown. Frequently, a pipe socket is threaded at the upper end, and the

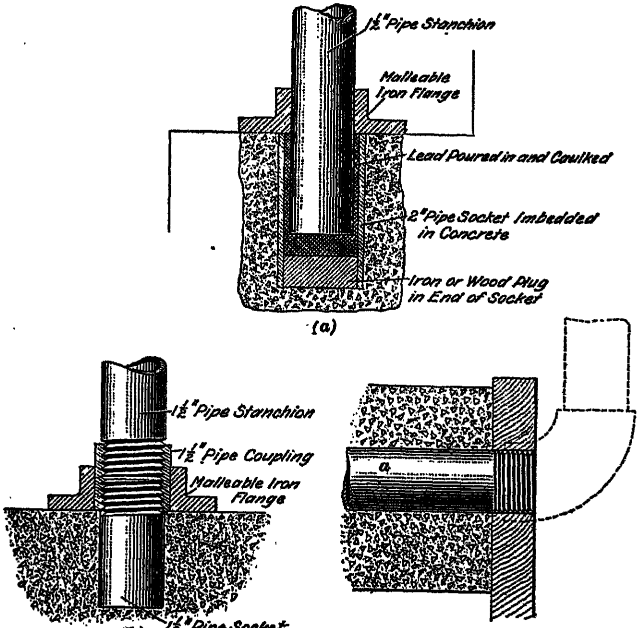


FIG. 42

stanchion is secured to it with a coupling and finished with a loose flange, as shown in (*b*). In each of these methods, the stanchions are placed on the top of the tread. If, however, it is desirable to gain the full width of the stairs, the stanchion is supported as shown in (*c*). Here, a threaded pipe socket *a* is placed through the form board used in molding the side, or cheek, of the steps; then, when the form board is removed, the threaded end of the socket projects and allows the stanchion to be supported by a threaded L fitting screwed on the socket.

Ornamental iron rails may be used on reinforced-concrete steps of a more pretentious character than the preceding. In such designs, the uprights of the rail are secured in pipe sockets by the use of melted lead, or holes are drilled in the concrete into which the uprights are inserted and leaded.

AREA AND VAULT WALLS

40. In conjunction with reinforced-concrete buildings, it is customary to make the area and vault walls of the same material. An *area wall* is a low wall holding up the ground in front of a basement window; a *vault wall* is the exterior wall of a basement and is often extended underneath the sidewalk.

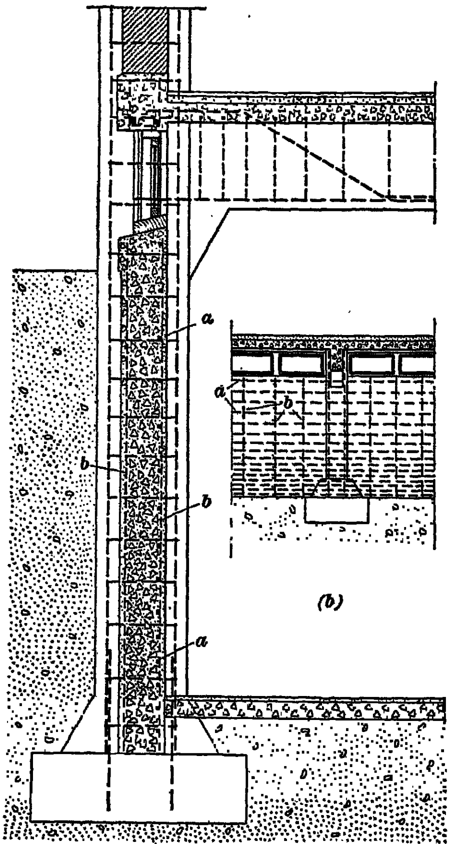


FIG. 43

Usually, vault walls extend between reinforced-concrete columns that hold them up against the pressure of the earth. Such a construction is shown in Fig. 43 (*a*), which is a vertical section laid midway between columns. The wall is reinforced with rods *a* extending horizontally from column to column; these rods are placed near the inner surface because this is the tension side of the wall under the pressure of the

earth. Since this pressure is greatest at the bottom, the rods are spaced closely at the bottom and farther apart in the upper portion of the wall. In addition to the reinforcing rods *a*, shrinkage rods *b* of $\frac{3}{8}$-inch round or $\frac{5}{16}$-inch square twisted bars should be inserted vertically throughout the wall.

As walls of this kind are frequently put in after the construction of the building has advanced, it is not always possible to continue the reinforcing rods *a* through the column, as shown in Fig. 43 (*b*). In such cases, the columns should be constructed with grooves in the sides so that the wall will bond in.

41. In order to gain all the available space in the basement, it is customary to extend the basement underneath the side-

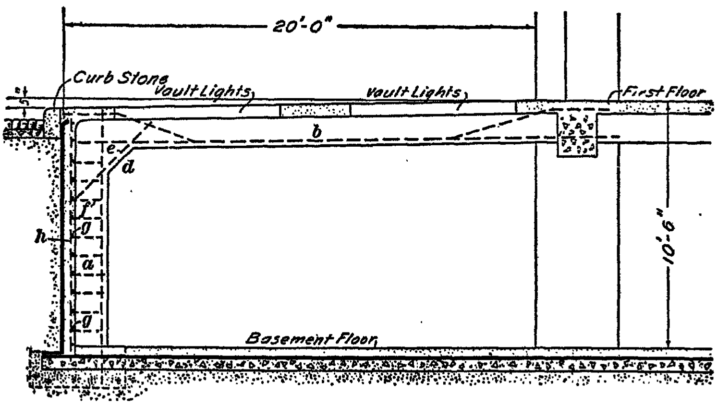


FIG. 44

walk, so that a vault is formed in the manner shown in Fig. 44. Here, the retaining wall is strengthened at intervals by buttresses *a*. These buttresses act as beams in resisting the thrust of the earth, and are connected with the concrete beams *b* that support the pavement. The buttresses are designed to resist the pressure of the earth on the concrete wall placed between. The wall may be made from 8 to 12 inches in thickness, depending on the height and distance from center to center of the buttresses. The buttress and beam are braced by brackets *d*.

The main reinforcing rods of the beams and buttresses may be arranged by bending so as to interlace. Diagonal rods *e* reinforce the brackets and the rods *f* are stirrups.

The wall between the buttresses acts as a slab supported by the buttresses, and the reinforcing rods *g* should therefore extend from buttress to buttress and preferably through the buttresses. There should be an ample number of shrinkage rods *h* extending vertically.

SPRINKLER TANKS

42. In nearly all buildings of an industrial or commercial nature there are installed sprinkler and fire-protection systems

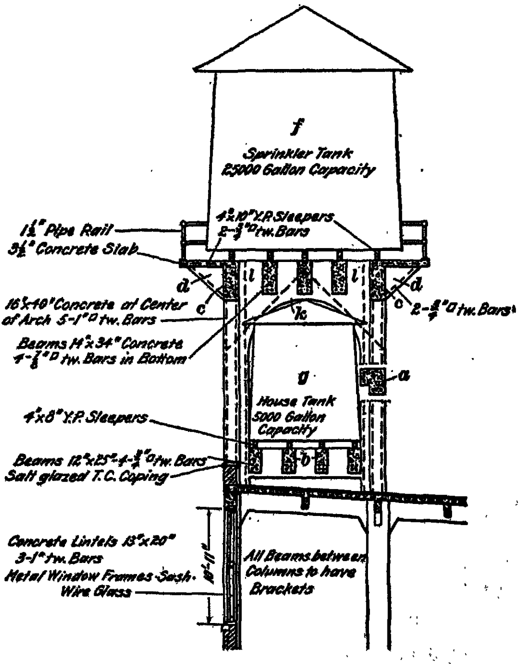


FIG. 45

that require large water tanks. These tanks are usually of wooden staves held together by bands of hoop iron. The

requirements of the fire-insurance underwriters generally demand that the tanks for sprinkler systems be supported at a distance of at least 20 feet above the highest point of the roof. These tanks range in capacity from 15,000 to 40,000 gallons, so that the weight to be supported is very great. The common practice in reinforced-concrete structures is to run four of the columns up above the roof as supports for the tank. Such columns are either made larger than the normal columns or they are more strongly reinforced. A typical reinforced-concrete tower for a tank for a commercial building is shown in elevation in Fig. 45 and in plan in Fig. 46. The columns supporting the tank *f*, Fig. 45, are of the sections indicated at *a*.

Usually, there is sufficient room beneath the sprinkler tank for a house supply tank *g*, so that a tier of beams and girders is introduced, as at *b*, which supports the house tank, and also greatly stiffens the tank tower. The top of the tower is formed with beams, and heavy girders *c* connect the columns on the four sides. The girders are usually arranged with strong brackets, or else a semicircular arch *k* is used, as shown. In order to provide a walk, or passageway, around the tank for painting and inspection, the slab at the top of the girders is extended and supported on cantilever brackets *d*.

43. In some instances, as in the construction shown in Fig. 45, the beams that support the sprinkler tank are made of rectangular section; that is, the slab is not run entirely across the top of the tank tower. In this way there are provided between the beams openings *l*, Fig. 46, that allow circulation of air under the tank, as the tank would otherwise be likely to rot. All the necessary reinforcement for the construction of a typical tank tower is illustrated in Fig. 45; this reinforcement consists of $\frac{3}{4}$-inch, $\frac{7}{8}$-inch, and 1-inch square twisted rods arranged as indicated in the figure.

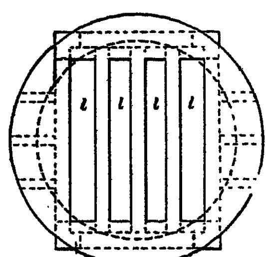


FIG. 46

DOME CONSTRUCTION

44. Reinforced concrete is a convenient material to use for the construction of domes and vaults. In many instances, the main supporting members of the dome or vault are made of structural steel and the shape of the dome, or sheathing, is worked out in reinforced concrete. Large domes have also been constructed by building up previously molded and properly shaped concrete blocks in concentric walls and filling the space between with a cinder concrete, which material has the advantage of being light in weight and sufficiently strong for the purpose. The best construction, however, if an opening is to be left for a lantern,* or skylight, is to use a properly reinforced concrete shell with a heavy ring at the spring of the dome and around the top.

Under most domes is arranged a ceiling dome placed some distance below the roof dome, and independent of it. This ceiling dome can likewise be constructed of reinforced concrete, and, where architectural treatment permits, it may be reinforced with ribs. These members strengthen the construction and provide a ground, or base, for working on the plaster moldings and finishings.

45. A typical reinforced-concrete construction for domes from 30 to 50 feet in diameter is illustrated in Fig. 47. Such domes must be heavily reinforced against spreading around the perimeter at the spring of the dome, as shown at *a*, and this ring of concrete must also be strongly reinforced. A similar stiffening ring must be provided around the top of the dome, as at *b*, provided a bull's eye, or lantern, is to be used at this place. It is usually convenient to make the dome shell of the same thickness throughout, keeping it as thin as possible so as to lessen the dead-weight. The reinforcement of the dome shell should consist of circular hoops *c* of light reinforcing metal placed at close intervals and reinforcing rods *d* running from

*As here used, the term **lantern** means a small tower or superstructure, as on a roof or dome, open below and admitting light from the sides; the term is sometimes applied to an upright skylight.

the top to the bottom of the dome, following the curve. These rods, however, are not so important as the hoop reinforcement. In the illustration, the inside layer *h* of concrete is shown partly broken away in order to lay bare the reinforcing rods, and the outside layer *g* is shown in position. The reinforcing rods are located near the inside of the dome because the concrete is in tension there.

The ceiling dome in the construction illustrated in Fig. 47 is shown at *e*. This dome is reinforced with rings *f* at the bottom

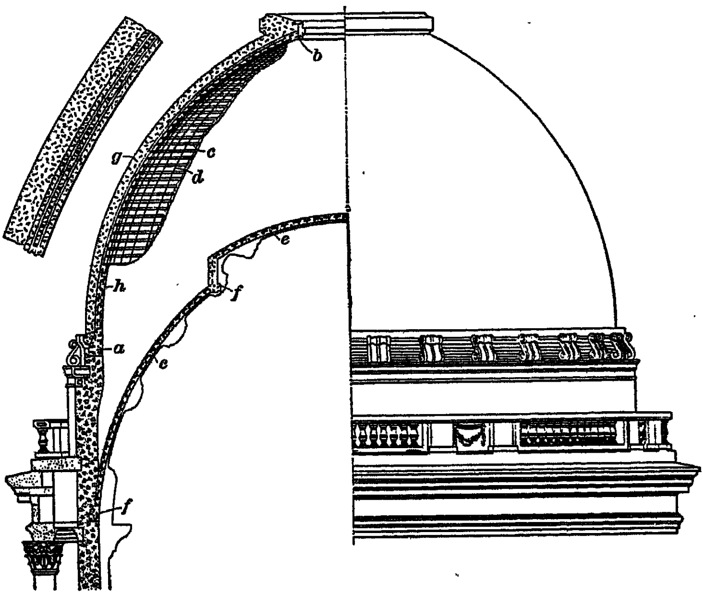


FIG. 47

and near the top, and, besides, it has reinforcement running in both directions, following the contour. It is best in the reinforcing of domes to use light rods spaced close, rather than heavy rods, because the lighter rods can be kept in position better while the concrete is being placed; also, they tend to strengthen the dome more uniformly throughout and provide greater elasticity to the construction, which is desirable in such structural features as domes and vaults.

SAW-TOOTH SKYLIGHT CONSTRUCTION

46. Saw-tooth skylights derive their name from the similarity of their contour to the tooth of a saw. They are extensively used for lighting the top floors of industrial buildings. The purpose of the saw-tooth skylight, an example of which is shown in Fig. 48, is to provide an upright skylight with an oblique face *a*, that is glazed throughout, is raised above

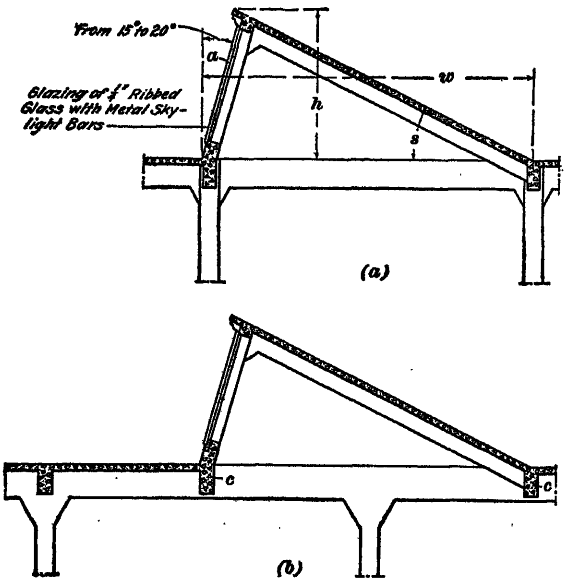


FIG. 48

the roof, and is usually facing north, so as to get the maximum amount of daylight without the glare of sunlight. The face *a* can usually be inclined at a small angle from the vertical, without admitting the direct rays of the sun. The angle of slope *s* of the roof is fixed by the width *w* of the skylight and the height *h*. The angle *s* must, however, in all cases be less than 35° if the roof is to be covered with a slag or gravel roofing.

In view (*a*) is indicated a skylight spaced so as to cover an entire bay extending from column to column. It is not neces-

sary, however, to arrange the skylights in this manner, as they can be placed in any position on the roof by providing cross-beams *c*, as shown in (*b*), and starting the face and roof slope of the skylight from these beams.

47. It is of advantage in saw-tooth skylight construction to use as few beams and girders beneath the skylight as possible, so that no unnecessary shadows will be cast in the room below. Owing to structural considerations, however, the building must be tied well together in both directions, and for this reason some connecting girders and beams must be employed.

The best framing for saw-tooth skylights is that shown in Fig. 49, which illustrates diagrammatically the framing plan and section of a typical construction. Plans of this kind are not drawn in conformance with ordinary drawing rules. For example, as shown in the figure, some of the parts, such as the girders, beams, and rafters, may be indicated by single dotted lines, while other parts, as columns and skylight supports, may be shown in cross-section.

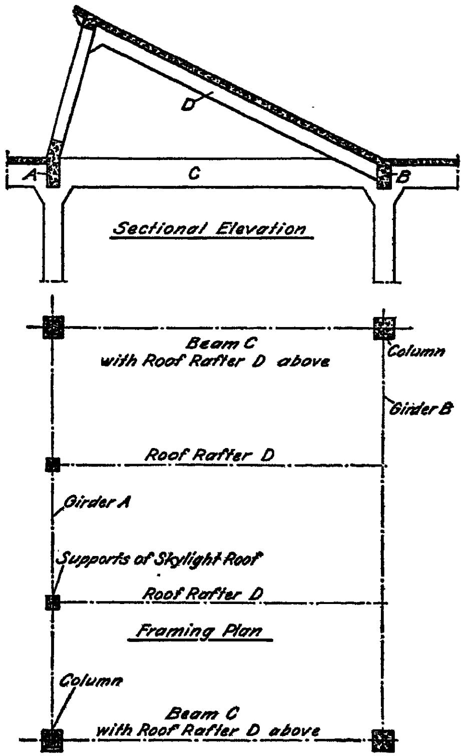


FIG. 49

FASTENINGS IN CONCRETE

48. As it is difficult to drill holes in reinforced concrete, *fastenings* or *sockets*, for securing shafting, equipment, or other installation to the slabs, beams, and girders, should be provided during the construction of the building. There are numerous devices intended to be embedded in the concrete work for the purpose of holding T-headed or tap bolts in a secure manner, so as to realize the full strength of the bolts. Such sockets must necessarily be arranged in the forms before the concrete is poured.

The sockets used are of two general classes, non-adjustable sockets and adjustable sockets.

49. Size and Spacing of Sockets in Concrete.—In nearly all large commercial or manufacturing buildings, it is necessary to fasten equipment or machinery to the ceiling, and as the cost of the sockets is comparatively small, a sufficient number of them should be provided in all beams or girders to answer any purpose. Ordinarily, the sockets should be tapped out for $\frac{3}{4}$-inch bolts, but where heavy machinery or hoisting apparatus is to be employed, it is better to use $\frac{7}{8}$-inch tappings in the sockets and to provide $\frac{7}{8}$-inch anchor bolts. Sockets or other fastenings should be provided near the end bearings of each beam and girder, and it is well to space sockets not farther apart than 4 or 5 feet along each beam and girder.

NON-ADJUSTABLE SOCKETS

50. Unit Socket.—The Unit socket, which is the salient feature of the unit system of girder-frame construction, is one of the best devices for bolting shafting and securing piping or other equipment to the soffit of beams or girders. This socket, a detail of which is shown in Fig. 50, is particularly efficient because it grips the steel reinforcement and cannot be pulled from the concrete without tearing away the reinforcing metal, which is securely bonded in the monolithic beam or girder by means of stirrups and its own bond.

The Unit socket shown is arranged for a narrow reinforced-concrete beam in which the four reinforcing bars are placed in pairs, one pair above the other. The socket *a*, which may be of either cast steel or malleable iron, is tapped out for a $\frac{3}{4}$-inch tap bolt *b*, by means of which the socket and the steel reinforcement to which it is attached by the bolt *c* are held in a correct position in the forms.

The lower part of the bolt *b* projects through a hole in the form bottom, to which it is temporarily clamped by means of a nut that is screwed up on the lower end. The bolt *c* is sufficiently long to extend between the reinforcing rods or bars and, by means of the heavy plate washer *d* and the casting *e*, clamps the rods of the girder frame. Thus, besides assisting in holding the rods together, this bolt secures the socket *a* to the steel reinforcement in such a manner as to prevent it from pulling away from the construction, making the weakest point of the device at the root of the $\frac{3}{4}$-inch threaded bolt.

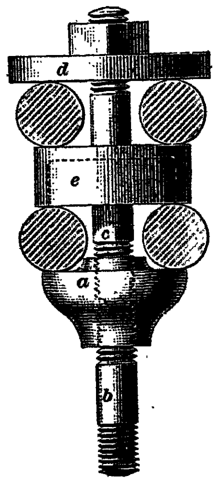


FIG. 50

This device is comparatively expensive and complicated to fix in place, but on account of its sure anchorage to the construction it is desirable for bolts of large size. A similar bolt anchorage can readily be improvised for attachment to any type of reinforcement. For this reason this method has a wide application. For the usual requirements in commercial practice, however, a cast-iron or malleable-iron socket of sufficient spread provides ample anchorage for a $\frac{3}{4}$-inch or $\frac{7}{8}$-inch bolt, and is more economical in cost and in placing.

51. Other Types of Non-Adjustable Sockets. There are many types of sockets; some of them have been patented, and others are merely expedients that have been designed from time to time to meet the requirements of building construction. In Fig. 51 are shown four forms of malle-

able-iron or cast-steel sockets. The socket shown in view (*a*) consists of a pronged, or forked, casting tapped out for the bolt required, as indicated at *a*. The prongs are cast with a turn at the ends to provide additional bond or grip in the concrete, and, when the socket is placed at the center of the beam or girder, this form is admirably adapted to reinforcement consisting of three rods, because the center rod can lie on the fork. These sockets are secured to the form board by means of the small lugs *b*, which are notched to receive nails driven into the bottom form board.

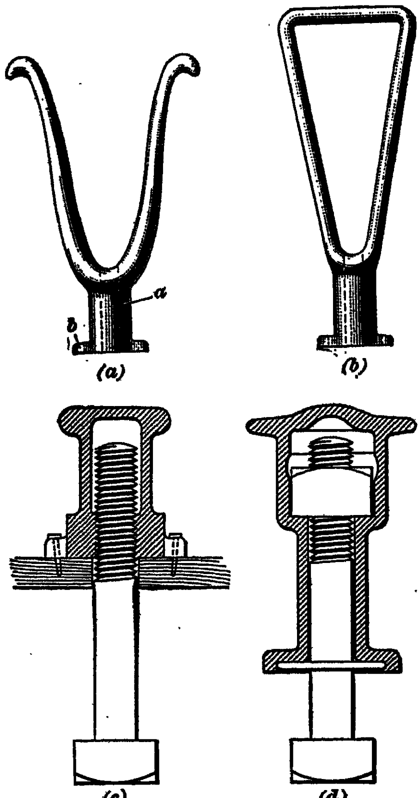


Fig. 51

The socket shown in (*b*) is somewhat similar in design to the forked socket, and its form is such as to secure a good hold in the concrete. This type of socket can have lugs cast on the side, as shown, or it can be held in place by supporting the top laterally against the beam form, in which case the lower end is held in place by driving a large nail into the form board and allowing the hole tapped out for the bolt to slip over it.

Simple *spool* sockets of the type shown in Fig. 51 (*c*) have been widely used for $\frac{5}{8}$-inch and $\frac{3}{4}$-inch bolts. They can be procured to order from local founders at comparatively low cost.

The *Philadelphia* socket shown in Fig. 51 (*d*) has the advantage that it needs no tapping, as the bolt nut is inserted.

52. Pipe Sockets and Bolt Fastenings.—The type of socket illustrated in Fig. 52 (*a*) consists of a piece of pipe swaged out or broken at the upper end, as indicated at *a*, with a solid wrought-iron bar or block welded to the other end, the solid portion of the block being tapped out for the bolt *b*, which passes through the bottom board *c* of the form.

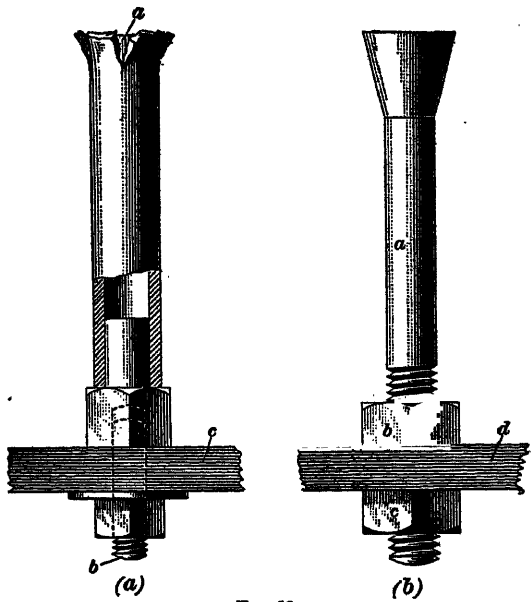


FIG. 52

The socket shown in Fig. 52 (*b*) can be used where there is no objection to the bolt end extending below the soffit of the beam or girder. It consists of a rod *a* threaded at one end and upset at the other. At the lower end it is provided with a nut *b*, which will form a bearing for placing the bolt in the forms. By screwing up the nut *b* on the threaded end and passing the latter through a hole in the bottom form board *d*, a nut *c* may be screwed on the projecting end, as shown. In this manner, the bolt can be secured to the form and held there

rigidly while the concrete is being placed. The upper end can be upset, split, and swaged out, or, if the beam or girder is of considerable depth, this end can be left plain, the bond in the concrete being sufficient to realize the full strength of the net section of the bolt.

53. Through Bolts.—Where it is necessary to install a heavy main, or head, drive from a large engine or power generator, or where a heavy motor is to be fastened to the ceiling, through bolts must be used instead of sockets. If the positions these bolts are to occupy can be determined beforehand, they may be placed in the forms and embedded in the concrete, in which case the threaded ends should project. This practice, however, is not so good as to place in the forms pieces of pipe having an inside diameter great enough to allow the bolts to pass through. Such pieces of pipe are more conveniently placed in the forms than are bolts, and, besides, they provide a neat, steel-lined hole in the concrete through which the bolts may be slid in and out with little friction. Such pipes have had an extensive use in certain large industrial buildings, where they have been placed horizontally through the stems of beams and girders to provide for shafting and other overhead installation.

54. If it is found necessary to provide bolt holes after the concrete has been placed, they can be drilled with a diamond-pointed drill. In all instances, such bolt holes should be located on the plans so as not to cut or damage the reinforcing rods. If rods are encountered, not only is the construction likely to be weakened by the cutting of the steel and the damaging of the concrete, but it will be very difficult to make the bolt hole accurate. It is also difficult to place any kind of fastening in the soffit of a reinforced-concrete beam unless provision is made in the placing of the concrete work. This is due to the fact that, in drilling into the concrete, the steel reinforcement is encountered at a distance of $1\frac{1}{2}$ to 2 inches from the surface, and, besides, such a fastening must be in the nature of an *expansion bolt,* which is a bolt or screw screwed in between

two metal plates inserted in the drilled hole. Many designs of expansion bolts are on the market but none are reliable when subjected to tension.

ADJUSTABLE SOCKETS

55. The non-adjustable sockets have been largely displaced by some form of concrete insert that permits of adjustment in position of the bolt inserted after the construction is completed. The adjustable socket is always advantageous when shafting, machinery, or other apparatus is to be installed.

In Fig. 53 is shown a type of adjustable socket known as the **Hancock insert.** This device consists of a cast- or malleable-

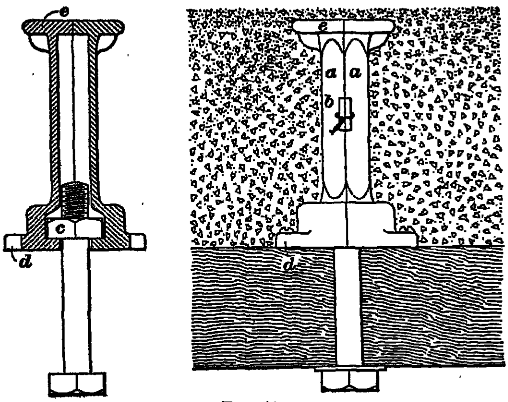


FIG. 53

iron casing that is made in loose halves *a* and is wired together through the lugs *b* cast on the sides. Before the two halves of the insert are wired together, however, there is placed in the recess a nut *c*, into which a bolt can be screwed from beneath. The castings are arranged with lugs, or flanges, *d* on the face end by means of which the insert may be screwed or nailed to the form board, and thus secured in an upright and secure position when the concrete is placed. The flange *e* at the top of the insert furnishes additional bond, or key, with the concrete.

The Hancock insert is made in 3-, 4-, and 6-inch lengths for $\frac{1}{4}$-, $\frac{3}{8}$-, $\frac{3}{4}$-, and 1-inch bolts. The bolts are capable of con-

siderable adjustment, on account of the play, or allowance made for the nut in the recess of the casting.

56. The adjustable concrete insert known as the **Bigelow socket** has probably seen as wide use as any, especially in concrete buildings having flat slab floors. The casting, as

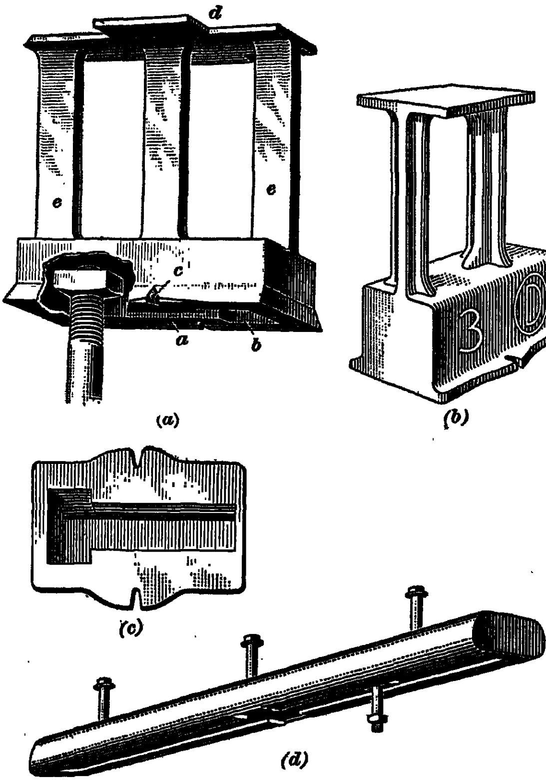


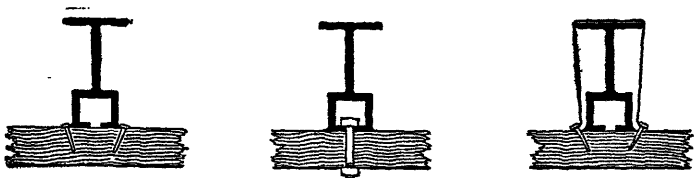
FIG. 54

shown in Fig. 54 (*a*), is made of malleable iron and arranged with a slot *a* that is enlarged at *b* so as to receive the head of the bolt. In order that the casting shall be securely held in the concrete, it is provided with a plate *d* and connecting pieces *e*. On the side of the socket at the face are cast notches *c* into which nails may be driven to fasten the device to the forms

of the slab centering. This socket is made for $\frac{1}{4}$-, $\frac{5}{16}$-, $\frac{3}{8}$-, $\frac{7}{16}$-, $\frac{1}{2}$-, and $\frac{3}{4}$-inch bolts.

A more recently devised socket that is similar in principle to that shown in Fig. 54 (*a*) is the **Dayton** socket, shown in Fig. 54 (*b*) and (*c*).

The **Truscon** insert, Fig. 54 (*d*), has a wide range of adjustment. On account of its length it should be placed in slabs in a



(*a*) (*b*) (*c*)

FIG. 55

direction parallel with the tension reinforcement, otherwise it occupies the place intended for reinforcing steel and thus reduces the effective depth of the slab.

57. Sockets of the types shown in Fig. 54 (*a*) and (*b*) may be secured to the forms by any of the three methods shown in Fig. 55. In the method shown in (*a*), the one generally employed, the socket is secured by nailing through the lugs on the side of the casting; in (*b*) a bolt with its head in the slot and passing through the form board is used; and in (*c*) a bar-iron strap, or stirrup, is passed over the top of the casting and its ends are secured with nails to the form boards. The bolts may be secured to the socket by means of locknuts as in Fig. 56.

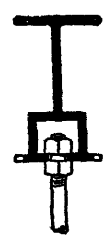


FIG. 56

58. Continuous Inserts for T-Headed Bolts.—In the construction of factory buildings it is sometimes desirable to arrange a slot in the bottom of a concrete beam or girder so that a T-headed bolt can be introduced at any point along the entire length. The Truscon insert, shown in Fig. 54 (*d*), is especially useful as a continuous insert. Several other methods are employed, the principal ones being illustrated in Fig. 57.

In Fig. 59 is shown another use of the sockets. In this case an I-beam track for a trolley hoist is supported by two light channel irons *d*. These channels are placed back to back, being

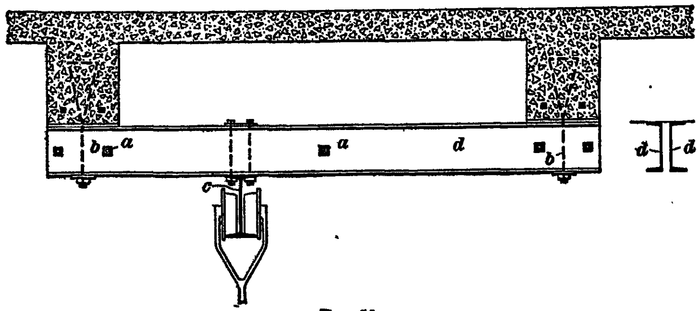
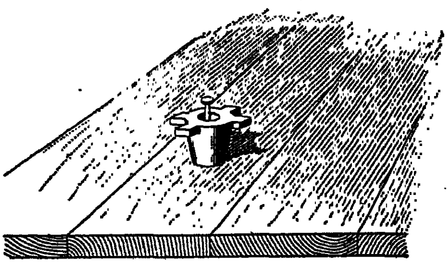


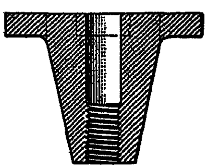
FIG. 59

spaced by means of separators and bolts *a* and secured to the concrete beam with bolts *b*. The trolley track *c* can be fastened to the channel irons at any point. Channels arranged in this way will securely support curves, frogs, or switches, and are sufficiently strong to carry the usual load placed upon hoists of this character.



(b)

SLAB SOCKETS



(a)
FIG. 60

61. It is good practice in reinforced-concrete work, especially in industrial plants where the electric-light and power wiring is to run upon the soffit of the floor slabs, to embed therein small cast-iron sockets tapped out to $\frac{1}{4}$ inch. For this purpose, the type of socket shown in cross-section in Fig. 60 (*a*) can be used. Several sockets can be

placed at intervals upon the form boards for the slab and each socket can be held in place with a nail, as shown in view (*b*), while the concrete is being tamped. Usually five or six sockets are sufficient for one panel.

Slab sockets must always be of a type that can be securely fastened to the slab centering, because they are subjected to such hard usage in placing and tamping the concrete that they are readily disturbed and displaced, which is liable to render them useless.

MISCELLANEOUS INSERTS

62. Electric Conduits.—Where the arrangement of the electric lights can be determined beforehand, the best practice is to embed conduits for the electric wiring in the reinforced-concrete work. The switch boxes are usually placed on the sides of the columns, and the outlet boxes on the soffits of the floor slabs or beams. The outlet boxes serve as fastenings for the electric fixtures.

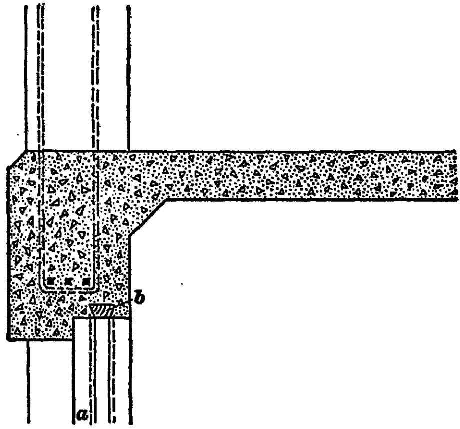


FIG. 61

The work is installed in such a manner that when the forms are removed the conduits are found buried entirely in the concrete, with only the switch boxes and outlet boxes showing flush with the surface. Care must be taken to have all the outlet boxes in place before the steel is placed. After the reinforcement has been placed and secured, the electrician returns to install his conduits, which are bent and arranged to interfere as little as possible with the steel reinforcement. The conduits should be kept below the neutral axis of the beam or slab in order to keep the compression area intact, but above the reinforcement.

63. Fastenings for Doors and Windows.—In addition to the rabbet usually provided for doors and windows, special nailing blocks are required to fasten them securely in place. In Fig. 61, the rabbet is indicated at *a*, the nailing block at *b*. The nailing block may be simply a beveled wooden strip, or any one of the many patent devices for receiving nails, screws, or bolts may be used. Such nailing blocks must, of course, be installed in the forms before the concrete is poured.

64. Pipe Sleeves.—Where a vertical pipe or duct is to pass through a concrete floor, an opening must be left in the concrete in order to avoid the expense of cutting through the completed work and the danger of cutting the reinforcement. In forming the opening, a sleeve of slightly larger diameter than the pipe is often used in order to allow for longitudinal expansion and contraction of the pipe. The annular space between pipe and sleeve is covered with a *floor plate* surrounding the pipe. In factory construction the sleeve is often simply a piece of pipe extending a few inches above the finished floor so that the water used for washing the floor will not drip into the story below.

PROTECTION OF EXPOSED CORNERS

65. It is well to remember that it is difficult to mold sharp corners in concrete, and that weak edges will pull off with the forms and require a great deal of difficult and unsatisfactory patching. Sharp corners are particularly undesirable in lintels and cornices, where ornamental effect is desired and where patching can hardly be allowed. It is therefore good practice to chamfer both the reentrant and projecting corners. For example, in Fig. 62 the projecting corner *a* has been chamfered by the simple expedient

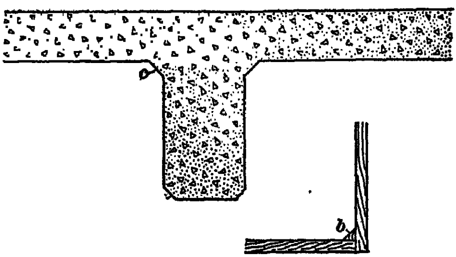


FIG. 62

of placing a bevel strip at *b* in the beam form. In the best type of design, the entrant corner at *c* is also beveled, but this is more expensive. Because of the difficulty of obtaining a good finish at sharp corners, molded concrete work should as far as possible be designed to have rounded corners or oblique angles.

FIG. 63

66. For columns and door openings, rounded or chamfered corners are also to be preferred in spite of their higher cost. Where there is much light traffic, as in schools and offices, the corners may be protected by means of a *corner bead*, Fig. 63, or by rounded metal strips with prongs, of which a large number of types are on the market. The corner beads are set in the forms and the concrete flowing around the prongs holds the bead in place. The beads may also be secured to the reinforcement by means of iron wire.

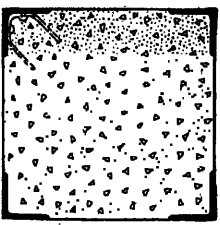


FIG. 64

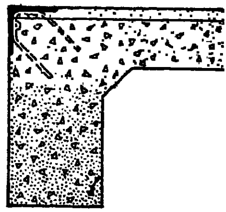


FIG. 65

Similar beads of heavier metal can be seen in place in the sidewalk curbs of many cities, especially at the street intersections. In reinforced-concrete buildings, the same design can be used to advantage for the edges of loading platforms and the like.

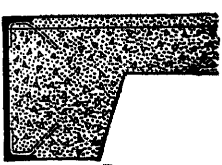


FIG. 66

Where heavy boxes and barrels are handled, and where trucks are used, it is better to protect the corners of the columns with *angle-iron guards*, as in Fig. 64, for a height of several feet above the floor level. The angle irons are kept in place by means of prongs, one of which is shown in the illustration, riveted to the angle irons; the rivets should be countersunk on the outside.

Similar devices are frequently used to reinforce the edge of the floor at elevator hatches, the top of the angle being set flush with the top of the floor finish as shown in Fig. 65.

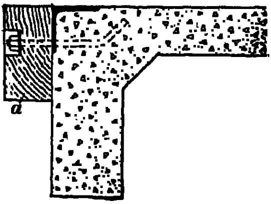


FIG. 67

Channel irons, embracing the entire depth of the floor, as shown in Fig. 66, are sometimes used at elevator hatches. Wooden stringers or bumpers, as shown in Fig. 67 at *a*, are not uncommon on concrete loading platforms. The bumper is provided more to protect the wagons than the concrete.

FLOOR FINISH IN CONCRETE BUILDINGS

67. As the top surface of a reinforced-concrete floor slab is not sufficiently smooth to form a finished floor, it is customary to supply a floor finish of some kind on top of the rough slab. Depending upon the contemplated use of the building, any one of a number of materials may be selected. That most commonly used is *granolithic finish,* or *granolithic,* as it is sometimes called, which is composed of cement and sand, and therefore seems particularly appropriate in a concrete building. Wooden floor coverings are also used extensively in spite of the fact that they are combustible. Less frequently used are coverings of asphalt, cork linoleum, magnesium composition, terrazzo, or tile.

GRANOLITHIC FINISH

68. By **granolithic finish** or *cement finish* is meant a coat of cement-and-sand mortar spread out over the rough concrete base and troweled to a smooth surface, as indicated in Fig. 68. The composition is usually 1 part of Portland cement, 1 part of coarse sand, and 1 part of crushed rock. The thickness varies from $\frac{3}{4}$ inch to 2 inches; 1 inch is commonly used. To make sure that the proper thickness is obtained, thin wooden guides, called *screeds,* are placed on the floor to serve as bases for the straightedge with which the finish is leveled off.

The coat may be applied before the base sets hard, in which case the coat unites readily with the base, or the coat may be placed weeks or even months after the base was put in, and in that case special precautions are necessary to insure a firm union between the two. On old surfaces a thicker finish coat is required, because it is more likely to stick than a thin one. Therefore, if a very cheap finish is desired for rough work, where a coat as thin as $\frac{1}{4}$ inch is sometimes used, it must be applied immediately after the base has been laid.

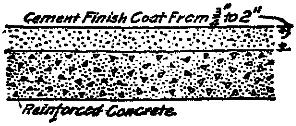


FIG. 68

69. Aggregate for Cement Finish.—It has been found too often that granolithic finish as ordinarily installed proves disappointing under heavy service. The surface in such cases wears down rapidly, and the minute particles worn off from the surface form dust, which is always objectionable. Investigation has taught that this condition is caused largely by the use of too fine aggregate, and that the best results are invariably obtained by using either coarse sand or crushed rock, or both together, containing under no circumstances particles so fine as to pass through a No. 30 sieve (with meshes of about $\frac{1}{60}$ inch in diameter). The upper limit is not so important, but should be from $\frac{1}{2}$-inch to $\frac{3}{4}$-inch size. The aggregate should be well graded between these limits and should consist of rock able to withstand abrasion. A finish coat made of 1 part of Portland cement and 2 parts of such aggregate will not be likely to cause trouble if laid with ordinary care. A fairly stiff mortar should be used, with just enough water so that it can be rammed into place with a light square tamper.

70. Installation of Cement Finish.—As stated before, the best bond between base and finish coat is obtainable where the cement finish can be laid before the base hardens. Yet many engineers and contractors prefer to place the finish coat after the building has been enclosed, rather than risk a sudden shower or unexpected frost on the green cement

finish, as this is certain to spoil the work. Also, the erection of falsework and forms for the next story is likely to mar the new cement finish, or even injure it. Moreover, leakage of mortar from the floor in process of erection above has often resulted in damage to the finish. Therefore, the cement finish is sometimes postponed to the very last, and it then becomes necessary to bond the new work to the old, which may be accomplished successfully by adhering to the following specifications suggested by Sanford E. Thompson (Transactions Am. Soc. Mech. Eng., Vol. 36, p. 387), which should be followed in the order indicated:

71. (*a*) Roughen the surface of the base concrete at the age of about 24 hours, so as to remove most of the surface scum.

(*b*) If surfaces have not been thus roughened, pick with a bush hammer to remove a part, but not all, of the surface skin.

(*c*) Spread dilute muriatic acid about one part acid to four parts water over the surface, allow to stand for a few minutes, then soak thoroughly with water, and wash off the surface.

(*d*) Sweep off the excess water on the surface of the concrete and spread on a coating of neat-cement paste, about $\frac{1}{8}$ inch thick, and broom it well into the concrete (do not use dry cement instead of the paste).

(*e*) Mix the granolithic finish in proportions of 1 part cement to $\frac{3}{4}$ part coarse sand, and $1\frac{1}{4}$ parts crushed granite screened through a $\frac{3}{4}$-inch-mesh screen and caught on a $\frac{3}{16}$-inch-mesh dust jacket.

(*f*) Make the consistency of granolithic rather stiff so that the water will just flush to the surface.

(*g*) Have the screeds laid parallel and level so that the granolithic can be spread even with a straightedge run over the screeds. See that plenty of material is being pushed ahead of the straightedge at all times so as to avoid pockets in the surface.

(*h*) Ram the granolithic with light square-faced tamper.

(*i*) Float the granolithic surface as soon as it begins to stiffen.

(*j*) Trowel the granolithic surface hard as soon as the proper stage has been reached, as determined by a competent cement finisher.

(*k*) Cover the surface of the granolithic about 24 hours after laying, with wet burlap or similar material which will hold water. Wet material each day, and oftener if necessary, for a period of 14 days.

72. Granolithic on Cinder Concrete.—Another method of installation is indicated in Fig. 69, where on top of the rough concrete slab is placed a 2-inch layer of cinder concrete, usually called *cinder-concrete fill,* or *filling.* This concrete is made up of 1 part of cement to 3 parts of sand and 7 parts of clean boiler cinders. The top of the cinder fill is finished with a 1-inch cement-finish coat, consisting of 1 part of Portland cement, 1 part of coarse sand, and 1 part of crushed rock. This finish coat is troweled smooth, and sometimes marked off in blocks. When constructed in this manner, the finish coat need not be placed at the same time as the concrete of the reinforced work. Care should be taken to have the cinder concrete *wet* before the finish coat is placed, as otherwise the porous cinders will draw the water away from the finish coat and prevent it from hardening.

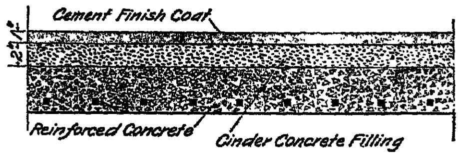


FIG. 69

73. Joints.—The floor surface is often marked off in blocks by shallow joints cut in the cement finish. It was formerly erroneously thought that these joints would eliminate shrinkage cracks in the granolithic surface, but where the finish is placed over a slab continuously reinforced with steel, these shallow joints have little effect on the cracking, because the top finish will crack whenever the base does.

One objection to joints is that in some cases the continuous striking of the iron wheels of trucks against the edge of a joint has started an innocent looking chipping of the finish which in time developed into a big patch. Hence it is far bet-

ter to place the granolithic finish in one continuous sheet from one end of the building to the other, without joints. If, however, expansion joints are cut *through the base,* these joints should be continued vertically through the finish coat, so that contraction and expansion can take place freely. Joints should also be used for all concrete flooring and covering placed directly on the ground, and especially where no reinforcement is used.

74. Dust Prevention.—Floors properly laid will not cause much trouble from dusting, and the longer they are used, the less they will dust, because of the wearing down to the hard rock forming the aggregate. But as a safeguard, patent compounds known as *floor hardeners,* for mixing with the finish coat or sprinkling over the top before troweling, are occasionally used. This process will give the finish better wearing resistance and help to make it dust-proof. Most of these compounds contain some form of iron ore or iron filings. Some of them have carborundum as an ingredient.

Although properly laid granolithic finish will give satisfaction, it may sometimes be found necessary to prevent dusting of old floors improperly laid. For this purpose, surface coatings of different kinds are used.

Paint is sometimes used, but it is hard to put on properly, is likely to come off quickly, and will, therefore, afford only temporary relief. A better method is to apply one or more coats of the various *liquid compounds* on the market. Some of these are intended to be mixed with the cement mortar, while others are applied afterwards as a surface wash. Where the concrete is porous so that the compound will penetrate a distance into it, satisfactory results have been obtained by washing with equal parts of water and *silicate of soda* solution. Solution of a compound known as *magnesium fluosilicate* have also been successfully used; the solution contains 7 to 15 per cent. of the compound. In cases where only the surface is soft, it may be removed by grinding away the soft layer and exposing the harder body underneath. The result of grinding the surfaces is described in the following article.

75. Ground Granolithic Finish.—A very pleasing effect is obtained by grinding the surface of granolithic finish with carborundum. A machine equipped with a circular grindstone revolving horizontally may be used, the purpose being to remove the thin surface coat. In this manner an excellent wearing surface is obtained, consisting of the grains of sand and stone ground flat.

76. Terrazzo is a ground granolithic finish obtained by using Portland cement and marble chips without sand. The grinding process leaves the outline of the chips exposed with a polished, smooth white surface. The cost of terrazzo is high and increases with the size of marble chips used, because the larger the chips, the deeper the cut must be in order to bring out the outline. Terrazzo is used mostly in halls of public buildings.

77. Steel grillage and cast-iron plates have been embedded in granolithic finish where heavy trucking makes exceptional demands upon the wearing surface of the granolithic finish.

WOODEN FLOOR FINISH

78. If the finished floor is to be of wood, it is usually constructed in the manner illustrated in Fig. 70. On the top of the reinforced-concrete slab are placed beveled sleepers. These are usually made of 2″×3″ yellow pine or chestnut, and are

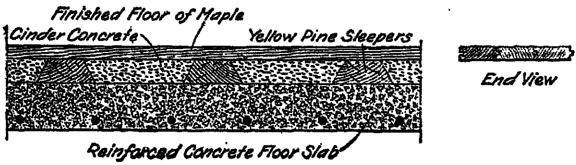


FIG. 70

placed 16 inches center to center, extending in a direction at right angles with the flooring. The space between the sleepers is filled with cinder concrete which is well tamped and made flush with the top of the sleepers. In commercial and manufacturing buildings, the flooring usually consists of 1-inch or

1¼-inch maple tongued-and-grooved flooring with 2¼-inch face. This flooring is nailed to the sleepers and left as laid, or, in some instances, the floor is planed.

Instead of the beveled sleepers, sleepers with straight sides may sometimes be used; these are held in the cinder-concrete fill by means of nails driven in the sides at intervals of from 2 to 3 inches. The sleepers to which a wooden floor finish is secured should never be embedded directly in the reinforced concrete, because such practice would decrease the strength of the work, by decreasing the compressional area of the concrete.

In some cases a ½-inch air space is left between the under side of the wooden floor and the top of the cinder concrete in order to create an air space, for the purpose of reducing the danger of dry rot. But such floors are very noisy, and it is much better to use either perfectly seasoned sleepers or patented metal sleepers, and then fill the cinder concrete flush with the bottom of the flooring.

79. Sometimes a double flooring is desired, in which case the under flooring is laid directly on the sleepers, and then the finish floor is nailed on. A layer of waterproof building paper is frequently placed between the two layers of wood. The under flooring may be old form lumber or other rough stuff. Sleepers have, in some cases, been dispensed with entirely where double or triple thickness floors were used. The layers are run diagonally in opposite directions and spiked, resting merely upon a sand cushion or upon sand and coal tar mixed. The sleepers, in other cases, have been laid in dry cinders. The construction shown in Fig. 70 has, however, been used very much more than any other, except the granolithic.

MISCELLANEOUS FINISHING MATERIALS

80. Asphalt, being soft, is not adapted for floors carrying heavy loads. It has found some use in chemical factories and laboratories, because it is unaffected by acids and other strong chemicals. Its dead black color and viscous nature have prevented its more general introduction.

81. Linoleum and cork carpets are fastened with nails on a smooth level base. As it is impossible to drive nails into a well-made concrete slab, wooden sleepers have sometimes been embedded in the concrete to receive the nails, or a special base of sawdust concrete has been spread under the carpet. In one instance it was found that a sawdust concrete composed of 1 part cement, 2 parts sand, and $\frac{3}{4}$ part sawdust gave good holding power to the nails.

These coverings are well adapted for hotel rooms, club houses, and similar structures.

82. Magnesium composition is used under various trade names in preference to granolithic finish where a more resilient and less noisy floor is wanted. Magnesium flooring has the added advantage that nails and screws may be fastened in it. Its use, however, has been confined to floors having light service, such as in offices, schoolrooms, and laboratories.

83. Tile is a word embracing a great variety of materials, from baked clay to rubber. The clay, or ceramic, tile, in spite of its high cost and weight, makes an ideal covering for flat roofs intended for roof gardens, for porch floors, and for lavatories, it being highly ornamental and practically indestructible. The tiles are laid in a cement mortar bed from 1 to 2 inches thick.

REINFORCED-CONCRETE BLOCK CONSTRUCTION

84. The construction of reinforced-concrete exterior wall piers and spandrels has been attended with several difficulties, namely, the obtaining of a good architectural finish, the cost of the wooden forms, and the difficulty of constructing this work in freezing weather without having the surface of the piers materially damaged by frost. In order to avoid these difficulties, a type of building constructed with piers built of hollow concrete blocks, the holes of which are filled with concrete reinforced with steel rods, has been designed, and, in several instances, erected.

85. In Fig. 71 (*a*) is shown a typical pier of this construction, two courses *a* and *b* of the pier being shown in cross-section in (*b*) and (*c*). As will be observed, each course consists of two hollow concrete blocks, with the holes in the blocks in each course coming over those in the course beneath. In this way, reinforcing rods *c* can be placed and the holes in the blocks filled with concrete. A composite pier is thus formed, consisting of the concrete blocks molded previously and laid in cement mortar, in a manner consistent with the best practice, and reinforced-concrete cores, which add to the bearing strength of the piers as well as to their lateral stability.

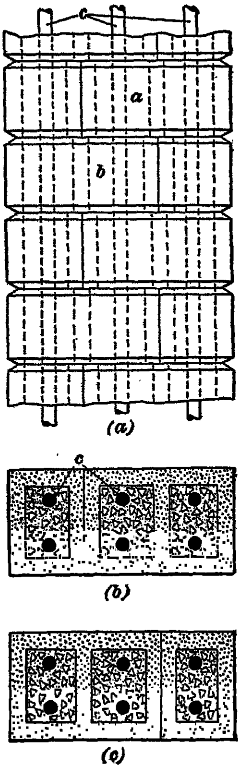


FIG. 71

In designing such piers, the standard size of blocks made by the particular concrete-block machine to be used must be adhered to, and the piers must be of a width and a thickness to correspond with the dimensions of the blocks. Many block machines are limited as to the height of block that they will hold, so that the story heights must be worked out to suit the height of the blocks when laid. This feature is important to observe, as the height of the blocks fixes the location of the beam, girder, and lintel bearings. This may be explained by means of Fig. 72, which shows the lintel and floor construction in conjunction with a concrete-block pier. The lintel supports a spandrel and is of such height as to permit the floor slab to come at the horizontal joint of the block course, as shown at *a*, and the girder that extends into the wall is of such depth that it will bear upon the next course but one below the lintel. It can thus be seen that the floor height is regulated by the height of the blocks.

86. If it is not possible to arrange the depth of the beams so that they will work out with the horizontal joint of the blocks, the end of the beam can be constructed in the manner shown in Fig. 73. By arranging the forms so that the fractional course *a* below the beams will be formed of concrete, the bearing of the beam is carried down to the top bed of the block below.

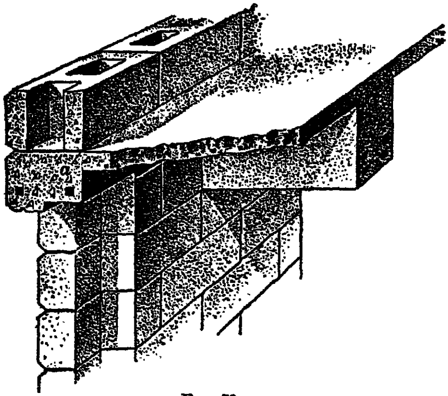


FIG. 72

The spandrels in this construction may be made either of solid concrete, as illustrated in Fig. 74 (*a*), or of hollow concrete blocks constructed as shown in (*b*). It is customary to provide on the top of the spandrel a solid artificial-stone sill made in short sections in an ordinary concrete - block machine.

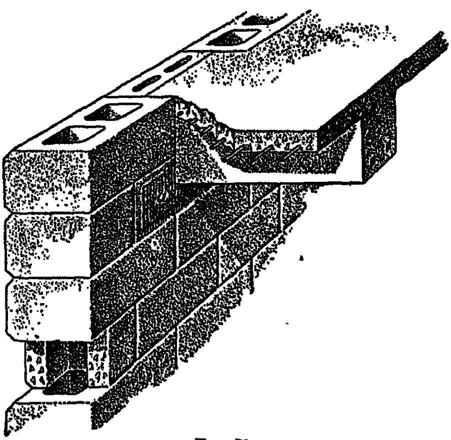
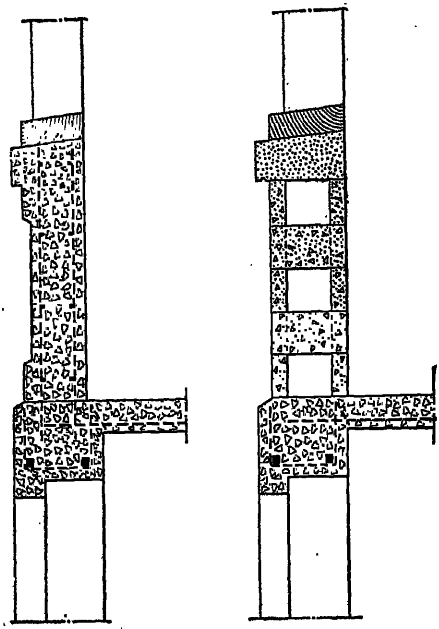


FIG. 73

87. A pier constructed in the manner just described will not have the same bearing strength as a monolithic pier of reinforced concrete, and should not be required to carry more than about 300 pounds per square inch. For while the reinforced-concrete cores, if solidly built, would be capable of resisting safely 500 pounds to the square inch, it is more than possible that this filling will not be carefully done.

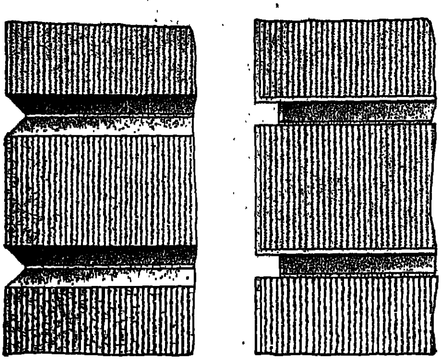
Piers of this character can be constructed in two ways. One method consists in first placing the hollow blocks to the full height required for the pier. Reinforcing rods are then inserted and the concrete filling of the cores is tamped in the best manner possible under the conditions. It is necessary to use almost a grout for this purpose. The other method consists in placing the reinforcing rods for the full height of the pier, threading the blocks over their top, and filling in each course with concrete as the work progresses. This construction is more troublesome than the one first mentioned, but is likely to result in better work.



(a) (b)

FIG. 74

88. In constructing buildings with blocks in the manner just explained, it is customary to use blocks that look like dressed stonework, and to cover the joints by molding the blocks with either rabbeted or chamfered edges, as shown in



(a) FIG. 75 (b)

Fig. 75. In forming the blocks with chamfered edges, as in (*a*), a slight bevel of about 1 inch on the side is generally used, so as to give good joint-line effects to the piers and not give the effect of rustication. Many machines are arranged so that the mold has to be filled from the top, in which case the metal edge that is placed in the form to give the chamfered effect is bothersome, as it interferes with the tamping of the cement. Where a rabbeted edge is used, the blocks are made as indicated in (*b*), the rabbet being provided on the lower edge of each block, so that all blocks are alike and may be laid up as indicated.

STEEL-SASH WINDOWS

89. The window sash used in reinforced-concrete buildings are of two kinds, made, respectively, of wood and of steel. In modern factory construction the use of wooden sash has largely been abandoned, and steel sash are used instead because of their greater fire-resistance, it having been found in many cases that a fire has gained access from a lower story of a burning building to an upper story because the wooden sash caught fire and burned out.

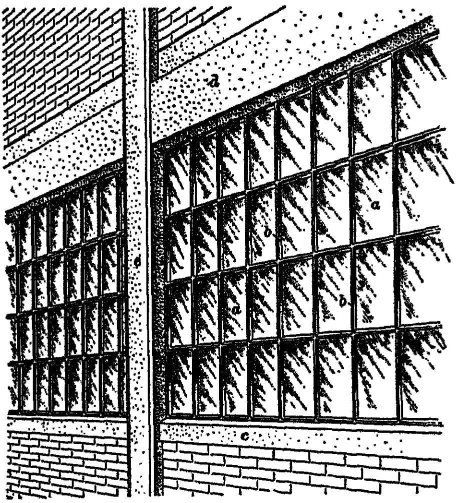


Fig. 76

90. If set in incombustible sash, window panes of even ordinary glass will retard the spread of the flames to a certain extent; if greater protection is desired, glass reinforced with woven metallic mesh, so-called *wire glass*, is used for the panes. Wire-glass

window panes are obtainable in various thicknesses and finishes; according to the character of its surface, window glass is known as smooth, rough, ribbed, or corrugated. Steel sash, glazed with wire-glass panes, furnish the best fire-resisting window construction known. In the following pages some of the features of steel-sash windows—or, as they are sometimes called, steel windows—will be described.

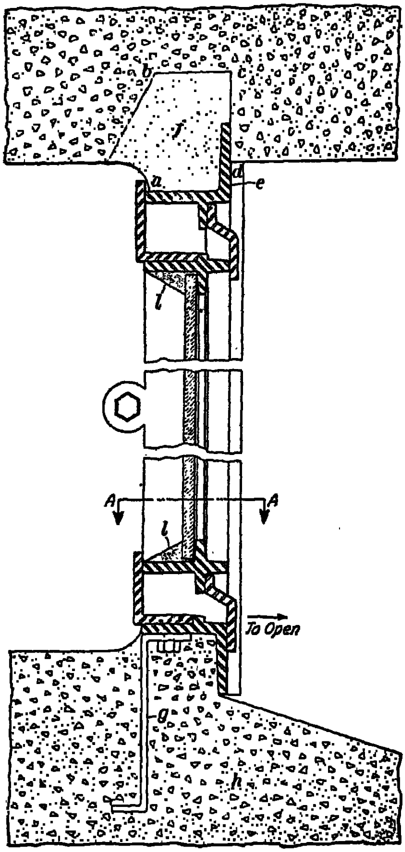


FIG. 77

91. There are on the market many types of steel windows differing in minor details of construction. Common to the majority of them is the construction shown in Fig. 76, where many small glass panes *a* are set in and supported by a framework of light horizontal and vertical sash bars *b*. This framework is supported on the concrete sill *c* and fastened at the top to the lintel *d*. At each side, support is also furnished by the concrete columns *e*.

92. Steel windows are built with either stationary or movable sash. When stationary sash are used, the sash bars are immovably mounted in a common frame to form a single unit; where it is desired to open the windows, a movable sash that swings on a pivot is used The movable sash may consist of a

single light, or of any number of lights, or the window may be composed of sash all of which are movable. Those parts of the stationary steel frame that bear against the concrete are known respectively as the *head* (at the lintel), the *jambs* (at each side), and the *sill* (at the bottom). Details of a form of construction with movable sash are shown in Fig. 77, which is a vertical section through the head and the sill, and in Fig. 78, which is a horizontal section through the jambs. In these figures the steel bars that constitute the window frame and the sash are shown section-lined; their arrangement differs somewhat in frames of different makes and will not be described here, since the main point of interest to the concrete engineer is the method used for fastening the window frames to the concrete.

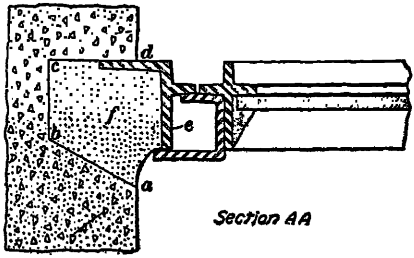


FIG. 78

In order to fasten the frames to the concrete, recesses *a b c d*, Figs. 77 and 78, are cast in the concrete of the lintel and the columns. The angle irons *e* of special design are set into these recesses, and the recess is then filled with cement mortar *f*. To fasten the frame at the bottom, strap-iron anchors *g* project downwards from the frame into the concrete sill *h* as shown in Fig. 77, or, as in Fig. 79, the angle iron *i* of the frame is partly embedded in the concrete sill *k*. In order to make this construction possible, the window frames are set before the concrete of the sill is poured; the plastic concrete of the sill flows around the anchor *g* of Fig. 77, or the angle iron *i* of Fig. 79, and after hardening keeps the frame securely in place.

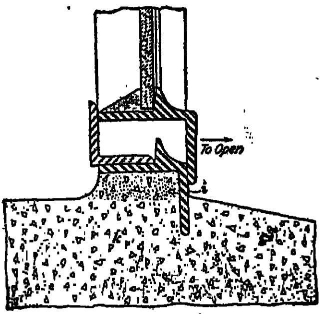


FIG. 79

While the concrete of the sill is being poured, the frame is supported and held in place, as shown in Fig. 80, by small

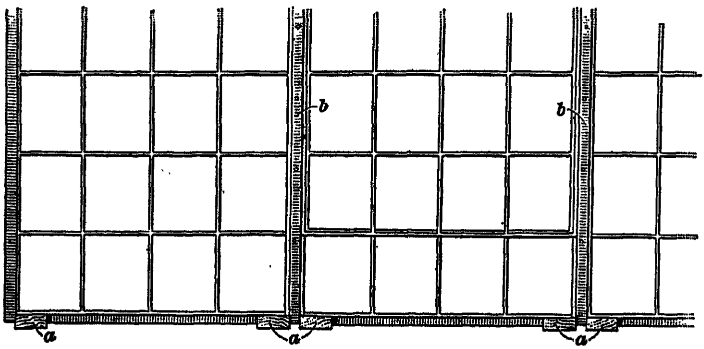


FIG. 80

wooden blocks *a* placed under the sash at the jambs and under the heavier uprights *b*, which are called the mullions.

93. While steel sash are very durable and satisfactory after they have been properly installed, they are nevertheless

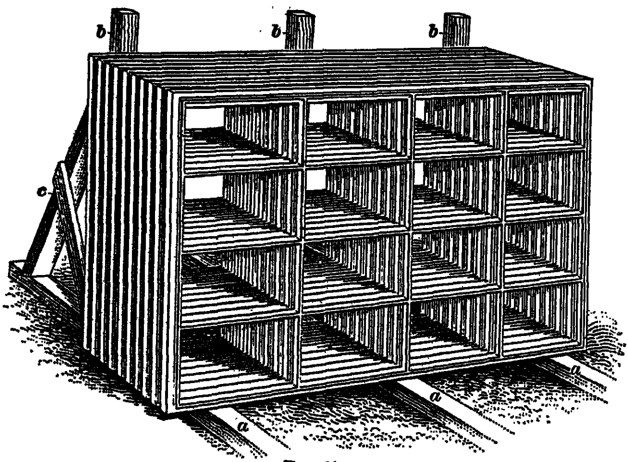


FIG. 81

easily damaged in handling and setting, owing to their slender dimensions. If for any reason the concrete openings and the

window frames do not fit well, adjustments must be made in the concrete, and since it is expensive to trim and cut the concrete, every care should be taken to have the concrete work true to line and of proper dimensions. In designing the work, advantage is usually taken of the fact that certain stock sizes are obtainable at lower cost than odd sizes, and the openings must be built accordingly.

In order to prevent the steel window frames from being bent or warped, they should be stored on the job as illustrated in Fig. 81, where a number of frames are shown resting with their longer sides upon timber sills *a* and supported in an almost upright position against timber posts *b* that are thoroughly braced by knee braces *c*.

94. The sash are received without glass, and are glazed after they have been erected; special clips are furnished to hold the lights in place, and, as shown in Fig. 77, putty is used to seal the *glazing rabbet l*. Most standard makes of steel sash are glazed from the inside.

The most commonly used standard sizes of glass are 10″×16″, 12″×18″, and 14″×20″. The sizes should not be mixed in the same building, nor are the dimensions interchangeable; thus, if 10″×16″ glass is used in one part of a building, it should be used throughout, and sizes such as 10″×18″ glass should never be used, because the sash to fit such sizes are difficult to obtain and more expensive.

OFFICE PRACTICE IN CONCRETE DESIGN

DETERMINATION OF DESIGN

FIXED CONDITIONS

1. The method of procedure in the design of reinforced-concrete buildings is as varied as the personality of the architect or the designing engineer. The practice in the offices of different engineers is not in any way uniform, so that while the information given in this Section may not conform to the practice in any particular office, an effort has been made to arrange the calculations and method of design so as to be consistent with the most universally adopted practice, and to systematize these calculations in such a manner as to shorten the work and arrive at satisfactory results.

2. Preliminary Data.—The problem worked out in successive steps in the ensuing articles of this Section will serve to illustrate the current office practice in the design of a reinforced-concrete building. As this type of construction is most suitable for buildings devoted to manufacturing, storage, or commercial purposes, the example will deal with the design of this class of structure.

It is proposed to build a five-story-and-basement factory building 42 feet wide and 146 feet long, outside to outside surface of brick walls, with the entire structural framework of the building of reinforced concrete. The spandrel filling, that is, the space beneath each window sill and above the

lintel of the window in the story below, is to be of brick masonry 12 inches in thickness. The exterior reinforced-concrete lintels are to be exposed on the face, or, in other words, are not to be veneered, or enclosed, with a brick facing. The piers are to be enclosed in brick, and are to project on the outside, like pilasters, 9 inches beyond the face of the wall.

It is decided that there shall be only one row of columns down the center of the structure. The building is to be used for heavy manufacturing purposes, requiring a floor construction capable of supporting a live load of 150 pounds to the square foot. The greatest amount of glass surface possible is to be provided in the windows, and consideration is to be paid to the construction of the lintels so that the window heads will be as close as possible to the under side of the slab. The building is to be lighted on both the sides and the ends.

Among the special features in the planning of the building is the strengthening of certain columns and wall piers and the roof, in order to form a support for a 25,000-gallon sprinkler tank to be placed on the roof. All the elevator shafts and stairways are to be enclosed with 9-inch brick walls, which are to be supported at each story by the reinforced-concrete construction.

CONDITIONS TO BE DETERMINED

3. Arrangement of Beams With Regard to Light. The general plan of the building is assumed to be as shown in Fig. 1, and the first step in the problem of design is to determine the best layout for the beams and girders; that is, the most economical layout and the one that will work out the best with regard to the lighting facility desired. It is practically determined by the conditions of the problem requiring maximum light that triple windows shall be used between the concrete wall piers, thus making the distance from center to center of wall piers 16 feet.

There are two possible beam framing plans for the building, as indicated in Fig. 2. By running a girder lengthwise of

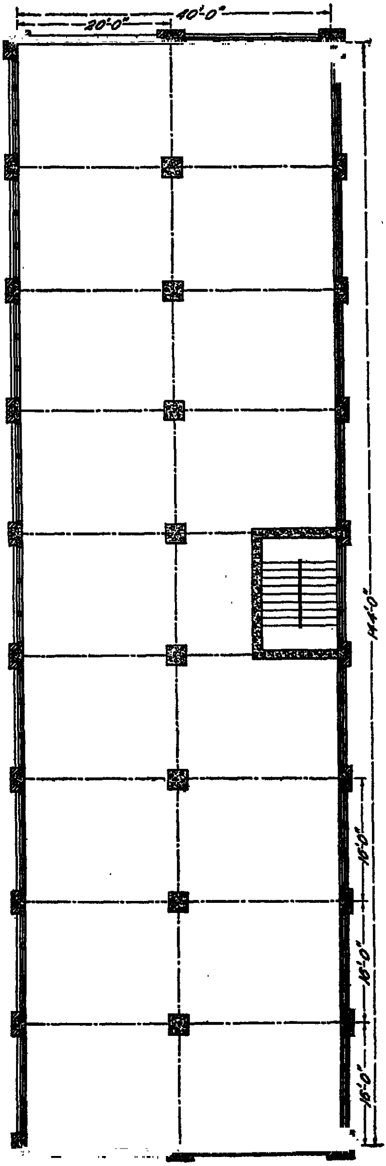


Fig. 1

the building and the beams across the building, lintels of considerable depth are required over the windows in order to receive the ends of the beams, as shown in (*a*). However, if the girders are run across the building, and the beams longitudinally, a lintel of sufficient strength to support only a portion of the floor slab and the spandrel over it will be required at the head of the window, as illustrated in (*b*). It will be noticed that this last system of beam framing will allow the window heads to be placed closer to the ceiling, giving more light in the building, but that the main supporting members or girders of the floor system have the longer span. With the other system, however, the beams, or secondary members, have the longer span and the girders the shorter span.

4. It is a question which of the preceding systems of beam framing is the more economical, and the only way this can be decided for a particular

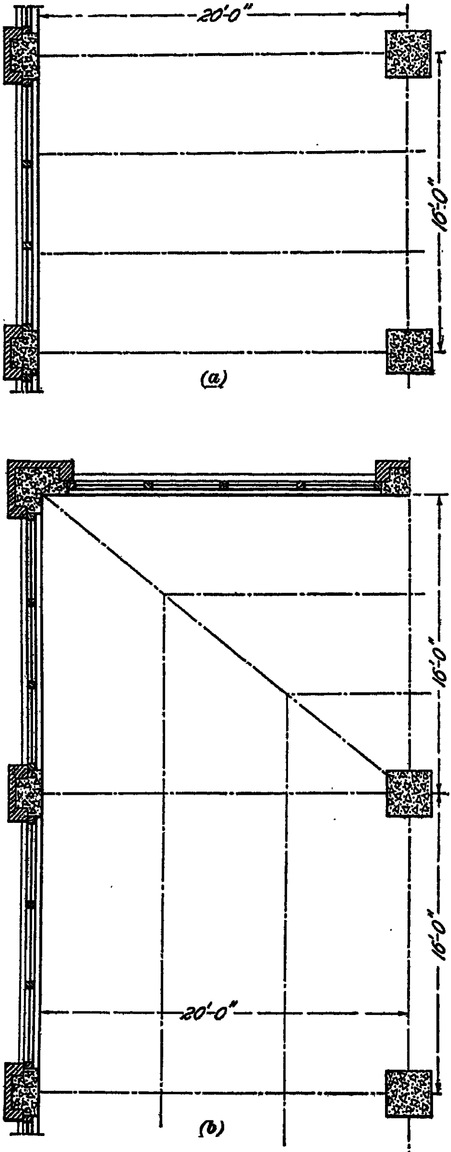


Fig. 2

instance is to make the preliminary estimates for the cost of one bay, comparing the two systems at the same unit prices. Before this can be done, however, preliminary calculations must be made to determine the approximate sizes of the beams and girders, as well as the thickness of the floor slab.

It will be assumed that these preliminary calculations are made, and that the sizes of the several beams and girders in both systems of framing have been approximately determined. When estimates of the cost of both systems are compared, it is found that the difference is slight, and as the preference is for the system that will allow the greatest height of window, the system shown in Fig. 2 (*b*) is adopted.

It is desired to follow this same sys-

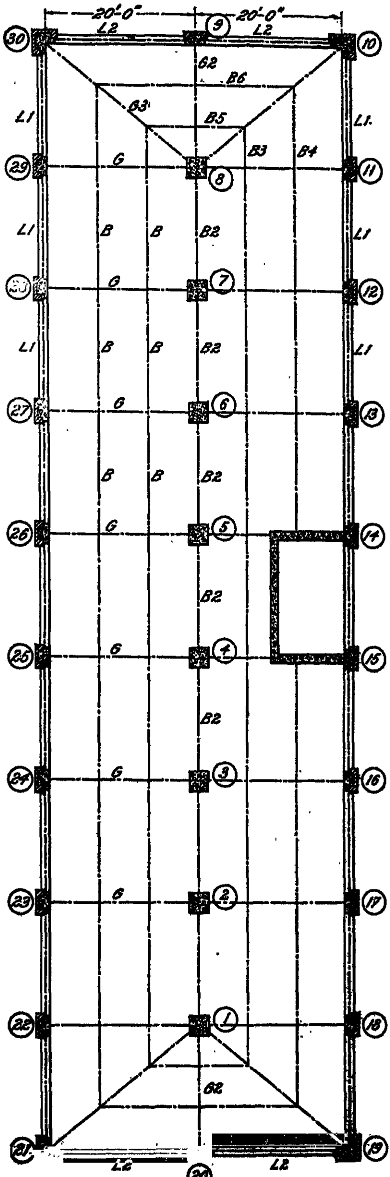


FIG. 3

tem of construction at the ends of the building as well as along the sides, and in order to do so, the end bays at each corner of the building must be formed by using an oblique girder and mitering beams running parallel with the sides and ends of the building into it, as shown in Fig. 2 (*b*). In this manner, the windows at the end of the building will be as high as those on the side.

5. Framing Plan. A typical framing plan of the floor system of the building is shown in Fig. 3. The beams marked *B* are the typical, or normal, beams of the framing plan, while those marked *B 2* are shorter than the normal beams and those marked *B 3*, *B 4*, *B 5*, and *B 6* are special beams. The normal girders are marked *G*, the girders of the end bays *G 2*, and the special diagonal girders *G 3*. The lintel beams are designated on the framing plan as *L 1* for the normal lintels on the sides, and *L 2* for the lintels on the ends of the building.

The several columns and piers are numbered so that a schedule may be made in which their sizes and reinforcement are given, or by which they may be referred to and identified in marking details. Every office has its own system of marking framing plans that may vary more or less from the above.

CALCULATION OF STRUCTURAL MEMBERS

DETERMINATION OF FLOOR LOADS

6. Live Load.—The live load is generally known from the inception of the structure. It is stipulated by the owners or determined by the architect or the engineer either from data taken by them from buildings used for the same purpose or from information given by the owners. More frequently, however, the live load is fixed by the building laws of the city in which the structure is to be erected.

In the problem under discussion, the live load is taken at 150 pounds per square foot. This load is a good average of the live loads usually employed in designing a factory, although for buildings used for light manufacturing purposes 120 pounds is considered by some engineers as sufficient.

7. Dead Load.—After the live load has been decided on, the next step is to find the dead load. The thickness of the floor slabs and the size of the beams depend on the total load, which includes the dead load. The dead load also depends on the size of the floor slabs and the beams. This state of affairs would seem to prevent the engineer from finding the dead load before the size of the slabs and beams is decided on, and also prevent him from finding the size of the slabs and beams before the dead load is decided on. As pointed out in *Concrete Beam and Column Design*, there are two ways out of this difficulty. The first is by using the tables given for that purpose, and the second is by assuming certain quantities.

8. Some architects and engineers do not use such tables, and in some cases it may be decided to use a certain mixture of concrete for which there is no table arranged. In cases like these, the second method of overcoming the difficulty, which consists in assuming certain values, must be employed. Either the weight of the concrete or the dimensions of the floor slabs and beams may be assumed.

If the weight is assumed, frequently a certain lump figure per square foot, based on precedent, is used. For instance, it is known that the dead weight of a concrete slab and beam system with cinder filling and a wooden floor weighs usually from 100 to 125 pounds per square foot of floor area. The designer would therefore design the slab and beams to carry 100 or 125 pounds per square foot plus the live load. After the correct thickness of slabs and the size of beams have been determined, the designer should calculate their actual weight to determine how nearly correct his first estimate is.

A better alternative is to assume the thickness of slabs and the size of beams and then find their weights. As this is the usual method of procedure for careful designers if tables similar to those mentioned are not to be used, it will be followed here. This method is more accurate than the one in which the weight is assumed first, because with a little experience the dimensions of a floor system will be found easier to approximate than will its weight. Then, again, this method is easier to check, and also the subsequent work can be more or less directed to make the first assumption come true.

9. To show different methods, the clear span has been sometimes used instead of the distance from center to center of supports. This practice was followed by some engineers, but is condemned by the Joint Committee; it is safer and better to follow the Joint Committee.

The distances from the center of the steel to the bottom of the beam or slab are different from those recommended by the Joint Committee. They are put in this Section to represent the practice of different engineers, although the values recommended by the Joint Committee should be used.

10. As an example of the method of calculating the dead load, the floor construction may be assumed to be as shown in Fig. 4. The maple flooring has been decided upon by the architect or the engineer and the owner. It is necessary to use at least 2 inches of cinder concrete to hold the sleepers to which the floor is to be nailed. The thickness of the slabs and the size of beams is accordingly assumed from the experience of the engineer. In the calculations, the weight of the wooden sleepers is neglected.

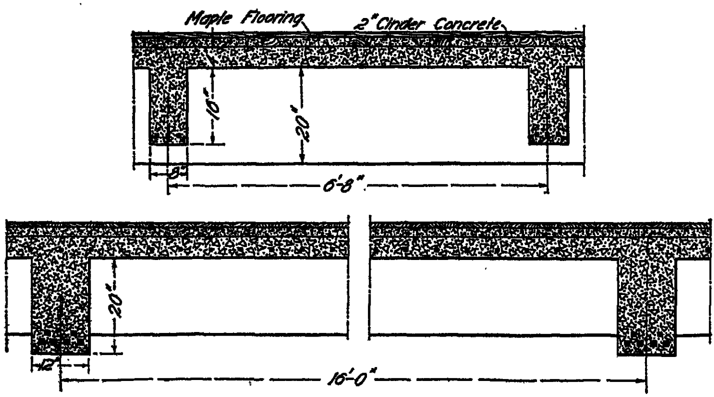


FIG. 4

The dead load per square foot to be withstood by the slab is calculated as follows:

	Pounds per Square Foot
Maple flooring	4
Cinder concrete	18
Concrete floor slab, 4 inches thick	50
Total dead load on slab	72

The beam section is assumed to be 8 inches wide and 16 inches deep, measured from the under side of the slab, so that the section of the concrete beams in parts of a square foot is equal to $.67 \times 1.33 = .891$ square foot. As 1 cubic foot of reinforced concrete weighs 150 pounds, each linear foot of the beam will weigh $150 \times .891 = 134$ pounds. Each beam supports, according to the detail, Fig. 4, a total width of floor of

6 feet 8 inches, measured from center to center of beam, so that the dead load per square foot due to the weight of the beam distributed over this area would equal $134 \div 6.67 = 20$ pounds, approximately.

The dead load in pounds per square foot of floor area on any normal beam is then equal to the following:

	Pounds per Square Foot
Total dead load on slab	72
Dead load due to weight of floorbeam	20
Total dead load on floorbeams	92

From Fig. 4 it is evident that the assumed size of the normal girder is 12 in. × 20 in., so that its weight per linear foot is $1 \times \frac{20}{12} \times 150 = 250$ pounds.

By a calculation similar to that just made for the beam, the added weight due to the girder is almost 16 pounds per square foot, and the total dead load, in pounds per square foot of floor area, is as follows:

	Pounds per Square Foot
Total dead load on slab	72
Dead load due to weight of floorbeam	20
Dead load due to weight of floor girder	16
Total dead load on girders	108

This value, as will be seen later, is used only in the design of columns and not in the design of the girders themselves.

11. Total Floor Loads.—After the live load has been decided on and the dead load has been calculated for each of the elements of the floor construction, the total load for calculating the reinforcement in the slab, beams, and girders may be tabulated as follows:

Floor Slab	Pounds per Square Foot
Live load	150
Dead load	72
Total load	222

Floorbeam	Pounds per Square Foot
Live load	150
Dead load	92
Total load	242

Floor Girder	
Live load	150
Dead load	108
Total load	258

12. Roof Loads.—In reinforced-concrete buildings, it is customary to allow a superimposed load of 30 pounds per square foot for snow and wind pressure combined. With ordinary construction, the dead load per square foot on a roof seldom exceeds from 60 to 70 pounds for the slab, 80 pounds for the slab and beam construction, and 90 pounds for the slab, beam, and girder construction. On these bases, the roof loads for the several structural elements in the roof construction will be as follows:

Roof Slab	Pounds per Square Foot
Roof load	30
Dead load	60
Total load	90

Roof Beam	
Roof load	30
Dead load	80
Total load	110

Roof Girder	
Roof load	30
Dead load	90
Total load	120

13. Special Loading.—After the several normal floor and roof loads have been determined, the lintel beams should be investigated to find the load that comes upon them, and then any special beams or girders should be analyzed, together with any columns that may differ from the normal column,

such columns, for instance, as those which are required to support a tank upon the roof. In designing all such special features of the construction, the greatest care should be exercised in order that in each case the dead load coming upon them may be properly determined, because these special beams and columns are not so likely to have the numerous checks applied to them as are the normal beams, girders, and columns. These special loadings will be discussed when the design of the members that support them are taken up.

DESIGN OF SLABS AND BEAMS

NORMAL MEMBERS

14. Calculations for Floor Slabs.—The total load per square foot upon a reinforced-concrete floor slab connecting beams and girders is not so great as the load per square foot superimposed upon the beams and girders. This was determined by the calculations made in Art. **11**.

The custom in office practice is to make the calculations for the floor slab first, so that if it requires a greater thickness than that assumed in the calculations for the dead loads upon the beams and girders, the assumed load upon these structural members may be adjusted to suit the new thickness of floor slab assumed.

From Fig. 4 it is evident that the distance from center to center of beams is 6 feet 8 inches, and if the beams are 8 inches in width, the clear span of the floor slab is 6 feet. In making the calculations for reinforced-concrete floor slabs, it is customary to consider a portion of the slab 12 inches in width and to figure the load on this width of slab, and likewise to determine the amount of reinforcement necessary in a section of the slab 12 inches wide.

Thus, in the example under consideration, the total load (see Art. **11**) is $1\times 6\times 222=1{,}332$ pounds, and the bending moment, by the formula $M=\frac{W\,l}{8}$, is $\frac{1{,}332\times 6}{8}=999$ foot-pounds, or $999\times 12=11{,}988$ inch-pounds.

As will be observed, in making these calculations, it is more convenient in using the formula for the bending moment to employ the span in feet instead of in inches, but as the bending moment must be determined in inch-pounds, the value found will have to be multiplied by 12.

15. After the bending moment has been found, the slab may be designed by Thacher's, Kahn's, or any other formula. Usually, in office practice, tables are arranged that give the resisting moments of slabs of ordinary thickness. These tables are employed to save the labor incurred by using the formulas every time a slab or beam is to be designed. They are calculated from Thacher's or some other formula. The formula used to calculate the tables depends on the preference of the engineer in charge and also on the recommendations of the building laws of the city in which the building is to be erected. As building laws are often very explicit about the design of reinforced concrete, it is necessary to consult them before going on with the work.

To make the problem under discussion entirely general, the slab will be designed by Kahn's formula. Although the unit stresses used in this formula are fairly high and may not be employed in some cities, yet, on account of its widespread adoption, it will serve the purposes of illustration throughout this Section.

16. Kahn's formula, from *Concrete Beam and Column Design*, is

$$M = 13{,}760\, d\, A$$

in which M = resisting moment, in inch-pounds;
d = effective depth of slab, in inches;
A = area of steel, in square inches.

If a 4-inch slab is to be used, it may be assumed that the depth from the center of the steel reinforcement to the top of the slab, is $3\frac{1}{4}$ inches. In Art. **14**, the required resisting moment was found to be 11,988 inch-pounds. Substituting these values in the equation, it is found that $11{,}988 = 13{,}760 \times 3.25\, A$. Solving for A, it is found that $A = .268$ square inch.

If $\frac{3}{8}$-inch square twisted bars are used, the area of each bar will be $\frac{3}{8}\times\frac{3}{8}=.141$ square inch. As the area of reinforcement required for each foot width of slab is .268 square inch, the spacing of the bars will be $12\div\frac{.268}{.141}=6.31$, say $6\frac{1}{4}$, inches.

Kahn's formula stipulates that the reinforcement must be less than 1 per cent. In this case the area is $12\times3\frac{1}{4}=39$ square inches. Then, $39\times\frac{1}{100}=.39$, which is more than .268. Therefore, less than 1 per cent. of reinforcement is used and the slab is consequently safe. It might be supposed that the reinforcement is so much less than 1 per cent. of the total area that the design would not be economical. Therefore, it might be considered advisable to assume a $3\frac{3}{4}$-inch slab, calculate the weights over again, and redesign the reinforcement. However, it is the custom of many designers to use, especially in floor slabs, considerably less than 1 per cent. of reinforcement. Although this at first sight would seem uneconomical, yet it has the advantage of reducing the concrete stresses, which is desirable because thin floors are difficult to cast. For this reason, it is decided to use a 4-inch slab reinforced with $\frac{3}{8}$-inch square bars spaced $6\frac{1}{4}$ inches apart.

17. Calculations for Beam Bending Moment.—The bending moment on the beams may now be calculated. From Fig. 2 it will be noted that the span is 16 feet. On account of the width of the girders, for very economical design the clear span might be taken at the actual distance between the girders instead of the center-to-center distance, but this practice, while sometimes used with slab design, as just illustrated, is seldom used with beams of considerable span.

In this case, the total load (see Art. **11**), is $6.67\times16\times242=25,826$ pounds, and the bending moment, by the formula $M=\frac{W\,l}{8}$, is $\frac{25,826\times16}{8}=51,652$ foot-pounds, or $51,652\times12=619,824$ inch-pounds.

In making this calculation, as has been mentioned, the distance from center to center of girders was used instead of the actual clear span. As some of the reinforcing rods are bent

up over the girders and extend into the beam on the opposite side, the beam may be considered as having fixed ends. In such cases, many engineers find the bending moment by the formula $\frac{W\,l}{10}$ instead of $\frac{W\,l}{8}$. Since, however, the latter is more conservative, it has been used in this problem; that is, the beams will be designed as simple beams.

18. Design of Beams.—Now that the bending moment on the normal beams has been found, these normal members may be designed by any convenient formulas or tables. As Kahn's formula was used to design the slab, it will be used here also. The depth of the beams will be taken to the top of the slab, because the beams and slabs are supposed to be placed at one time and well bonded together by means of stirrups.

As the total depth of the beam is 20 inches, it may be assumed to have an effective depth (from center of steel to top of slab) of 18.25 inches. Substituting the correct values in the formula, it is found that $619{,}824 = 13{,}760 \times 18.25 \times A$; therefore, $A = 2.47$ square inches. The area of the beam is $8 \times 18.25 = 146$ square inches; 1 per cent. of the area of the beam is therefore $146 \times \frac{1}{100} = 1.46$ square inches. The area required is more than this value, but inasmuch as the beam is really of **T** section, the amount of reinforcement will probably not be excessive. The stresses produced should be checked by the method given in *Concrete Beam and Column Design.*

The kind of reinforcement to use is left to the designer. It is of course necessary to use commercial sizes of bars. The combined area of three ¾-inch round bars and three ⅝-inch square bars is 2.497 square inches, which is only slightly in excess of the amount required.

The details of the reinforcement, such as the placement of stirrups, will not be discussed here. These features can be designed by applying the practical rules given in *Concrete Beam and Column Design.*

19. Calculations for Girder Bending Moment. Before attempting to calculate the girder bending moment,

it should be borne in mind that the normal girders support their own weight, which, as has been found in Art. **10**, is 250 pounds per linear foot; also, that these girders support the beams at two points, so that each girder may be considered as supporting two concentrated loads.

Referring to Fig. 1, it will be observed that the span from the center of the column to the edge of the wall pier is 20 feet. As the dimensions of the center columns have not been determined, the beams cannot be exactly located on the clear span of the girder. However, it is close enough for practical purposes to consider the conditions to be the same as those diagrammatically shown in Fig. 5.

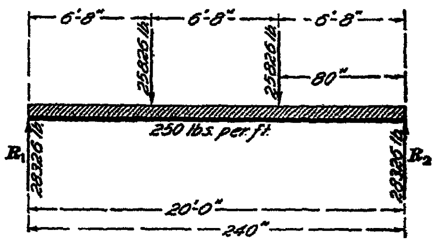


FIG. 5

Since the loads are symmetrical, each reaction is equal to half the total load and the maximum bending moment is in the middle. This bending moment may be found by the ordinary method. Thus, $M = 28{,}326 \times 120 - 250 \times 10 \times 60 - 25{,}826 \times 40 = 2{,}216{,}080$ inch-pounds.

20. Design of Girders.—The total depth of the girder to the top of the slab is 24 inches, and its effective depth may be taken at 22 inches. Substituting the correct values in Kahn's formula, it is found that $2{,}216{,}080 = 13{,}760 \times 22\,A$; therefore, $A = 7.321$ square inches.

This girder is of a **T** section and the above method of calculation assumes that the beam is rectangular. The girder should be finally checked as a **T** beam and for shear, and if necessary the slab should be assisted by steel placed in the top of the girder so as to make it doubly reinforced as well as of **T** section. After the size of columns and piers has been decided upon, the bending moment must be checked and the secondary reinforcement may be placed.

SPECIAL BEAMS

21. Calculations for Lintel Bending Moment. Having figured the slabs and normal beams and girders, the attention of the designer should be directed to the lintel beams along the side walls that carry the spandrel beneath the window sills. The proposed detail of the construction of this portion of the building is shown in Fig. 6, the view in (*a*) being in elevation and that in (*b*) in section. It will be observed from this figure that little benefit is derived from the floor slab acting in conjunction with the concrete of the lintel. The lintel beams, consequently, must be figured not as beams of **T** or **L** section, but as simple rectangular beams.

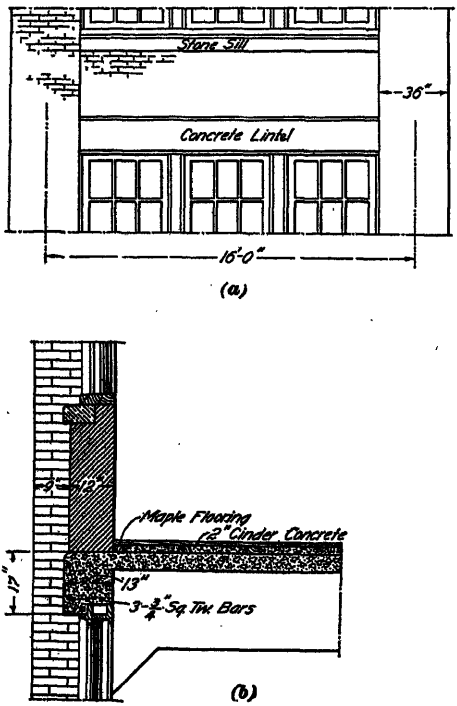


FIG. 6

22. The clear span of the lintel beam, since the piers are 36 inches in width, is equal to the difference between 16 feet and 3 feet, or 13 feet. It will be considered that the lintel beam supports a portion of the slab load equal to one-half the span of the slab extending from the wall to the first beam, and that besides it supports the stone sill and all the brickwork forming the spandrel beneath the window. The calculations will therefore be as follows:

		Pounds
Floor load $= \frac{6.67}{2} \times 13 \times 222$ (see Art. 9) ...	=	9,625
Spandrel load, including weight of lintel itself (approximately)	=	6,500
Total load	=	16,125

The maximum bending moment is, therefore, $\frac{16,125 \times 13}{8}$ $= 26,203$ foot-pounds, or $26,203 \times 12 = 314,436$ inch-pounds.

23. Design of Lintel Beams.—As rectangular beams are most economical when the steel used for their reinforcement is fully stressed and the concrete is not overstressed, but is stressed up to the allowable stress, an effort should be made in designing ordinary beams to approach this condition. The percentage of steel required for these ends varies with the nature of the concrete, as well as with the ratio of the moduli of elasticity of the concrete and steel. According to Kahn's formula, 1 per cent. of steel is allowable; but, let it be assumed that a somewhat lower stress in the concrete is desired, which would reduce the allowable percentage of steel. In this case, therefore, the desired percentage of steel will be taken at $\frac{3}{4}$ of 1 per cent.

The width of the lintel is governed by the thickness of the brick wall, and as the object is to make the lintel project slightly from the wall, it will be made 13 inches wide. Its effective depth may be called d. The area of steel required is then approximately $\frac{3}{4} \times \frac{1}{100} \times 13\,d$. Substituting the correct values in Kahn's formula it is found that $314,436 = 13,760 \times \frac{3}{4} \times \frac{1}{100} \times 13\,d \times d$. Therefore, $d^2 = 234.37$, or $d = 15.31$. It is thus evident that the total depth of the lintel would be about 17 inches.

The area of steel required is then $\frac{3}{4} \times \frac{1}{100} \times 15.5 \times 13 = 1.51$ square inches, which is about the area of three $\frac{3}{4}$-inch square twisted bars.

24. Design of Miscellaneous Beams and Girders. The main features of the design for the reinforced-concrete

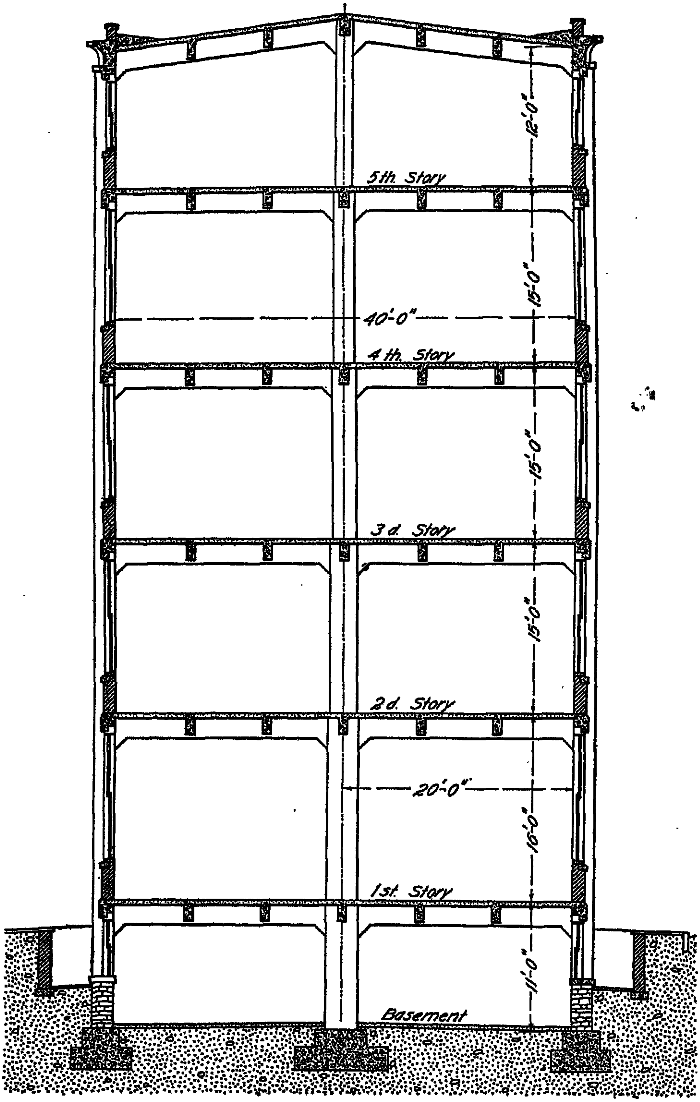

5th. Story
4 th. Story
3 d. Story
2 d. Story
1st. Story
Basement
40'-0"
20'-0"
12'-0"
15'-0"
15'-0"
15'-0"
16'-0"
11'-0"

FIG. 7

beams and girders used in the building under discussion have been worked out in a manner consistent with the usual office practice, and it remains only to work out the sizes and steel reinforcement for the roof and other special beams and girders, such as those carrying the water tank, and to decide on such details as the spacing of the stirrups, ties, shrinkage rods, etc., and such secondary reinforcement as may be required to complete the design. These details, with the exception of the stirrup spacing, are usually laid out according to the judgment of the designer. A method of spacing stirrups is considered in *Concrete Beam and Column Design.*

DESIGN OF COLUMNS

NORMAL COLUMNS

25. In calculating the loads upon the columns, it is best to tabulate the figures in such a way that the load on the column of any floor may be determined at once by inspection. The framing plan of the building under consideration is shown in Fig. 3, and the sectional elevation is illustrated in Fig. 7.

From the framing plan, it will be observed that the total area on one floor supported by a normal column is 16 ft. × 20 ft., or 320 square feet, and that the roof area is practically the same.

26. In office practice, it is customary to use a table that gives the strength of reinforced-concrete columns of different sizes. These tables may be calculated by means of any suitable formula. It has been decided in this case to allow a given number of pounds of safe load per square inch of column section, as suggested by the Philadelphia building laws.

Table I gives the safe load on square reinforced-concrete columns for various unit stresses. It will be observed on reference to the table that both the weight for 1 linear foot of column and the amount of concrete required for 1 linear foot of column are given. These values will be found useful in

TABLE I

SAFE LOAD OF REINFORCED-CONCRETE COLUMNS OF SQUARE SECTION, IN THOUSAND POUNDS

Weight for 1 Foot in Height Pounds	Cubic Yard of Concrete in 1 Foot in Height	Size Inches	Area Square Inches	Unit Compressive Stress, in Pounds			
				350	500	650	750
66.7	.016	8×8	64	22.4	32.0	41.6	48.0
84.4	.021	9×9	81	28.4	40.5	52.7	60.8
104.2	.026	10×10	100	35.0	50.0	65.0	75.0
126.0	.031	11×11	121	42.4	60.5	78.7	90.8
150.0	.037	12×12	144	50.4	72.0	93.6	108.0
176.0	.043	13×13	169	59.2	84.5	109.9	126.8
204.2	.050	14×14	196	68.6	98.0	127.4	147.0
234.4	.058	15×15	225	78.8	112.5	146.3	168.8
266.7	.066	16×16	256	89.6	128.0	166.4	192.0
301.0	.074	17×17	289	101.2	144.5	187.9	216.8
337.5	.083	18×18	324	113.4	162.0	210.6	243.0
376.0	.093	19×19	361	126.4	180.5	234.7	270.8
416.7	.103	20×20	400	140.0	200.0	260.0	300.0
459.4	.113	21×21	441	154.4	220.5	286.7	330.8
504.2	.124	22×22	484	169.4	242.0	314.6	363.0
551.0	.136	23×23	529	185.2	264.5	343.9	396.8
600.0	.148	24×24	576	201.6	288.0	374.4	432.0
651.0	.161	25×25	625	218.8	312.5	406.3	468.8
704.2	.174	26×26	676	236.6	338.0	439.4	507.0
759.4	.187	27×27	729	255.2	364.5	473.9	546.8
816.7	.202	28×28	784	274.4	392.0	509.6	588.0
876.0	.216	29×29	841	294.4	420.5	546.7	630.8
937.5	.231	30×30	900	315.0	450.0	585.0	675.0
1,001.0	.247	31×31	961	336.4	480.5	624.6	720.8
1,067.0	.263	32×32	1,024	358.4	512.0	665.6	768.0
1,134.0	.280	33×33	1,089	381.2	544.5	707.9	816.8
1,204.0	.297	34×34	1,156	404.6	578.0	751.4	867.0
1,276.0	.315	35×35	1,225	428.8	612.5	796.3	918.8
1,350.0	.333	36×36	1,296	453.6	648.0	842.4	972.0

making calculations concerning columns, although the weight of columns is often neglected in calculating their strength.

27. The loads upon the columns in the problem under discussion, together with the corresponding sizes required to carry the loads, as obtained from Table I, are arranged as shown in Table II. The allowable unit stress is taken at 500 pounds per square inch.

TABLE II

CALCULATIONS FOR NORMAL COLUMNS

Story Locating Column	Area Carried by One Column Square Feet	Total Load per Square Foot Pounds	Total Load on Column for One Story Pounds	Total Load, Including Load From Column Above Pounds	Size of Column From Table I Inches
Fifth........	320	120	38,400	38,400	9×9
Fourth......	320	258	82,560	120,960	16×16
Third.......	320	258	82,560	203,520	21×21
Second......	320	258	82,560	286,080	24×24
First........	320	258	82,560	368,640	28×28
Basement....	320	258	82,560	451,200	31×31

28. If the preceding method of calculation is to be used, the columns should be reinforced with at least 1 per cent. of steel reinforcement, though, for convenience in handling the steel, nothing less than $\frac{3}{4}$-inch bars or rods should be used. However, $\frac{5}{8}$-inch bars or rods might be used for the 9″×9″ column supporting the roof. Such a small column is very seldom used, however, it being customary, on account of the difficulties attending the pouring of the concrete, not to use any column smaller than 10 inches square.

29. Many owners will not permit columns that are as large as 31 inches and 28 inches to be used in the basement and the first floor, respectively, in which event it will be

necessary to use steel cores in these columns. In figuring the steel cores, the concrete is not considered as resisting any compression, and the steel is proportioned by allowing a given safe unit compressive stress. This unit stress is generally taken at 16,000 pounds, so that for the basement columns, the sectional area required for the steel core will be equal to $451,200 \div 16,000 = 28.2$ square inches, and for the first-story column, $368,640 \div 16,000 = 23$ square inches. These areas can be made up of any convenient steel sections and then be surrounded with concrete to make a column that is, say, 18 inches square.

SPECIAL COLUMNS

30. Columns Supporting Tank.—Besides the normal columns just discussed, there are many special columns, such as those which support the water tank on the roof and the wall piers themselves.

The columns used to carry the sprinkler tank will be considered first. It is necessary to decide where this tank is to be placed. Let it be assumed that the tank is to be located on the roof and at the corner of the building, and that it is to be 20 feet long and 16 feet wide. Then, three corners of the tank will be supported by wall piers, and the fourth corner will be supported by one of the central columns of the building. The tank is to contain 25,000 gallons of water, which will weigh about 208,000 pounds.

The weight of the tank itself cannot be determined until it is designed. It may be estimated, however, at 150,000 pounds. The total weight of the tank when filled with water is therefore 358,000, say 360,000, pounds. Since each column supports one-fourth of this amount, the interior column will carry $360,000 \div 4 = 90,000$ pounds. This load is in addition to the load that the normal column carries, as given in Table II. Although the column under consideration is not exactly a normal column, since it does not support normal girders, yet it supports the same floor area, so that its load is practically the same as that of a normal column.

Table III is then formed in practically the same manner as was Table II, giving the total loads at the different stories that the interior column that carries one corner of the sprinkler tank must support.

If desired, the size of this column may be reduced in the lower stories by putting in a steel core.

The columns for the other corners of the tank may be designed in the same way; that is, by taking the loading of the wall pier under normal circumstances and adding to it one-fourth the load of the tank. This operation will not be worked out here, however, as it should present no difficulties.

TABLE III

CALCULATION FOR INTERIOR COLUMN CARRYING SPRINKLER TANK

Story Locating Column	Floor and Roof Load Pounds	Tank Load Pounds	Total Load Pounds	Size of Column From Table I Inches
Fifth	38,400	90,000	128,400	17×17
Fourth	120,960	90,000	210,960	21×21
Third	203,520	90,000	293,520	25×25
Second	286,080	90,000	376,080	28×28
First	368,640	90,000	458,640	31×31
Basement	451,200	90,000	541,200	33×33

31. Wall Piers.—The wall piers on the sides will next be considered. These members carry the loads half way to the center column; that is, each normal wall pier supports an area of 10 ft. × 16 ft., or 160 square feet. This area is equal to half the area supported by the middle columns; therefore, the normal wall piers support half the load. Besides this load, the wall piers carry the load of the spandrel filling—one-half a spandrel on each side. As assumed in Art. **22**, this load will be taken as 6,500 pounds. Table IV may now be constructed for the calculation of the normal wall piers. As these piers will not be square, the required area of the piers

as given in the last column of the table is found by dividing the load to be carried by the safe unit stress to be used. The width and thickness of the pier have been decided on, as shown in Fig. 6. The pier is to be veneered with brick on the outside.

32. The rabbet for the window frames is determined usually by the size of the frame. Let it be assumed that the frame has been designed and that the cross-section of the pier is as shown in Fig. 8. The area of the concrete in this pier is

TABLE IV

CALCULATION FOR NORMAL WALL PIERS

Story Locating Column	Load From Spandrel at Each Floor Pounds	Total Load From Spandrel Pounds	Total Floor Load, Being Half Values Given in Table II Pounds	Total Load at Each Floor Pounds	Area of Pier Required at 500 Pounds per Square Inch Square Inches
Fifth........			19,200	19,200	38.4
Fourth......	6,500	6,500	60,480	66,980	134.0
Third.......	6,500	13,000	101,760	114,760	229.5
Second......	6,500	19,500	143,040	162,540	325.1
First........	6,500	26,000	184,320	210,320	420.6
Basement....	6,500	32,500	225,600	258,100	516.2

378 square inches. As it is not often considered necessary to reduce the area of the pier in the upper stories of a building, the section shown in Fig. 8, with, say, at least 1 per cent. of steel reinforcement, will be large enough for the fifth, fourth, third, and second stories. The pier, however, will not be sufficiently strong for the basement or first floor, and some additional resistance is necessary.

In this case, a steel core may be used, as it is not very practicable to enlarge the pier itself. As an example, consider the pier in the basement. The load to be carried is 258,100

pounds. If the allowable unit stress in the core is taken at 16,000 pounds per square inch, the area of core required will be $258,100 \div 16,000 = 16.1$ square inches. This area of steel may be inserted in the form of angles if desired. A convenient method might be to insert four **Z** bars near the center of the pier. The pier on the first floor is designed in the same manner.

33. Miscellaneous Columns.—After the partition about the elevator and all the stair and elevator details have been laid out, the columns that support these constructive details may be designed by exactly the same methods that have just been used.

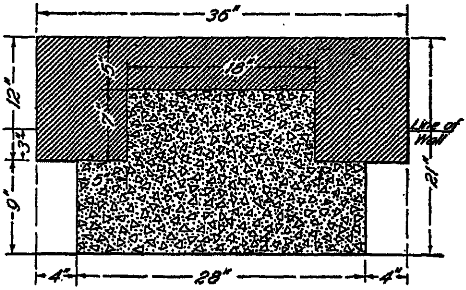


FIG. 8

The end and corner wall piers are also designed by similar methods.

The cornices and other small constructive details are not usually designed entirely for strength. Many questions arise that experience alone can answer. The designs given in *Reinforced-Concrete Buildings*, Part 1, furnish excellent models for this class of design.

34. The $1\frac{1}{2}$ inches of fireproofing should then be put on the outside of each column. After all the details have been decided upon, the loads on each column, including the weights of the columns themselves, should be recalculated.

DESIGN OF SPREAD FOUNDATIONS
(PART 1)

SPREAD COLUMN FOOTINGS

INTRODUCTION

1. Definitions.—The term *spread footings* is applied to footings which are shallow in depth, spread over a large area of soil, and project far enough beyond the upper portion of the foundation to require special provision for resisting the bending moments developed. The two types of spread footings most extensively used are the steel-beam grillage footing, shown in Fig. 1 (*a*), and the reinforced-concrete footing shown in Fig. 2. In the grillage footing, the resistance to bending is provided by the steel beams, and in the reinforced-concrete footing, by the concrete slab and the reinforcing rods. Both footings accomplish the same result; namely, they provide a footing shallow in depth, spreading over a large bearing area on the soil, and possessing sufficient transverse strength to transfer a great pressure from a comparatively small area of column base to a large area of foundation bed.

2. Conditions That Favor Adoption of Spread Footings.—In providing foundations for heavy buildings where bed-rock is at a considerable distance below the surface, it is often found most economical to rest the foundation on the comparatively plastic soil. This is especially the

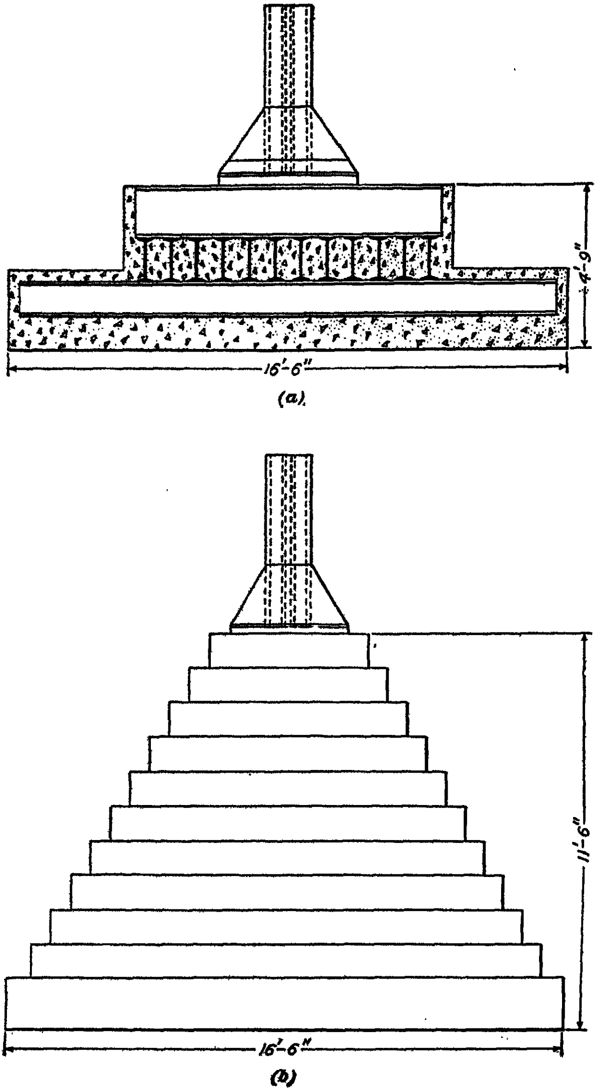


Fig. 1

case where a fairly hard surface stratum is underlaid with a much softer stratum of great depth, which precludes the possibility of driving piles. In the business section of Chicago, for instance, the surface stratum of made ground extends to a depth of about 14 feet below the street surface, and is followed by a stratum of hard clay of 6 to 12 feet in depth, below which the clay is much softer and extends to a depth of 75 feet or more.

Tall buildings of the skeleton type concentrate great loads on the basement columns, and the problem of transferring

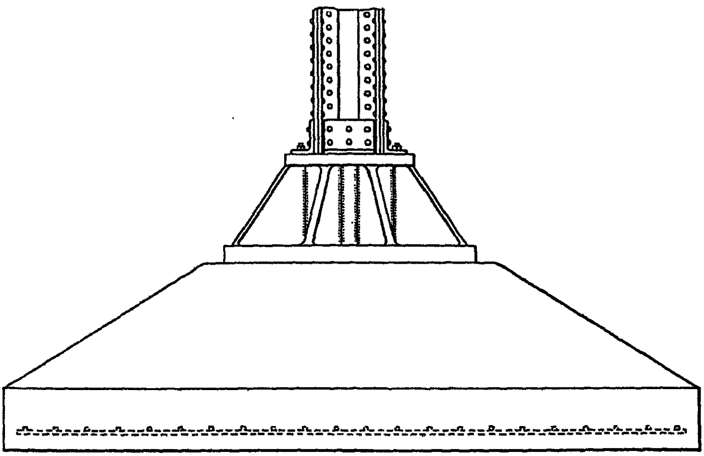


FIG. 2

these loads to a plastic and compressible soil is not a simple one. Each footing must spread over a sufficiently large area of the foundation bed so that the unit pressure which the foundation soil can safely bear will not be exceeded. Furthermore, since a building resting on a compressible foundation bed invariably settles during and after construction, it is essential to proportion the foundation so that whatever settlement will occur shall be uniform.

3. In light buildings the footings for columns or walls may be plain concrete piers. In heavy buildings, if plain concrete footings having the proper proportions were used,

they would be of such great depth and dimensions as to raise the cost of materials and excavation quite high. Furthermore, their weight would add considerably to the already heavy loads transferred to the foundation soil from the columns. Where a relatively hard, but not deep, surface stratum is followed by a much softer stratum of great depth, the footings cannot be carried much below the surface and must extend into the basement, occupying valuable space that might be used for engine rooms, cafés, shops, etc. The logical solution under such circumstances is the adoption of the spread footing. Fig. 1 illustrates the obvious advantages that the spread footing possesses over the plain concrete footing. In (*a*) is shown a spread footing which supports a column of a tall building, and in (*b*) a plain-concrete pier designed for the same load and soil.

4. Recapitulation.—The conditions that lead to the adoption of spread footings are as follows:

1. The soil conditions preclude the possibility of carrying the footings to rock or hardpan, or of driving piles for their support.
2. Sufficient bearing area can be provided under each footing so as not to exceed the bearing capacity of the soil.
3. The foundation and footings must be shallow so as not to penetrate or impair the bearing capacity of a thin but hard stratum underlaid with a soft stratum of great depth.
4. No foundation piers of great bulk should occupy space in the basement that may be otherwise utilized.
5. The weight of the footing should constitute but a small percentage of the entire load it transfers to the soil, in order that a considerable portion of the footing area will not be taken up in carrying the weight of the footing.

5. Principal Factors in Design.—The principal factors to be considered in designing the foundation for a building are: (*a*) the magnitude of the loads that may be expected to come on it, (*b*) their distribution, and (*c*) the bearing capacity of the soil. These factors will be discussed in the following articles.

BEARING AREAS OF FOOTINGS

LOADS ON FOOTINGS

6. Classification of Loads.—The loads that are transferred by the footings to the foundation bed are:

(*a*) The *dead load*, or the weight of the completed structure and all permanent fixtures in it.

(*b*) The *live load*, or the expected movable load on the floors and roof of the building.

(*c*) The *wind load*, or the effect of an assumed horizontal wind pressure against any side of the building, the wind blowing in any one direction at a time.

7. Dead Load.—It is generally agreed that the dead load is most effective in causing settlement. It is practically the only loading transferred to the foundation before the building is occupied, or it is the sole factor in initial settlement. From the day the structure is completed it exerts its full pressure on the soil and remains constant thereafter. Its actual value may be predetermined quite accurately by calculating the weights of the materials composing the floors, walls, partitions, and other permanent construction and fixtures. Usually, the minimum weight of a fireproof floor for an office building is taken at not less than 75 pounds per square foot.

8. Live Load.—The live load is assumed according to the character of the building. It is usually specified by the city building codes. Thus, the New York building code specifies a minimum live load of 60 pounds per square foot on floors of office buildings and 100 pounds on floors of assembly rooms; for roofs inclined 20° or less it specifies a live load of 40 pounds per square foot of surface, and for roofs inclined more than 20° a live load of 30 pounds.

Since it is hardly probable that all floors and the roof of a building will ever be loaded simultaneously with the full assumed live load, it is good practice in designing columns

to consider that only a percentage of the total live load reaches the columns. Thus, according to the New York building code, in designing columns for buildings more than five stories high, the full specified live load should be considered for the top floor and roof, 95 per cent. of the specified live load for the floor below the top floor, 90 per cent. for the next lower floor, and a decrease of an additional 5 per cent. for each succeeding floor. In no case, however, should less than 50 per cent. of the specified live load be assumed. The Chicago building code specifies that the columns shall be assumed loaded with the full live roof load at the roof, 85 per cent. of the specified live floor load at the top floor, 80 per cent. at the floor below, and 5 per cent. less for each succeeding floor until 50 per cent. is reached, which shall be used for all remaining floors.

The live load that comes on a column footing is naturally the live load transferred by the lowest section of the column. It is, therefore, an assumed load. The actual live load is nearly always much smaller than the assumed live load, and it seldom acts with its maximum intensity for a long period of time.

9. Wind Load.—The wind load is also an assumed quantity, which is usually taken as a horizontal pressure of 30 pounds per square foot on any side of the building, acting in one direction at a time. The action of the wind is only for relatively short intervals of time, and in high buildings it is improbable that it will ever exert its full pressure over the entire area of one side, even when the building is not partly shielded by adjoining structures.

In the average tall building ample wind bracing is provided to prevent deformation due to wind pressure. The wind, therefore, tends to overturn the entire structure, causing an uplift at the footings on the windward side and pressure on the foundation on the leeward side. The footings of interior columns are therefore but slightly affected by wind stresses.

The New York building code specifies that all buildings over 150 feet high, and buildings or parts of buildings in

which the height is more than four times the minimum horizontal dimension, shall be designed to resist a horizontal wind pressure of 30 pounds per square foot of exposed surface, allowing for wind in any direction.

SAFE BEARING CAPACITIES OF SOILS

10. Examination of Foundation Bed.—Before designing the footings for any large or heavy building, sufficient borings and loading tests should be made on the proposed site to ascertain the bearing value of the foundation bed over the entire site. It should not be taken for granted that because the soil of the adjoining property is stable and capable of bearing a specified load, that the soil of the proposed site will carry the same load. Frequently, a concealed spring or the bed of an old, unexplored stream or creek under the proposed site is not discovered until foundation work has begun, and in such cases many of the footings must be redesigned. Borings will reveal the kind and depth of material on which the building is to be erected.

11. Allowable Unit Pressures.—Most cities specify in their building codes the maximum allowable unit pressures on various foundation soils. These codes are the results of careful investigation and they may be safely followed in designing foundations for ordinary buildings. On the larger building projects, however, too much reliance should not be placed upon the unit pressures allowed by codes, which at best are only average values.

Safe bearing values of different foundation soils, which, according to the New York building code, should not be exceeded, are given in Table I.

12. Loading Tests on Soils.—Whenever the magnitude of the project justifies the expense, loading tests on the soil should be made to determine its bearing capacity. Preferably, these tests should be fairly representative of the manner in which the foundation bed will be loaded. However, due to the expense involved, tests on comparatively large

areas are rarely made. In the usual tests a short column about 1 foot square in cross-section, carrying a loading platform and resting on a leveled portion of the foundation bed, is loaded with specified loads and the consequent settlement and soil changes are observed. Care should be taken to keep

TABLE I

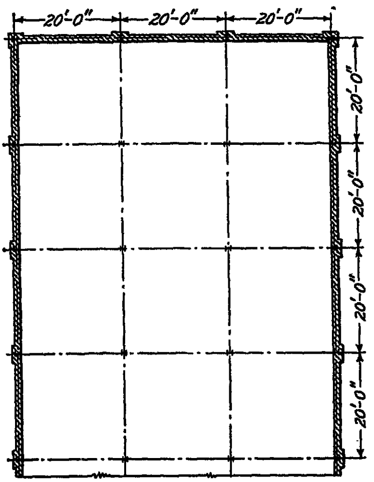
SAFE LOADS ON EARTH FOUNDATION BEDS

Kinds of Material	Loads in Tons per Square Foot
Hard rock	40
Medium rock	15
Hardpan	10
Soft rock	8
Gravel	6
Sand, firm and coarse	4
Clay, hard and dry	4
Sand, fine and dry	3
Ordinary firm clay	2
Sand and clay, mixed or in layers	2
Sand, wet	2
Clay, soft	1

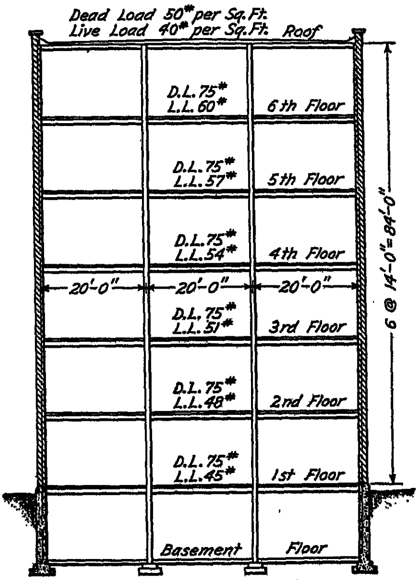
the column in a vertical position by means of horizontal guys, which, however, must not impede its vertical motion. The test loads on the platform should be applied without undue jar and should be concentric with the column throughout the test. In conducting such tests, it is customary to apply first an initial load which is permitted by the building code for the particular soil, and it is allowed to remain undisturbed for 48 hours, observations of settlement being taken every 24 hours. A 50- or 100-per-cent. excess load is then added and allowed to remain for 6 or 7 days. If the settlement of the total load is not excessive, the initial load may be safely assumed as the proper bearing capacity of the soil.

As an illustration of the advantage of making soil tests, the following case may be cited: The code of New Orleans

allows a maximum soil pressure of 1,400 pounds per square foot on the alluvial formation underlying the whole city, and yet tests performed in connection with an important project to determine the actual bearing capacity of the soil showed that for the particular site chosen only 650 pounds per square foot could safely be sustained by the soil.



PLAN
(a)



TYPICAL SECTION
(b)

FIG. 3

PROVISIONS FOR UNIFORM SETTLEMENT

13. Main Consideration.—In designing footings for a compressible soil, the ideal condition desired is so to proportion the footings that the unit pressure on the foundation bed will always be the same under all footings. Such a condition would insure even settlement throughout the building. However, since the loads transferred to the different footings vary, the desired ideal condition cannot be practically attained. Nevertheless, it should be approached as nearly as practical considerations will permit.

14. Minimum Proportions.—The minimum proportions of footings are determined by the maximum load they may have to sustain and the bearing capacity of the soil. Each footing should be so proportioned that the maximum load coming on it will not cause a unit pressure on the soil greater than its bearing capacity. The minimum area of each footing will, therefore, be determined by dividing the maximum assumed load on the footing by the safe bearing value of the soil.

EXAMPLE.*—Let the building shown in Fig. 3, in plan in (*a*) and in section in (*b*), be loaded with a dead load of 50 pounds per square foot of roof area and 75 pounds per square foot of floor area, and with a live load of 40 pounds per square foot on the roof, and 60 pounds per square foot on each floor. The columns are to be designed for a live load of 60 pounds per square foot on the top floor, 60 pounds less 5 per cent., or 57 pounds per square foot, on the floor below, and 5 per cent. less for each succeeding floor. The partly glazed exterior 13-inch walls are carried by lintels at each floor above the first. These lintels are supported by the exterior columns, and they may be assumed to carry a uniform load of 1,100 pounds per linear foot of wall at each floor above the first, and 500 pounds per linear foot at the roof. Since the height of the building is less than four times the minimum horizontal dimension, no wind load need be assumed. What are the minimum bearing areas of the column footings required to sustain the above loads on a soil of fine and dry sand?

SOLUTION.—According to Table I, the allowable soil pressure is 3 tons per square foot. The minimum bearing areas for the footings of the various columns are found in the following manner:

Corner Column: The span of each lintel is 20 ft. and the load on it is 1,100 lb. per ft., at each of the five floors between the first floor and roof, and 500 lb. per ft. at the roof. The end reactions of each lintel are, therefore, $1,100 \times 10 = 11,000$ lb. at the intermediate floors, and $500 \times 10 = 5,000$ lb. at the roof. Since each corner column carries the end reactions of two lintels at each floor above the first, the total load from the lintel reactions, or due to the weight of the wall, which is transferred to the footing of a corner column, is

$$2 \times 5 \times 11,000 + (2 \times 5,000) = 120,000 \text{ lb.}$$

Each corner column also carries the dead load coming on $10 \times 10 = 100$ sq. ft. of area of each floor and the roof. The dead load per square foot on each of the six floors being 75 lb., and on the roof 50 lb., the dead load from the floors and roof transferred to the footing of a corner column, is

$$(75 \times 6 + 50) \times 100 = 50,000 \text{ lb.}$$

* The examples in this Section were computed by means of the slide rule and the results are therefore correct to the nearest three significant figures.

The live load per square foot on the roof is 40 lb., on the top floor 60 lb., on the fifth floor 57 lb., on the fourth floor 54 lb., on the third floor 51 lb., on the second floor 48 lb., and on the first floor 45 lb. Since each corner column carries the live load coming on $10\times10=100$ sq. ft. of area of each floor and the roof, the total live load transferred to the footing of a corner column is

$$(40+60+57+54+51+48+45)\times100=35{,}500 \text{ lb.}$$

A summary of the loads transferred to the footing of a corner column is as follows:

Dead load from weight of wall.....	120,000 lb.	
Dead load from floors and roof....	50,000 lb.	
Total dead load..................		170,000 lb.
Total live load..................		35,500 lb.
Total load....................		205,500 lb.

The minimum footing area required is $\dfrac{205{,}500}{6{,}000}=34.3$ sq. ft. Ans.

Intermediate Exterior Column: Each intermediate exterior column carries the end reactions of two lintels, and hence the load transferred to the footing from the lintel reactions, or due to the weight of wall, is the same as for a corner column, or 120,000 lb.

The area of each floor and the roof which transfers dead load to an intermediate exterior column is $20\times10=200$ sq. ft., or twice that of a corner column. Hence, the dead load from the floors and roof transferred to the footing of an intermediate column is $2\times50{,}000=100{,}000$ lb.

The area of each floor and the roof which transfers live load to an intermediate exterior column also being 200 sq. ft., the total live load transferred to the footing of an intermediate exterior column is twice the total live load transferred to a corner column, or $2\times35{,}500=71{,}000$ lb.

A summary of the loads transferred to the footing of an intermediate column is as follows:

Dead load from weight of wall.....	120,000 lb.	
Dead load from floors and roof.....	100,000 lb.	
Total dead load..................		220,000 lb.
Total live load..................		71,000 lb.
Total load....................		291,000 lb.

The minimum footing area required is $\dfrac{291{,}000}{6{,}000}=48.5$ sq. ft. Ans.

Interior Column: The dead load transferred to each interior column is that carried by $20\times20=400$ sq. ft. of floor or roof area, and, therefore,

the total dead load transferred to the footing of an intermediate column is four times that of a corner column, or $4\times 50{,}000=200{,}000$ lb.

The total live load transferred to the footing of an intermediate column is likewise four times the total live load transferred to a corner column, or $4\times 35{,}500=142{,}000$ lb.

The summary of the loads transferred to the footing of an intermediate column is as follows:

Total dead load	200,000 lb.
Total live load	142,000 lb.
Total load on footing	342,000 lb.

The minimum footing area required is $\frac{342{,}000}{6{,}000}=57$ sq. ft. Ans.

The results obtained in the example of this article are arranged below in tabular form. For the sake of future comparisons, the ratio of live load to dead load transferred to each footing is also included.

Column	Loads in 1,000 Pounds			Ratio Live to Dead	Minimum Footing Area in Square Feet
	Dead	Live	Combined		
Corner	170	35.5	205.5	.21	34.3
Intermediate					
Exterior	220	71.0	291.0	.32	48.5
Interior	200	142.0	342.0	.71	57.0

15. In the example of the preceding article the dead load per square foot under a corner column is $\frac{170{,}000}{34.3}=4{,}960$ pounds; under an intermediate exterior column, $\frac{220{,}000}{48.5}=4{,}540$ pounds, and under an interior column $\frac{200{,}000}{57}=3{,}510$ pounds. It is, therefore, obvious that if the minimum footing areas were used, the settlement of the building before occupancy

would be greater for the exterior columns than for the interior columns. Also after occupancy, since the actual live load would seldom equal the assumed live load, and would rarely act with its maximum intensity throughout the building, there would still be a tendency for the exterior columns to settle more than the interior columns. The amount of settlement involved would naturally depend on the compressibility of the soil. For soils of relatively great compressibility, the uneven settlement would cause cracks in the floors and in extreme cases it might lead to serious consequences. There are many designers who proportion column footings on compressible soils according to the combined loads, but such a policy is not to be recommended.

METHODS OF PROPORTIONING BEARING AREAS OF COLUMN FOOTINGS

16. To provide for even settlement some designers allow smaller soil pressures under exterior than under interior column footings. However, there is no scientific basis for such practice, and it is not to be recommended. Nearly all rational methods for proportioning column footings assume that the full dead load and only part of the live load are effective in causing settlement. Some of these methods will be discussed in the following articles.

17. Schneider Method.—According to Schneider's specifications, the bearing areas of the footings of a structure should have the same ratio to each other as the total dead loads which they sustain. The footing which has the largest ratio of live to dead load is first chosen as the index footing and its bearing area is determined by dividing the combined dead and live load by the allowable soil pressure which gives the minimum footing area, as found in Art. **14**. The dead load on this footing is then divided by the minimum footing area, and a reduced unit working pressure for determining the footing areas of the other columns is thus obtained. The dead loads of the other footings divided by this reduced unit working pressure give the required bearing areas of the footings.

18. If the Schneider method were used in proportioning the footings in the example in Art. **14**, an interior column footing would be chosen as the index footing because the ratio of live to dead load for that footing, .71, is larger than the ratios for the intermediate exterior and corner column footings, .32 and .21, respectively. The required bearing area for the interior column footing is the same as the minimum footing area, or 57 square feet. The reduced unit working pressure is, therefore, $\frac{200{,}000}{57}=3{,}510$ pounds per square foot. The required footing area for a corner column is then $\frac{170{,}000}{3{,}510}=48.4$ square feet, and for an intermediate exterior column it is $\frac{220{,}000}{3{,}510}=62.7$ square feet.

The Schneider method is being used quite extensively in practice; it forms the basis of the specifications for the design of footings in the present New York building code. The chief objections to the Schneider method are that it is too conservative, involving the use of excessively large footings, and that it does not take into consideration the live load which is always present to some extent after occupancy of the building.

19. McCullough Method.—According to the method proposed by Ernest McCullough, the bearing areas of footings are proportioned also for dead load only. However, in his method the footing which has the smallest ratio of live to dead load is taken as the index footing and its bearing area is found by dividing the combined dead and live load by the allowable soil pressure. The dead load of that footing is then divided by the determined bearing area to obtain the reduced unit working pressure, and the footings for the other columns are proportioned by dividing their dead loads by the reduced unit working pressure.

In the example of Art. **14**, the index column is a corner column. The reduced unit working pressure is $\frac{170{,}000}{34.3}=4{,}960$ pounds per square foot. The footing area of a corner

column remains 34.3 square feet, but the corresponding footing area for an intermediate exterior column is $\frac{220{,}000}{4{,}960}=44.4$ square feet, and for an interior column $\frac{200{,}000}{4{,}960}=40.3$ square feet.

The principal objection to the McCullough method is that it gives footing areas smaller than the minimum footing areas, and hence comes in conflict with several building codes which expressly state that the pressure under footings due to combined dead and live load shall not exceed the safe bearing capacity of the soil. Furthermore, like the Schneider method, the McCullough method totally disregards the live load.

20. Fleming Method.—A method that is apparently more rational than either one of the two foregoing methods, was proposed by R. Fleming.* According to his method, the areas of footings are proportioned to have the same ratio to each other as the total dead load and one-third the live load coming upon them. The footing which has the largest ratio of live to dead load is proportioned for combined dead and live load. The dead and one-third live load on this footing is then divided by the area previously found, and the reduced unit working pressure is thus obtained. The sums of the dead loads and one-third live loads coming on the other footings are then divided by the reduced unit working pressure to obtain their bearing areas.

In the example of Art. 14, the interior column would be chosen as the index column. The reduced unit working pressure would then be $\frac{200{,}000+47{,}300}{57.0}=4{,}340$ pounds per square foot. The footing area for an intermediate exterior column is $\frac{220{,}000+23{,}700}{4{,}340}=56.1$ square feet, and for a corner column it is $\frac{170{,}000+11{,}800}{4{,}340}=41.9$ square feet.

*In Engineering News-Record of July 29, 1920.

21. Comparison of Methods.—The bearing areas of the column footings of the example in Art. **14**, as determined by the various methods previously discussed, are arranged in tabular form as follows:

Column	Minimum Footing Area Square Feet	Schneider Method	McCullough Method	Fleming Method
Corner......	34.3	48.4	34.3	41.9
Intermediate Exterior ..	48.5	62.6	44.4	56.1
Interior	57.0	57.0	40.3	57.0

The areas determined by the Schneider method are the largest, while those determined by the McCullough method are the smallest. However, as it will be observed, for intermediate exterior and interior column footings, the areas found by the McCullough method are smaller than the minimum footing areas, thus violating the provisions of many building codes.

22. Proportioning According to Character of Structure.—Some building codes specify that column footings shall be proportioned for the full dead load and a percentage of the live load, the specified percentage being different for the various structures. For instance, the Chicago building code specifies that in proportioning foundations for columns, piers, and walls, the full dead load and the following percentages of the live load shall be considered: for warehouses and stores, 75 per cent.; for office buildings, 50 per cent.; and for churches, theaters, and schoolhouses, 25 per cent. Other building codes specify that in addition to the dead load, a certain uniform live load on the floors shall be assumed as effective in causing settlement, and that all foundations shall be designed for the combined dead load and the uniform live load. This uniform live load is different from the live load specified for designing the floors of the structure. The Indianapolis building code specifies that the load carried by the soil shall be the

total dead load and a uniform live load of not less than 10 pounds per square foot on the entire floor area of office buildings and tenement houses, 20 pounds per square foot on the entire floor area of buildings used for mercantile purposes, and 60 pounds per square foot on the entire floor area of warehouses.

EXAMPLES FOR PRACTICE

1. The dead loads on the footings of three columns A, B, and C, where A is a corner column, B is an intermediate exterior column, and C is an interior column, are, respectively, 218,000 pounds, 276,000 pounds, and 232,000 pounds, and the live loads are, respectively, 51,500 pounds, 103,000 pounds, and 206,000 pounds. What are the minimum bearing areas of these footings on a soil of 2.5 tons bearing capacity?

$$\text{Ans.}\begin{cases}\text{Area } A = 53.9 \text{ sq. ft.}\\ \text{Area } B = 75.8 \text{ sq. ft.}\\ \text{Area } C = 87.6 \text{ sq. ft.}\end{cases}$$

2. Find the bearing areas of the column footings in example 1 if they are proportioned by Schneider's method.

$$\text{Ans.}\begin{cases}\text{Area } A = 82.3 \text{ sq. ft.}\\ \text{Area } B = 104.1 \text{ sq. ft.}\\ \text{Area } C = 87.6 \text{ sq. ft.}\end{cases}$$

3. What are the bearing areas of these footings in example 1 according to Fleming's method?

$$\text{Ans.}\begin{cases}\text{Area } A = 68.5 \text{ sq. ft.}\\ \text{Area } B = 90.4 \text{ sq. ft.}\\ \text{Area } C = 87.6 \text{ sq. ft.}\end{cases}$$

STEEL-BEAM GRILLAGE FOOTINGS

GRILLAGE FOOTINGS SUPPORTING ONE COLUMN

GENERAL DETAILS OF CONSTRUCTION

23. Steel Rails and I Beams.—The beams originally used in the construction of grillage footings were steel rails crossed in alternate layers. Undoubtedly they were adopted on account of being readily obtainable and also because cf their shallow depth and the considerable resistance they offer to transverse stress. At the present time, however, steel I beams may be obtained with equal facility and are being used almost exclusively in the construction of grillage foot-

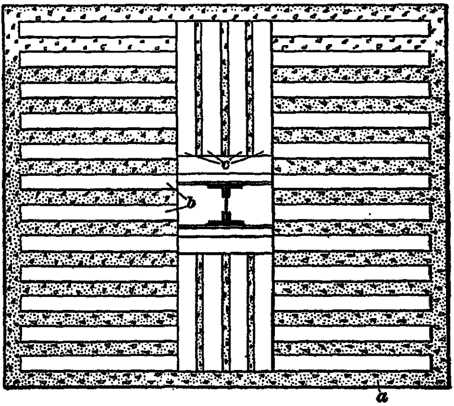
ings. In such construction the **I** beam has a decided advantage over the rail on account of the greater resistance to bending which it offers for a given weight per foot of length. For instance, the section modulus of an A. S. C. E. rail weighing 90 pounds per yard, or 30 pounds per foot, is 12.19, while the section modulus of a 10-inch **I** beam of the same weight per foot is 26.7, or more than twice as great. For heavy column loads, riveted girders have often been used economically in grillage footings.

24. Properties of Standard I Beams.—In Table II are given the properties of standard **I** beams which are employed in the usual design of grillage footings. In column 1 is given the depth of the beam, *d*; in column 2 the weight of the beam per foot of length; in column 3 the section modulus of the beam about axis 1–1, *s*; in column 4 the thickness of the web of the beam, *t*; in column 5 the width of the flange of the beam, *b*. The application of these properties in the design of grillage footings will be given later.

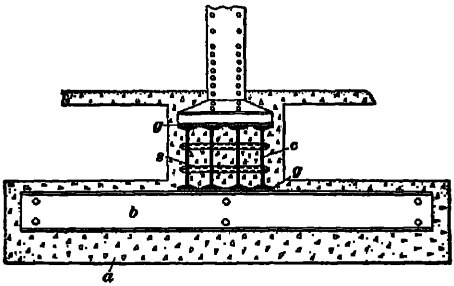
25. Shape of Footing.—For the support of a single column, a square footing is most advantageously employed. In exterior columns it is frequently found that square footings would project beyond the building line or would interfere with the foundations of adjoining structures, and rectangular footings are then substituted. To insure uniform distribution of the column load over the foundation bed, and hence even settlement of the footing, the center of gravity of the footing should, whenever possible, be made to coincide with the axis of the column.

26. Concrete Mat.—A typical grillage footing is shown in Fig. 4, in plan at (*a*) and in elevation at (*b*). On the properly leveled foundation bed is laid a mat *a* of plain concrete, usually 12 inches thick and composed of one part of Portland cement, two parts of sand, and four parts of broken stone or gravel. On this concrete mat, after it has hardened sufficiently, are placed the grillage beams *b* and *c* in tiers or layers, the beams of each tier being placed parallel to each

other and at right angles to those of the tier below. The outside dimensions of the mat are proportioned so as to give the required area of footing. The beams of the lowest tier are then assumed to cover an area somewhat less than the area of the concrete mat, the projection of the mat being



(a)



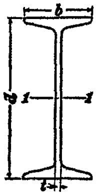
(b)

Fig. 4

capable of resisting some bending. The length that the concrete mat may safely project beyond the edges of the beams depends on the soil pressure. However, a projection of one-half the thickness of the slab will be found permissible for the safe loads on earth foundation beds commonly used in the design of spread footings.

TABLE II

PROPERTIES OF STANDARD I BEAMS



1	2	3	4	5
Depth of Beam, in Inches	Weight per Foot, in Pounds	Section Modulus Axis 1-1	Thickness of Web, in Inches	Width of Flange, in Inches
d		s	t	b
24	115.0	245.0	.737	7.987
	110.0	239.1	.675	7.925
	105.9	234.3	.625	7.875
24	100.0	197.6	.747	7.247
	95.0	191.8	.686	7.186
	90.0	185.8	.624	7.124
	85.0	180.0	.563	7.063
	79.9	173.9	.500	7.000
20	100.0	164.8	.873	7.273
	95.0	160.0	.800	7.200
	90.0	155.0	.726	7.126
	85.0	150.2	.653	7.053
	81.4	146.6	.600	7.000
20	75.0	126.3	.641	6.391
	70.0	121.4	.567	6.317
	65.4	116.9	.500	6.250
18	90.0	139.6	.796	7.236
	85.0	135.2	.714	7.154
	80.0	130.8	.632	7.072
	75.6	126.9	.560	7.000
18	70.0	101.9	.711	6.251
	65.0	97.5	.629	6.169
	60.0	93.1	.547	6.087
	54.7	88.4	.460	6.000
15	75.0	91.6	.868	6.278
	70.0	87.9	.770	6.180
	65.0	84.3	.672	6.082
	60.8	81.2	.590	6.000

TABLE II—(*Continued*)

1	2	3	4	5
Depth of Beam, in Inches	Weight per Foot, in Pounds	Section Modulus Axis 1–1	Thickness of Web, in Inches	Width of Flange, in Inches
d		s	t	b
15	55.0	67.8	.648	5.738
	50.0	64.2	.550	5.640
	45.0	60.5	.452	5.542
	42.9	58.9	.410	5.500
12	55.0	53.2	.810	5.600
	50.0	50.3	.687	5.477
	45.0	47.3	.565	5.355
	40.8	44.8	.460	5.250
12	35.0	37.8	.428	5.078
	31.8	36.0	.350	5.000
10	40.0	31.6	.741	5.091
	35.0	29.2	.594	4.944
	30.0	26.7	.447	4.797
	25.4	24.4	.310	4.660
9	35.0	24.7	.724	4.764
	30.0	22.5	.561	4.601
	25.0	20.3	.397	4.437
	21.8	18.9	.290	4.330
8	25.5	17.0	.532	4.262
	23.0	16.0	.441	4.171
	20.5	15.1	.349	4.079
	18.4	14.2	.270	4.000
7	20.0	12.0	.450	3.860
	17.5	11.1	.345	3.755
	15.3	10.4	.250	3.660
6	17.25	8.7	.465	3.565
	14.75	7.9	.343	3.443
	12.5	7.3	.230	3.330
5	14.75	6.0	.494	3.284
	12.25	5.4	.347	3.137
	10.0	4.8	.210	3.000
4	10.5	3.5	.400	2.870
	9.5	3.3	.326	2.796
	8.5	3.2	.253	2.723
	7.7	3.0	.190	2.660
3	7.5	1.9	.349	2.509
	6.5	1.8	.251	2.411
	5.7	1.7	.170	2.330

27. Concrete Filling.—Good practice requires that as the beams of each tier are placed they should be encased in concrete to a thickness of at least 4 inches, the spaces between them being also thoroughly tamped with concrete. The purpose of the concrete filling is to protect the beams from fire or rust and to help them act in unison. To allow proper tamping, the beams of each tier should be placed so that the clear distance between their flanges is not less than $2\frac{1}{2}$ inches. Also, to insure uniform distribution of the load on the beams of each tier and to obviate bending stresses in the concrete filling, the clear distance between the flanges of the beams should not exceed one and one-half flange widths. When finally completed, the entire footing, including the column base, should be covered by not less than 4 inches of concrete to protect the steel. The beams for grillage footings should not be painted, in order that the concrete may better adhere to the steel surfaces. Furthermore, since the concrete cover acts as an excellent protection for the steel against rust, an additional preservative is not necessary.

28. Grout Filling.—To obviate the necessity for extreme care in setting the beams of each tier truly level, so as to insure uniform bearing of the column base on the top tier and each tier on the underlying tier, the beams of each tier and the column base are set $\frac{3}{4}$ inch above the beams of the tier below as at *g* in Fig. 4, and the $\frac{3}{4}$-inch spaces are filled with grout.

29. Length and Width of Tiers.—The length of the beams of each tier above the bottom tier is usually made equal to the distance between the outside flange edges of the outside beams of the tier below. The extreme width of the top tier is determined by the extreme dimensions of the column base, the beams being placed so that their outside flange edges coincide with the edges of the column base.

30. Separators.—In order that the beams of each tier may be kept the proper distance apart, they are secured to each other by means of $\frac{3}{4}$-inch bolts and separators, *s* in Fig. 4.

These separators may be either of gas pipe, as shown in Fig. 5, or of cast-iron as shown in Fig. 6. The cast-iron separators, besides holding the beams the proper distance apart, also help support their webs against buckling, but many engineers object to their use because they tend to break the continuity of the concrete filling. Gas-pipe separators are therefore used more extensively.

The separators should be placed from 5 to 6 feet apart throughout the length of the beams and not more than 6 inches from the ends of the beams. When gas-pipe separators are used, they should be placed so that the number of rods and their spacing will be practically the same as for standard cast-iron separators. Thus, in beams less than 12 inches deep, the gas-pipe separators should be placed midway between flanges, while in all beams 12 inches or more in depth they should be placed in pairs spaced vertically the same distance apart as the rods for standard cast-iron separators.

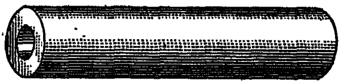


FIG. 5

FIG. 6

METHODS OF DESIGN

31. Preliminary Considerations.—In Fig. 7 are shown two side views of a grillage footing supporting one column. The pressure transmitted to the footing through

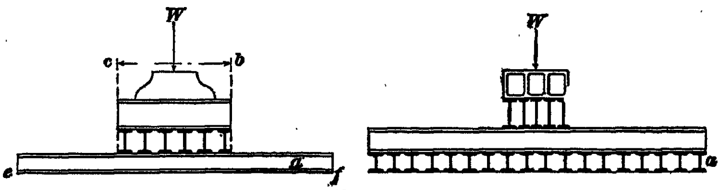


FIG. 7

the column is known and may be designated by W. The area of the footing has been determined as previously described. The dimensions of the column base are practically fixed by the design of the column, or they may be originally assumed

and increased or reduced, as conditions warrant. The number and size of beams in each tier are yet unknown. It is, therefore, best to compute the bending moment in the beams of an entire tier, and then provide beams of sufficient stiffness to resist that bending moment and of sufficient web thickness to resist buckling and shear.

32. Bending.—For the lower tier *a* of Fig. 7, the total load acting on the top of the beams is equal to the entire load from the column, which is assumed to be uniformly distributed over the length *c b*, while the reaction acting upwards on the bottom of the beams in opposition to the load from the column is assumed to be uniformly distributed over the length *e f*. The condition of loading that exists then on the beams is diagrammatically shown in Fig. 8, in which the load on the top of the beams is equal in amount to the upward reaction. The greatest bending moment under such a condition of loading does not occur at the edge of the first tier of beams, *g* in Fig. 8, but at the center of the beams, *c*, where the shear is zero. Then, considering the point *c* as the center of moments, the bending moment of the forces on the left half of the span at that point will be equal to the difference between the moment of the reaction $\frac{W}{2}$ acting upwards and that of the load $\frac{W}{2}$ on the beams acting downwards. The lever arm of the upward reaction is the distance from the center of gravity of that load to the center of moments, and is equal to $\frac{l}{4}$. The lever arm of the downward load is equal to $\frac{a}{4}$. Therefore, the positive moment is equal to $\frac{W}{2}\times\frac{l}{4}=\frac{W\,l}{8}$, the negative moment is

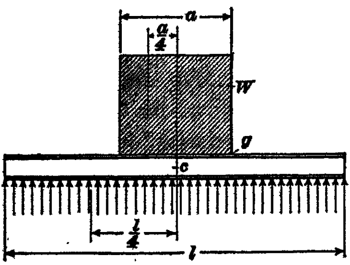


FIG. 8

equal to $\frac{W}{2}\times\frac{a}{4}=\frac{W\,a}{8}$, and the bending moment $M=\frac{W\,l}{8}-\frac{Wa}{8}$,

or

$$M=\frac{W}{8}\,(l-a)$$

The method for obtaining the bending moment as indicated by the formula just given may be stated as follows:

Rule.—*The maximum bending moment on any tier of beams in a grillage footing is equal to one-eighth of the entire load on the column, multiplied by the difference between the length of the tier and the width of either the base plate or of the tier of beams above.*

EXAMPLE.—In the grillage shown in Fig. 4, the column load is 650,000 pounds, the extreme width of the top tier is 36 inches and its length is 11 feet. What is the maximum bending moment in the bottom tier beams if their length is also 11 feet?

SOLUTION.—Here $W=650{,}000$ lb., $l=11$ ft. $=132$ in., and $a=36$ in.; hence,

$$M=\frac{W}{8}\,(l-a)=\frac{650{,}000}{8}\times(132-36)=7{,}800{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

33. Buckling.—Besides bending the beams, the load W also tends to buckle the webs in the portions of the beams directly under the base plate or the upper tier of beams. The unit stress that tends to buckle the webs in the portions of the beams directly under the loaded area, or the *actual unit buckling stress*, may be found by the formula

$$f_a=\frac{W}{n\,t\,a} \qquad (1)$$

in which f_a = actual unit buckling stress, in pounds per square inch;

n = number of beams in tier;

t = thickness of each web, in inches;

a = length of the portions of the webs under the loaded area, in inches.

The actual unit buckling stress should never exceed the *allowable unit buckling stress* which may be found by the formula

$$f_b = 16{,}000 - 200\,\frac{d}{t} \qquad (2)$$

in which f_b = allowable unit buckling stress, in pounds per square inch;
d = depth of beam, in inches;
t = thickness of each web, in inches.

EXAMPLE.—If the upper tier beams c in Fig. 4 are 24-in. I's 90 lb., and the column base plate is 36 inches square, what is (*a*) the actual unit buckling stress, and (*b*) the allowable unit buckling stress in these beams? The column load is 650,000 pounds.

SOLUTION.—(*a*) In this case the number of beams $n=4$. According to Table II, for a 24-in. I 90 lb. the web thickness $t=.624$ in. The length of the portions of the webs directly under the loaded area, $a=36$ in. Hence, by formula **1**, the actual unit buckling stress is

$$f_a = \frac{W}{n\,t\,a} = \frac{650{,}000}{4 \times .624 \times 36} = 7{,}230 \text{ lb. per sq. in. Ans.}$$

(*b*) The depth of the beams, $d=24$ in., and therefore, applying formula **2**, the allowable unit buckling stress is

$$f_b = 16{,}000 - 200\,\frac{d}{t} = 16{,}000 - 200 \times \frac{24}{.624} = 8{,}310 \text{ lb. per sq. in. Ans.}$$

Since the actual unit buckling stress, 7,230 lb. per sq. in., is less than the allowable unit buckling stress, 8,310 lb. per sq. in., the design is safe.

34. Shear.—In beams loaded as in Fig. 8, the maximum shear V occurs at g, the edge of the superimposed load, and its value may be found by the formula

$$V = \frac{W\,(l-a)}{2\,l} \qquad (1)$$

To complete the investigation of the beams, it is necessary to find whether their webs can safely resist this maximum shear. Since the intensity of shear is greatest at the neutral axes of the beams and least at their flanges, many designers assume that the shear is resisted by the webs of the beams between the ends of the fillets of the flanges only. However, for all practical purposes the average unit shear in the webs may be found by the formula

$$v = \frac{V}{n\,t\,d} \qquad (2)$$

The unit shear thus found must not exceed 10,000 lb. per square inch.

EXAMPLE.—What is the average unit shear in the upper tier beams in the example of Art. **33**?

SOLUTION.—Since $W=650{,}000$ lb., $l=132$ in., and $a=36$ in., by formula **1**, the maximum shear in the beams is

$$V=\frac{W\,(l-a)}{2\,l}=\frac{650{,}000\times(132-36)}{2\times132}=236{,}000 \text{ lb.}$$

Also, since $n=4$, $t=.624$ in., and $d=24$ in., the average unit shear is

$$v=\frac{V}{n\,t\,d}=\frac{236{,}000}{4\times.624\times24}=3{,}940 \text{ lb. per sq. in.} \quad \text{Ans.}$$

35. Governing Features in Design.—In designing the lower tier beams for ordinary grillage footings, the bending moment will invariably be found to govern the design, because there are usually a large number of beams of shallow depth to resist buckling and shear. However, the beams in the upper tiers are comparatively few in number and of relatively great depth, and their design will often be governed by buckling or shear.

When the bending moment governs the design and there is a choice between two beams of equal weight, the deeper beam is usually preferred because of its greater stiffness. Thus, if both a 9-in. **I** 30 lb. and a 10-in. **I** 30 lb. offer the required resisting moment, the 10-in. **I** 30 lb. would be preferred, its section modulus being 26.7, while that of the 9-in. **I** 30 lb. is only 22.5.

When the shear or buckling stress governs the design, in a choice of two beams of equal weight, the shallower beam is usually preferred because of its thicker and stiffer web. Thus, in a 9-in. **I** 30 lb., the thickness of the web is .561, the web area $t\,d=.561\times9=5.049$ sq. in., and the ratio $\frac{d}{t}=\frac{9}{.561}=16$; in a 10-in. **I** 30 lb. the web thickness is .447, the web area $t\,d=.447\times10=4.47$ sq. in., and the ratio $\frac{d}{t}=\frac{10}{.447}=22.4$. Shallower beams also require less concrete filling.

Nevertheless, in spread footings where stiffness is greatly desired, it is often better to use the deeper beam even if the cost is somewhat greater and the web resistance less.

PRACTICAL APPLICATIONS

36. Summary of Steps to be Followed in Design of Grillage Footings.—In order to design a grillage footing for the support of a single column, it is first necessary to establish the dimensions of the footing mat by determining the required bearing area of the footing and assuming an appropriate thickness, usually 12 inches. The next step is to compute the area of the column base, if it has not been previously determined.

The number of tiers of beams is then assumed according to the magnitude of the column load and the allowable soil pressure. For light loads two tiers may be safely assumed, while for heavy loads it is necessary to make two or more designs with different numbers of tiers and to choose the most economical arrangement. Three tiers of beams are usually required for heavy column loads or when the footing area is very large. For soils of high bearing capacity and light loads, one tier of beams will often prove sufficient. The experienced designer usually has little difficulty in making the right assumption from the start.

37. The next step is to determine the bending moment in the upper tier beams by the formula in Art. **32**, and to provide the proper number and size of beams to resist that bending moment. The actual and allowable unit buckling stresses in the beams are then computed by the formulas in Art. **33**, in order to investigate whether the beams are safe in buckling. Finally, the maximum shear in the beams is found by formula **1**, and the average unit shear by formula **2**, in Art. **34**, to determine whether the average unit shear is below the allowable 10,000 pounds per square inch.

The design of the lower tier beams then follows. First, the bending moment in the tier is determined and the proper

number and size of beams provided to resist that bending moment. As previously stated, in the usual designs it is not necessary to investigate these beams for buckling and shear.

The design of a grillage footing for the support of a single column is illustrated by the following example:

EXAMPLE.—Design a square grillage footing for the intermediate exterior column of Art. **14**, the footing area being proportioned by Schneider's method. The safe unit fiber stress in the beams is 16,000 pounds per square inch.

SOLUTION.—According to the table in Art. **21**, the required bearing area of the footing is 62.6 sq. ft. A concrete mat each side of which is $\sqrt{62.6}$, say 8 ft., will, therefore, be used, as shown in Fig. 9. According to the table in Art. **14**, the combined load on the column is 291,000 lb. Assuming that the grout under the column base can safely carry 400 lb. per sq. in., the required area for the column base is $\frac{291,000}{400}=727.5$ sq. in., and a square plate 27 in.×27 in.=729 sq. in. is ample.

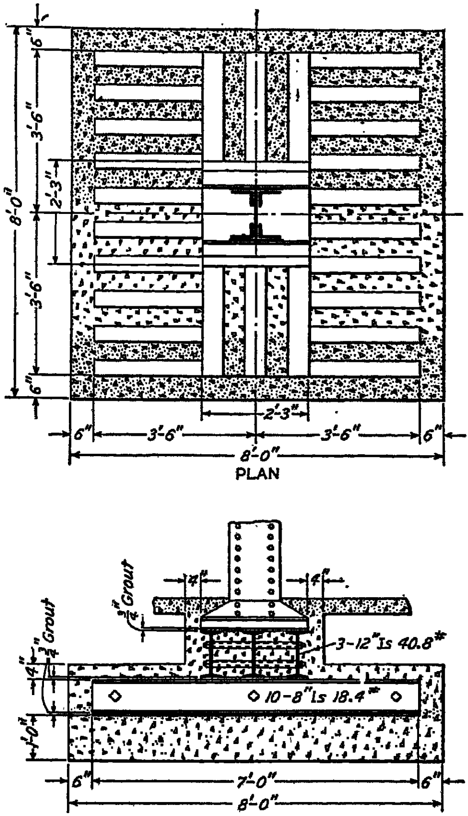


FIG. 9

Since the footing is of relatively small area, only two tiers of beams will be required. The concrete mat will be made 12 in. thick and will be assumed to project 6 in. beyond the steelwork. The length of the bottom tier beams is, therefore, 8 ft.−2×6 in.=7 ft. The width of the bottom tier is also 7 ft., which makes the length of the top tier beams 7 ft. The width of the top tier is the same as the width of the column base,

or 27 in. For the top tier beams, $W=291{,}000$ lb., $l=84$ in., and $a=27$ in.; hence, applying the formula of Art. 32,

$$M=\frac{W}{8}(l-a)=\frac{291{,}000}{8}(84-27)=2{,}070{,}000 \text{ in.-lb.}$$

The required total section modulus $=\dfrac{2{,}070{,}000}{16{,}000}=129.4$.

Since the width of the tier is 27 in., it is evident that either three or four beams can be used to provide the required section modulus. If three beams are used, the section modulus of each should be not less than $\dfrac{129.4}{3}=43.1$, and if four beams are used, the section modulus of each should be not less than $\dfrac{129.4}{4}=32.4$. According to Table II, either three 12-in. I's 40.8 lb. or four 12-in. I's 31.8 lb. are suitable. The combined section modulus of three 12-in. I's 40.8 lb. is $3\times44.8=134.4$; their combined weight per foot of length is $3\times40.8=122.4$ lb., and the clear distance between flanges is $\dfrac{27-(3\times5.25)}{2}=5\frac{5}{8}$ in. The combined section modulus of four 12-in. I's 31.8 lb. is $4\times36=144$; the combined weight is $4\times31.8=127.2$ lb. per ft. and the clear distance between flanges is $\dfrac{27-(4\times5)}{3}=2.33$ in. The three 12-in. I's 40.8 lb. will therefore be used.

The actual unit buckling stress on the webs of the beams, by formula **1** in Art. **33**, is

$$f_a=\frac{W}{nta}=\frac{291{,}000}{3\times.46\times27}=7{,}810 \text{ lb. per sq. in.,}$$

which is below the allowable unit buckling stress, found by formula **2** in Art **33**,

$$f_b=16{,}000-200\frac{d}{t}=16{,}000-200\times\frac{12}{.46}=10{,}800 \text{ lb. per sq. in.}$$

The maximum total shear, by formula **1** in Art. **34**, is

$$V=\frac{W(l-a)}{2l}=\frac{291{,}000\ (84-27)}{2\times84}=98{,}700 \text{ lb.}$$

and the average unit shear, by formula **2** in Art. **34**, is

$$v=\frac{V}{ntd}=\frac{98{,}700}{3\times.46\times12}=5{,}960 \text{ lb. per sq. in.,}$$

which is below the permissible 10,000 lb. per sq. in.

For the bottom tier beams in this problem W, l, and a have the same values as for the top tier beams. The total bending moment and section modulus are therefore the same as before, or $M=2{,}070{,}000$ in.-lb., and the total section modulus is 129.4. The width of the bottom tier is 7 ft. $=84$ in. According to Table II, the combined section modulus of seven 9 in. I's 21.8 lb. is $7\times18.9=132.3$, their combined weight per foot is

$7\times21.8=152.6$ lb., and clear distance between their flanges is $\frac{84-(7\times4.33)}{6}$ $=9$ in. The total section modulus supplied by ten 8-in. I's 18.4 lb. is $10\times14.2=142$, their combined weight per foot is $10\times18.4=184$ lb., and the clear distance between flanges is $\frac{84-(10\times4)}{9}=4.9$ in. The total section modulus supplied by thirteen 7-in. I's 15.3 lb. is $13\times10.4=135.2$, their total weight per foot is $13\times15.3=198.9$ lb., and their clear distance between flanges is $\frac{84-(13\times3.66)}{12}=3$ in. A comparison of the various combinations of beams is made in tabular form as follows:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
7	9-in. I 21.8 lb.	18.9	.290	4.33	132.3	152.6	9.0
10	8-in. I 18.4 lb.	14.2	.270	4.00	142.0	184.0	4.9
13	7-in. I 15.3 lb.	10.4	.250	3.66	135.2	198.9	3.0

The combination of seven 9-in. I's 21.8 lb. is the lightest, but the clear distance between flanges is greater than one and one-half times the width of each flange. The ten 8-in. I's 18.4 will therefore be chosen.

The actual unit buckling stress on the webs of these beams,

$$f_a=\frac{W}{nta}=\frac{291,000}{10\times.27\times27}=3,990 \text{ lb. per sq. in.,}$$

which is considerably below the allowable unit buckling stress,

$$f_t=16,000-200\frac{d}{t}=16,000-200\times\frac{8}{.27}=10,100 \text{ lb. per sq. in.}$$

The maximum total shear is the same as for the top tier beams, 98,700 lb., and the average unit shear in the webs,

$$v=\frac{V}{ntd}=\frac{98,700}{10\times.27\times8}=4,570 \text{ lb. per sq. in.,}$$

which is below the permissible 10,000 lb. per sq. in.

The required grillage footing is, therefore, as shown in plan and elevation in Fig. 9.

38. In the example of the preceding article the column load and the required footing area were comparatively small, therefore two tiers of beams proved ample for the grillage

footing. Where the column load is heavy, and the required footing area is large, three tiers of beams are generally required, as in the following illustrative example.

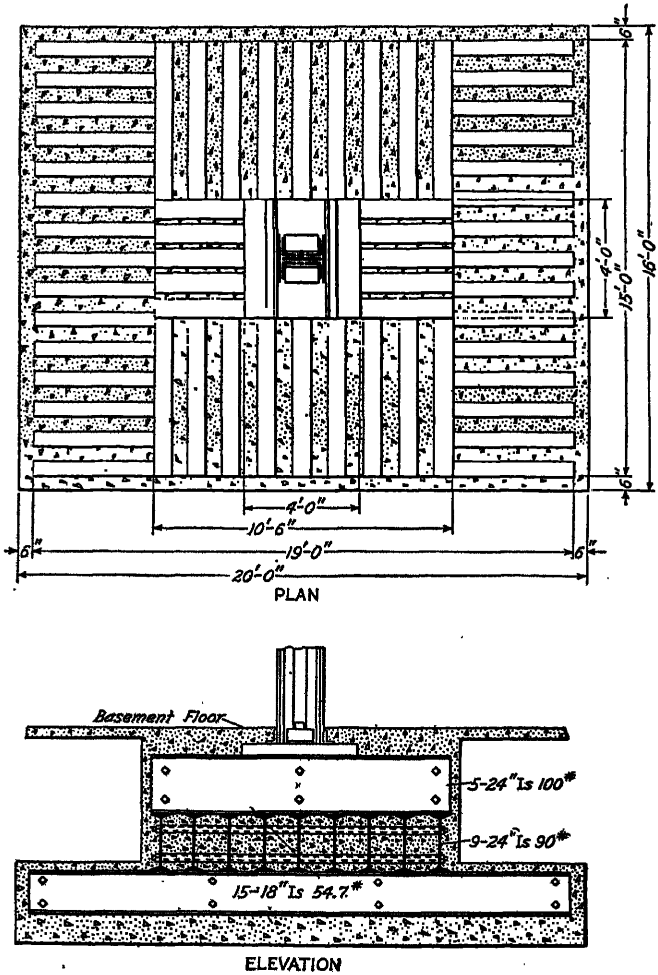


Fig. 10

Example.—Design a grillage footing to support a column load of 800 tons on a soil with an allowable bearing capacity of $2\frac{1}{2}$ tons per square foot. The column base rests directly on the flanges of the top tier beams and is

4 feet square. Architectural features limit the width of the footing to 16 feet.

Solution.—The required bearing area is $\frac{800}{2.5}=320$ sq. ft. For a width of 16 ft. the corresponding length is $\frac{320}{16}=20$ ft. Assuming a concrete mat 12 in. thick, and allowing it to project 6 in. at each end, the dimensions of the grillage are 15 ft. in width by 19 ft. in length.

If two tiers of beams are assumed, the width of the top tier is 4 ft., its length is 15 ft., and the loaded distance $a=4$ ft. By the formula of Art. 32, the maximum bending moment in the top tier beams is

$$M=\frac{1{,}600{,}000\times(15-4)\times 12}{8}=26{,}400{,}000 \text{ in.-lb.,}$$

and the required total section modulus is $\frac{26{,}400{,}000}{16{,}000}=1{,}650$. This section modulus cannot be supplied by any combination of beams given in Table II. For instance, the heaviest beam given in the table is the 24-in. I 115 lb. Seven such beams give a total section modulus of $245\times 7=1{,}715$, but their flanges alone occupy about $7\times 8=56$ in., or more than 48 in., which is the extreme width of the tier. It is, therefore, obvious that either riveted plate girders must be used for the top tier, or three tiers must be assumed to allow the use of shorter beams in the top tier. The latter assumption will be made and the beams will be arranged as shown in Fig. 10.

Let five 24-in. I's 100 lb. be assumed for the top tier. In an extreme width of 48 in., these beams offer a clearance between flanges of $\frac{48-(5\times 7.247)}{4}=2.94$ in., and a total section modulus of $197.6\times 5=988$. Their resisting moment is $988\times 16{,}000=15{,}810{,}000$ in.-lb. For economic design the maximum bending moment should equal the resisting moment, or

$$\frac{1{,}600{,}000\times(l-4)\times 12}{8}=15{,}810{,}000 \text{ in.-lb.;}$$

therefore,

$$l-4=\frac{15{,}810{,}000\times 8}{1{,}600{,}000\times 12}=6.59 \text{ ft.}$$

and

$$l=6.59+4=10.59 \text{ ft.}$$

A length of 10.5 ft. will be assumed.

The actual unit buckling stress in the webs of the beams,

$$f_a=\frac{1{,}600{,}000}{5\times .747\times 48}=8{,}920 \text{ lb. per sq. in.,}$$

which is below the allowable unit buckling stress,

$$f_b=16{,}000-200\times\frac{24}{.747}=9{,}570 \text{ lb. per sq. in.}$$

The maximum shear in the beams,

$$V=\frac{1{,}600{,}000\ (10.5-4)}{2\times 10.5}=495{,}000\text{ lb.,}$$

and the average unit shearing stress in the webs,

$$v=\frac{495{,}000}{5\times .747\times 24}=5{,}520\text{ lb. per sq. in.,}$$

which is below the allowable 10,000 lb. per sq. in.

The width of the intermediate tier is 10.5 ft., its length 15 ft., and the loaded distance $a=4$ ft. The maximum bending moment in the beams of the intermediate tier,

$$M=\frac{1{,}600{,}000\times(15-4)\times 12}{8}=26{,}400{,}000\text{ in.-lb.}$$

and the required total section modulus is $\frac{26{,}400{,}000}{16{,}000}=1{,}650.$

This section modulus is supplied by the following combinations of beams:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
9	24-in. I 90 lb.	185.8	.624	7.12	1672.2	810	7.7
10	24-in. I 79.9 lb.	173.9	.500	7.00	1739.0	799	6.2
15	20-in. I 65.4 lb.	116.9	.500	6.25	1753.5	981	2.3

The combination of ten 24-in. I's 79.9 lb. seems to be the best. However, it is found that the actual unit buckling stress,

$$f_a=\frac{1{,}600{,}000}{10\times .5\times 48}=6{,}670\text{ lb. per sq. in.}$$

is greater than

$$f_b=16{,}000-200\times\frac{24}{.5}=6{,}400\text{ lb. per sq. in.}$$

The nine 24-inch I's 90 lb. will therefore be tried. The actual unit buckling stress is

$$f_a=\frac{1{,}600{,}000}{9\times .624\times 48}=5{,}940\text{ lb. per sq. in.}$$

which is below the allowable unit buckling stress,

$$f_b=16{,}000-200\times\frac{24}{.624}=8{,}310\text{ lb. per sq. in.}$$

The maximum shear in the beams,

$$V=\frac{1{,}600{,}000\times(15-4)}{2\times 15}=587{,}000\text{ lb.}$$

and the average unit shearing stress in the webs,

$$v = \frac{587{,}000}{9 \times .624 \times 24} = 4{,}360 \text{ lb. per sq. in.,}$$

which is below the allowable 10,000 lb. per sq. in. The nine 24-in. I's 90 lb. will therefore be adopted.

The width of the bottom tier is 15 ft., its length is 19 ft., and the loaded distance $a = 10.5$ ft. The maximum bending moment in the beams,

$$M = \frac{1{,}600{,}000 \times (19 - 10.5) \times 12}{8} = 20{,}400{,}000 \text{ in.-lb.}$$

and the required total section modulus is $\frac{20{,}400{,}000}{16{,}000} = 1{,}275$. The combinations of beams that supply the required total section modulus are as follows:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
15	18-in. I 54.7 lb.	88.4	.460	6.00	1326	820.5	6.4
16	15-in. I 60.8 lb.	81.2	.590	6.00	1299	972.8	5.6
22	15-in. I 42.9 lb.	58.9	.410	5.50	1296	943.8	2.8

The combination of fifteen 18-in. I's 54.7 lb. is the lightest and will be adopted. Since the lower tier beams offer much more web area to resist buckling and shear than the upper tier beams, no further investigation need be made.

The arrangement and spacing of beams will be as shown in Fig. 10.

EXAMPLES FOR PRACTICE

1. A grillage footing is composed of two tiers of steel beams. The column base, which is 30 inches wide and 42 inches long, supports a load of 300,000 pounds. The beams in the first tier are five in number and the tier is 42 inches in width, measuring between the outside edges of the flanges of the outside beams, and the length of the beams in this tier is 150 inches. The number of beams in the bottom tier is fifteen and their length is also 150 inches. Provided that an allowable unit fiber stress of 16,000 pounds is assumed, what will be the economical sizes of the beams in each tier?

Ans. $\begin{cases} \text{Upper tier, 15-in. I's 42.9 lb.} \\ \text{Lower tier, 9-in. I's 21.8 lb.} \end{cases}$

2. Determine the economic sizes of steel beams required for a grillage footing composed of three tiers of steel beams. The bottom tier is 12 feet

6 inches long and 10 feet wide, the intermediate tier has a length equal to the width of the bottom tier and a width of 6 feet 6 inches, while the top tier extends across the intermediate tier and is 3 feet 6 inches wide. The cast-iron base is rectangular in plan, being equal in length to the width of the top tier and 3 feet wide. Five beams compose the top tier, while in the intermediate and bottom tiers there are ten and fifteen beams, respectively. The load on the footing is from an interior column and amounts to 320 tons. The assumed allowable unit fiber stress is 16,000 pounds per square inch.

Ans. { Top tier, 12-in. I's 40.8 lb.
Intermediate tier, 12-in. I's 40.8 lb.
Bottom tier, 10-in. I's 25.4 lb.

GRILLAGE FOOTINGS SUPPORTING TWO COLUMNS

39. Provision for Uniform Soil Pressure.—In providing grillage footings for columns, it is often found necessary to rest two or more columns on one continuous footing. This may be due to various conditions, such as exist when an exterior column is so near the building line that it is impossible to design a symmetrical footing for it, which necessitates supporting the exterior and the nearest interior columns on one continuous footing. Such a footing would also become necessary when two columns are so placed that their separate footings would touch or overlap. Fig. 11 illustrates the arrangement of the grillage beams in a footing supporting two columns, in plan at (*a*) and in elevation at (*b*).

To insure even settlement throughout the length of a combined footing, the area of the footing should be proportioned so that the pressure on the soil will be uniform. In a footing supporting two columns, a uniform soil pressure may be obtained by designing the base of the footing so that the resultant of the superimposed column loads will pass through the center of gravity of the footing. The resultant of the superimposed loads and the resultant of the soil pressure then act in the same line.

40. Footings Supporting Two Equally Loaded Columns.—When the two columns supported on a combined footing carry equal loads, the resultant of the loads acts midway between the column axes, as at *c*, Fig. 11. The foot-

ing base should then be made rectangular in section and provided with projections x of equal length, in which case the footing is symmetrical about c. This arrangement will make the line of action of the resultant pass through the center of gravity of the footing, and a uniform distribution of pressure over the entire length of the footing will be obtained.

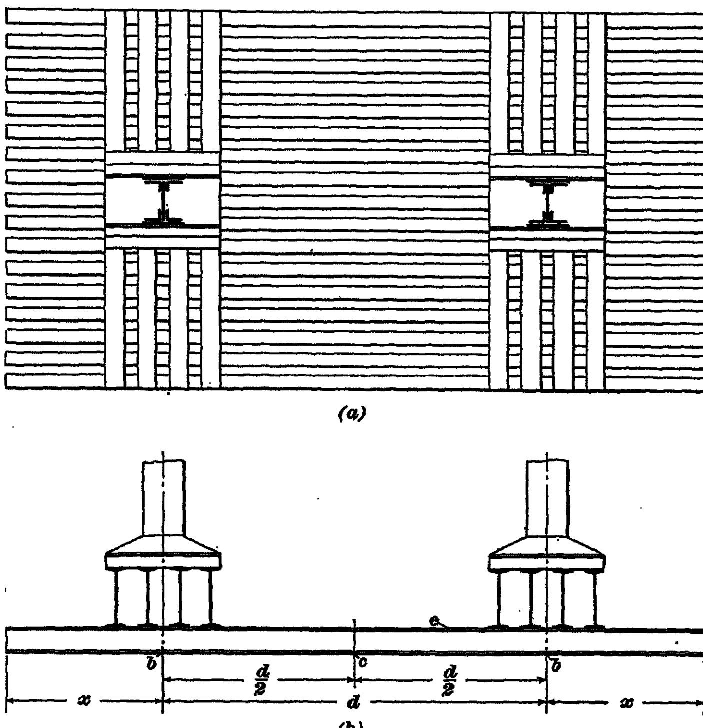


FIG. 11

The diagram of loading on the lower tier beams e and the manner in which they tend to deflect is shown in Fig. 12. In such cases the greatest bending moment in the beams occurs either at c, the center of the span d, or in the projections under the columns, depending on the ratio of the length of the projections x to the distance between the column axes d.

When the projections x are short as compared with the span d, the maximum bending moment occurs at c, but when the projections are comparatively long, the maximum bending moment may occur at b. The length of the projections of the footing is frequently limited by building conditions. But, where no limitations exist, maximum economy in design is obtained by choosing the distance x so that the bending moment at the middle of the span is equal to the bending moment in the projections at the center of either column. When the projections cannot be made of economic length, it is necessary to compute the maximum bending moment in

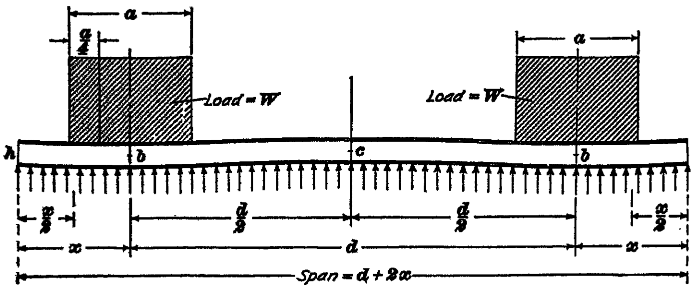


Fig. 12

the middle of the span and the maximum bending moment in either projection and design the beams for the greater of the two bending moments.

The soil reaction on each unit length of the beams shown in Fig. 12 is

$$p = \frac{2W}{d+2x}$$

If moments of the loads on the projection $b\,h$ about the center line of the column b are taken, the upward soil pressure is $p\,x$, its lever arm is $\frac{x}{2}$, and the moment of the load is $\frac{p\,x^2}{2}$; the downward load is $\frac{W}{2}$, its lever arm is $\frac{a}{4}$, and the moment of the load is $\frac{W\,a}{8}$; hence, the bending moment is

$$M_1=\frac{px^2}{2}-\frac{Wa}{8}=\frac{Wx^2}{d+2x}-\frac{Wa}{8};$$

whence,
$$M_1=\frac{W}{8}\left(\frac{8x^2}{d+2x}-a\right) \qquad (1)$$

The bending moment M_1 is positive because it produces tensile stresses in the lower fibers of the beams and compressive stresses in the upper fibers.

To derive a formula for the bending moment at c, the moments of the loads on the half-span hc about c are found. The clockwise moment of the upward soil pressure, which is positive, is $W\left(\frac{d+2x}{4}\right)$, while the counter-clockwise moment of the downward column load, which is negative, is $\frac{Wd}{2}$. When $\frac{Wd}{2}$ is less than $W\left(\frac{d+2x}{4}\right)$, which occurs when $\frac{d}{2}$ is less than $\frac{d+2x}{4}$, the bending moment at c is positive; the stresses then produced in the beam are tensile in the lower fibers and compressive in the upper fibers, the point c deflecting downwards. When the two moments are equal, there is no bending moment or deflection at c; but when $\frac{Wd}{2}$ is greater than $W\left(\frac{d+2x}{4}\right)$, or when $\frac{d}{2}$ is greater than $\frac{d+2x}{4}$, the bending moment at c is negative, compressive stresses being produced in the lower fibers and tensile stresses in the upper fibers, and the beam deflects in the manner shown in Fig. 12. It is this negative bending moment at c that enters in the usual designs, because a positive bending moment at c occurs only when the projections are excessively long. The value of the negative bending moment M_2 at c is found by subtracting $W\left(\frac{d+2x}{4}\right)$ from $\frac{Wd}{2}$, or $M_2=\frac{Wd}{2}-W\left(\frac{d+2x}{4}\right)$; whence,

$$M_2=\frac{W}{4}(d-2x) \qquad (2)$$

For economic design, $M_2 = M_1$, or

$$\frac{W}{4}(d-2x) = \frac{W}{8}\left(\frac{8x^2}{d+2x} - a\right)$$

$$16x^2 - 2ax - ad - 2d^2 = 0$$

and
$$x = \frac{a + \sqrt{a^2 + 16ad + 32d^2}}{16} \qquad (3)$$

If the projections of the footing are given the length computed by formula **3**, a footing will result in which the maximum positive bending moment at the center of the columns will equal the maximum negative bending moment in the middle of the span.

41. Illustrative Example.—A typical design of a grillage footing supporting two equally loaded columns is given in the following example:

Example.—Design a grillage footing to support two interior columns each carrying 600,000 pounds and spaced 14 feet center to center. The allowable soil pressure is 2 tons per square foot. The column bases are each 3 feet 6 inches square.

Solution.—In this example $W = 600{,}000$ lb., $d = 14$ ft., and $a = 3.5$ ft. The required bearing area of the footing is $\frac{2 \times 600{,}000}{4{,}000} = 300$ sq. ft.

To obtain equal moments in the bottom tier beams, formula **3** in Art. **40** is applied. Hence,

$$x = \frac{a + \sqrt{a^2 + 16ad + 32d^2}}{16}$$

$$= \frac{3.5 + \sqrt{3.5^2 + 16 \times 3.5 \times 14 + 32 \times 14^2}}{16} = 5.47 \text{ ft.},$$

and a length of 5 ft. 6 in. will be adopted. Since the columns are 14 ft. center to center, and the projections are each 5 ft. 6 in. long, the required length of the bottom tier beams is 14 ft. $+ 2 \times 5.5 = 25$ ft. If a 12-in. concrete mat is assumed, it may be allowed to project 6 in. beyond the ends of the lower tier beams. The total length of the footing would then be $25 + (2 \times .5) = 26$ ft., and since the required bearing area of the footing is 300 sq. ft., the required width of the footing is $\frac{300}{26} = 11.5$ ft. The width of the bottom tier, therefore, is $11.5 - (2 \times .5) = 10.5$ ft. $= 126$ in.

By formula 1 in Art. **40**, the maximum bending moment in the projections at the center lines of the columns,

$$M_1=\frac{W}{8}\left(\frac{8\,x^2}{d+2\,x}-a\right)$$
$$=\frac{600{,}000}{8}\times\left(\frac{8\times5.5^2}{25}-3.5\right)$$
$$=463{,}000\text{ ft.-lb.}=5{,}560{,}000\text{ in.-lb.}$$

By formula **2**, the maximum bending moment at the center of the beams,

$$M_2=\frac{W}{4}(d-2\,x)$$
$$=\frac{600{,}000}{4}\times(14-(2\times5.5)$$
$$=450{,}000\text{ ft.-lb.}=5{,}400{,}000\text{ in.-lb.}$$

The slight difference between the two moments is due to the assumption $x=5$ ft. 6 in. instead of 5.47 ft., the computed value.

The required total section modulus $=\frac{5{,}560{,}000}{16{,}000}=347$, which is supplied by the following combinations of beams:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
10	12-in. I 31.8 lb.	36.0	.350	5.00	360.0	318.0	8.4
15	10-in. I 25.4 lb.	24.4	.310	4.66	366.0	381.0	4.0
19	9-in. I 21.8 lb........	18.9	.290	4.33	359.1	414.2	2.4

The fifteen 10-in. I's 25.4 lb., being the lightest combination that offers proper clearance between flanges, will be adopted.

The top tier beams under each column are 10 ft. 6 in. long and 3 ft. 6 in. wide. Their maximum bending moment is determined as for the top tier beams of a footing supporting one column, or

$$M=\frac{600{,}000}{8}(10.5-3.5)=525{,}000\text{ ft.-lb.}$$
$$=6{,}300{,}000\text{ in.-lb.}$$

The total section modulus $=\frac{6{,}300{,}000}{16{,}000}=394$, which is supplied by the following combinations of beams:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
3	20-in. I 81.4 lb.	146.6	.600	7.00	439.8	244.2	10.5
4	20-in. I 65.4 lb.	116.9	.500	6.25	467.6	261.6	5.7
5	18-in. I 54.7 lb.	88.4	.460	6.00	442.0	273.5	3.0

The four 20-in. I's 65.4 lb. will be adopted, although they are somewhat heavier than the three 20-in. I's 81.4 lb., because they offer a greater section modulus and better beam spacing for a relatively small increase in weight.

The actual unit buckling stress on the webs of the beams is

$$f_a=\frac{600{,}000}{4\times.50\times42}=7{,}140 \text{ lb. per sq. in.,}$$

which is below the allowable buckling stress,

$$f_b=16{,}000-200\times\frac{20}{.50}=8{,}000 \text{ lb. per sq. in.}$$

The maximum shear on the webs of the beams,

$$V=\frac{600{,}000\ (10.5-3.5)}{2\times10.5}=200{,}000 \text{ lb.,}$$

and the corresponding unit shearing stress,

$$v=\frac{200{,}000}{4\times.5\times20}=5{,}000 \text{ lb. per sq. in.,}$$

which is below the allowable 10,000 lb. per sq. in. The construction of this footing is shown in Fig. 13.

42. Rectangular Footings Supporting Unequally Loaded Columns.—When a combined footing supports two columns carrying unequal loads, the line of action of the resultant of these loads is nearer to the axis of the column carrying the heavier load. To insure uniform soil pressure, the resultant of the superimposed loads must pass through the center of gravity of the footing. This may be accomplished by making the footing base rectangular and allowing it to project a sufficiently longer distance beyond the axis of the heavier column than beyond the axis of the lighter column. In Fig. 14 (*a*) is shown a diagram of the loading on the lower

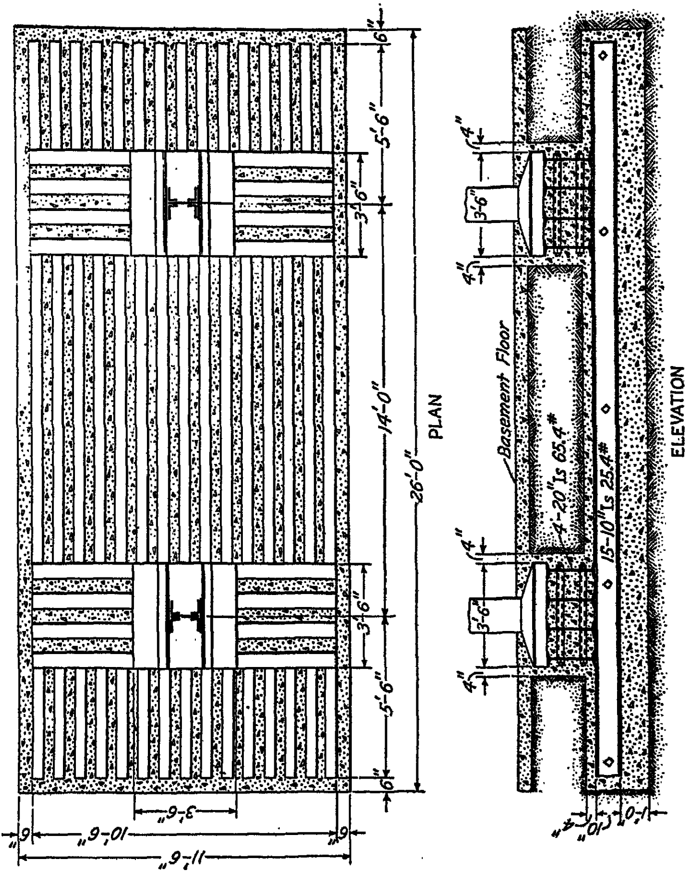

PLAN
ELEVATION
Basement Floor
4-20" Is 65.4#
15-10" Is 25.4#
26'-0"
14'-0"
5'-6"
3'-6"
6"
4"
11'-6"
10'-6"
1'-0"
10"

Fig. 13

tier beams of a rectangular footing supporting two unequal column loads, W_1 and W_2, the distance between center lines of columns being d. The resultant of the two loads is

$$R = W_1 + W_2 \qquad (1)$$

To obtain the distance x of the point of application of the resultant from the center line of W_1, moments are taken about the point g, giving $(W_1+W_2)\,x - W_2\,d = 0$; whence,

$$x = \frac{W_2\,d}{W_1 + W_2} \qquad (2)$$

To insure uniform soil pressure, the projections should be proportioned so that $x+k=y+h$, in which $y=d-x$.

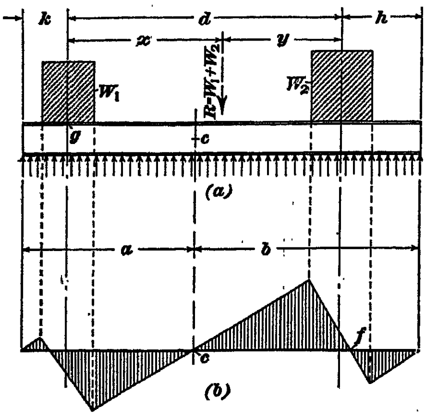


FIG. 14

The shear diagram for the loading in (a) is shown in (b). The point of zero shear in span d being at c, at a distance a from the left end of the tier, the upward soil pressure per unit length of tier, p, multiplied by the distance a equals the downward column load W_1, or $W_1 = a\,p$. Hence,

$$a = \frac{W_1}{p} \qquad (3)$$

Similarly, the distance of the point of zero shear in span d from the right end of the tier, or distance b, may be found by dividing the column load W_2 by p. Hence,

$$b = \frac{W_2}{p} \qquad (4)$$

The lower tier beams are designed for the greatest bending moments in them, which may occur either beneath one of the columns or at the point of zero shear between the columns. Although, as the shear diagram in Fig. 14 (*b*) indicates, the point of zero shear, and hence of maximum moment, under either column is not at the center line of the column, for all practical purposes it may be assumed to be there. The reason for this is that the maximum bending moments under the columns are large when the projections of the footing are long, and when such conditions prevail, the points of zero shear are near the center lines of the columns, as point *f* in (*b*). The upper tier beams under each column are designed in the same manner as the upper tier beams of footings supporting equally loaded columns. However, since the two column loads in this case are of different magnitudes, the number and size of beams under each column will differ.

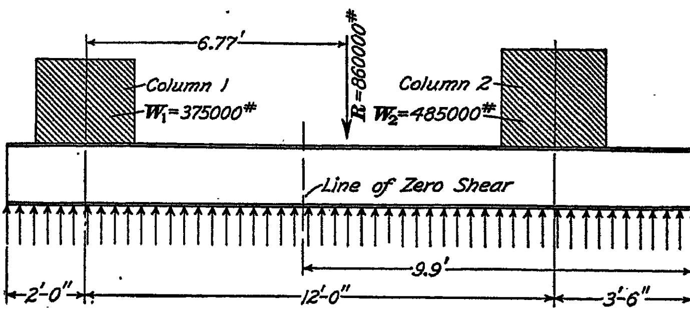


FIG. 15

43. Illustrative Example.—A typical design of a rectangular grillage footing, supporting two unequally loaded columns is given in the following example:

EXAMPLE.—Design a column footing for the support of two columns, 1 and 2, spaced 12 feet center to center. Column 1 carries a load of 375,000 pounds and its base plate is 28 inches square; column 2 carries a load of 485,000 pounds and its base plate is 32 inches square. The required footing area is 172 square feet and it must not project more than 2 feet 6 inches beyond the axis of the lighter column.

SOLUTION.—The diagram of loading on the lower tier beams of the footing is shown in Fig. 15. The resultant of the two loads W_1 and W_2 is

$$R = W_1 + W_2 = 375{,}000 + 485{,}000 = 860{,}000 \text{ lb.}$$

By formula 2 in Art. 42, the distance of the point of application of the resultant from the axis of the lighter column is

$$x=\frac{W_2 d}{W_1+W_2}=\frac{485{,}000\times 12}{860{,}000}=6.77 \text{ ft.}$$

To secure uniform pressure on the soil, the resultant of the column loads should pass through the center of gravity of the footing. This condition may be obtained by making the footing symmetrical about the point of application of the resultant. Since the footing must not project

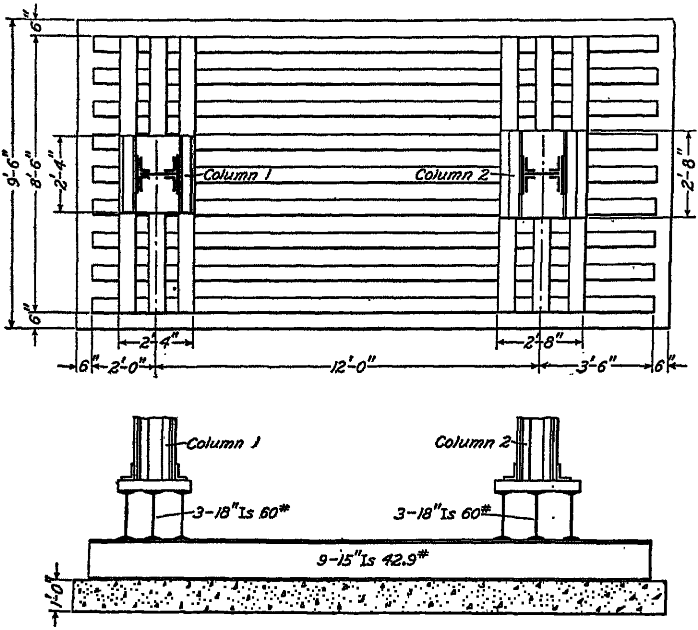


Fig. 16

more than 2 ft. 6 in. beyond the axis of the lighter column, the distance from the line of action of the resultant to the left end of the footing is 6.77 +2.5=9.27 ft., and the entire length of the footing is 2×9.27=18.54 ft., say 18.5 ft. The projection of the footing beyond the axis of the heavier column is therefore 18 ft. 6 in.−14 ft. 6 in.= 4 ft. The required width of the footing is $\frac{172}{18.5}=9.3$ ft., say 9 ft. 6 in., as shown in Fig. 16.

Assuming the concrete footing mat to be 12 in. thick, 6 in. projections beyond the steelwork may be allowed, and the bottom tier is therefore,

8 ft. 6 in. wide by 17 ft. 6 in. long. The reaction per unit length of tier on the bottom tier beams is

$$p=\frac{860,000}{17.5}=49,100 \text{ lb. per ft.}$$

The maximum bending moment that will govern the design is obviously either beneath column 2 or at the point of no shear between the columns. The maximum bending moment beneath column 2 is found by taking moments about the center line of column 2 of the loads to the right of that center line, or

$$M_1=49,100\times3.5\times\frac{3.5}{2}-\frac{485,000}{2}\times\frac{2.67}{4}=139,000 \text{ ft.-lb.}$$

The distance of the point of zero shear from the right end of the lower tier beams

$$b=\frac{485,000}{49,100}=9.9 \text{ ft.}$$

The bending moment at the point of zero shear,

$$M_2=49,100\times9.9\times\frac{9.9}{2}-485,000\times(9.9-3.5)=-700,000 \text{ ft.-lb.},$$

which is the greatest bending moment and will govern the design. The required total section modulus in the lower tier beams is, therefore, $\frac{700,000\times12}{16,000}=525$, which is best supplied by nine 15-in. I's 42.9 lb., spaced about 12 in. center to center, as shown in Fig. 16 (*a*).

The maximum bending moment in the upper tier beams under column 1 is $\frac{375,000}{8}\times(8.5-2.33)\times12=3,470,000$ in.-lb. The required section modulus is $\frac{3,470,000}{16,000}=217$, which is supplied by the following combination of beams:

No. of Beams	Size of Beams	Section Modulus	Thickness of Web Inches	Width of Flange Inches	Total Section Modulus	Total Weight Pounds per Foot	Clearance Between Flanges Inches
3	18-in. I 60.0 lb.	93.1	.547	6.087	279.3	180.0	4.9
3	18-in. I 54.7 lb.	88.4	.460	6.000	265.2	164.1	5.0
3	15-in. I 60.8 lb.	81.2	.590	6.000	243.6	182.4	5.0

If the three 18-in. I's 54.7 lb. are chosen as the most economical combination, the unit buckling stress in their webs is

$$f_a=\frac{375,000}{3\times.460\times28}=9,700 \text{ lb. per sq. in.},$$

which exceeds the allowable unit buckling stress,

$$f_b=16{,}000-200\times\frac{18}{.46}=8{,}170 \text{ lb. per sq. in.}$$

The three 18-in. I's 60 lb. will therefore be adopted. The unit buckling stress in their webs,

$$f_a=\frac{375{,}000}{3\times.547\times28}=8{,}150 \text{ lb. per sq. in.,}$$

which is less than the allowable unit buckling stress,

$$f_b=16{,}000-200\times\frac{18}{.547}=9{,}420 \text{ lb. per sq. in.}$$

The greatest shear,

$$V=\frac{375{,}000\ (8.5-2.33)}{2\times8.5}=136{,}000 \text{ lb.,}$$

and the unit shear,

$$v=\frac{136{,}000}{3\times.547\times18}=4{,}600 \text{ lb. per sq. in.,}$$

which is below the allowable 10,000 lb. per sq. in.

The maximum bending moment in the upper tier beams under column 2 is

$$M=\frac{485{,}000}{8}\ (8.5-2.67)\times12=4{,}240{,}000 \text{ in.-lb.}$$

The required section modulus is $\frac{4{,}240{,}000}{16{,}000}=265$, which is best supplied by three 18-in. I's 60 lb., spaced about 13 in. center to center. The unit buckling stress in their webs is

$$f_a=\frac{485{,}000}{3\times.547\times32}=9{,}240 \text{ lb. per sq. in.,}$$

which is below the previously computed value

$$f_b=9{,}420 \text{ lb. per sq. in.}$$

The greatest shear,

$$V=\frac{485{,}000\ (8.5-2.67)}{2\times8.5}=166{,}000 \text{ lb.,}$$

and the unit shear,

$$v=\frac{166{,}000}{3\times.547\times18}=5{,}610 \text{ lb. per sq. in.,}$$

which is amply safe.

The design and the grillage footing is shown in plan and in elevation in Fig. 16, the concrete filling and protection above the mat being removed.

44. Trapezoidal Footing Base.—When the distance the footing may project beyond the axis of both columns is limited, the resultant of the two unequal loads may be made to

pass through the center of gravity of the footing base by assuming a trapezoidal shape for the latter.

Combined grillage footings of trapezoidal shape are seldom used, and, therefore, will not be explained here. The chief difficulty in the design of such footings is in proportioning the proper shape and in computing the bending moments and shears, and these will be explained in the next Section in

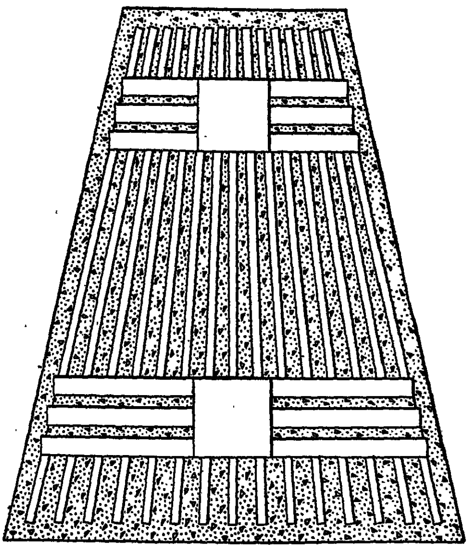


FIG. 17

connection with the design of trapezoidal footings of reinforced concrete. The design of combined grillage footings of trapezoidal shape is similar to the design of combined grillage footings of rectangular shape. The trapezoidal arrangement of the beams in the lower tier is usually obtained by placing the end beams parallel to the sloping edges of the mat and spacing the other beams so that their ends are at equal distances apart at both ends of the footing, as in Fig. 17. Sometimes this purpose is accomplished by placing the beams parallel to each other and adding several shorter beams in the wider part of the footing.

EXAMPLES FOR PRACTICE

1. Determine the size of the steel beams necessary in the bottom tier of a grillage foundation to support two columns, each carrying 200 tons. The distance between the centers of the columns is 10 feet. Each upper tier of beams is 3 feet wide and the number of beams in the bottom grillage is 12. Assume a safe unit fiber stress of 16,000 pounds per square inch and a safe bearing capacity of soil of 2 tons per square foot.

Ans. 8-in. I 18.4 lb.

2. Two columns supporting loads of 310 tons each rest on tiers of steel beams that are supported on a bottom tier. The distance from center to center of columns is 12 feet and the distance from the center of columns to the end of the lower tier is in this case 5 feet at each end. If the width of each upper tier between the outside flange edges is 40 inches and an allowable unit fiber stress of 16,000 pounds is assumed, what will be the size of the steel beams in the lower tier if ten beams are used?

Ans. 12-in. I 31.8 lb.

3. Two columns A and B, spaced 11 feet on centers, are supported on one footing. Column A carries a load of 315,000 pounds and its base plate is 30 inches square, while column B carries a load of 412,000 pounds and its base plate is 33 inches square. If the required footing area is 165 square feet and it must not project more than 2 feet beyond the axis of the lighter column, what will be the number and size of beams in the lower tier for a unit fiber stress of 16,000 pounds per square inch?

Ans. Ten 12-in. I's 40.8 lb.

DESIGN OF SPREAD FOUNDATIONS
(PART 2)

REINFORCED-CONCRETE SPREAD COLUMN FOOTINGS

INTRODUCTION

1. Advantages.—As compared with plain-concrete footings, reinforced-concrete spread footings produce a saving in materials and excavation and effect a considerable reduction in the bulk and weight of the foundation. Spread footings of reinforced concrete compare favorably also with grillage footings. While in a grillage footing the steel I-beams only are relied on to resist the transverse stresses, the concrete filling being used merely as a protection, in a reinforced-concrete footing both the concrete and the steel are effective in resisting stresses, each according to its capacity. Thus, the concrete is expected to carry all compressive stresses and the steel reinforcement all tensile stresses. The saving from such an arrangement is quite obvious. Furthermore, reinforcing rods are more readily obtainable than I-beams, and can be placed with greater ease and without the use of derricks.

In buildings that are to be constructed mainly of reinforced concrete, spread footings of the same material are particularly desirable because they may be effectively connected to the superimposed columns and walls by means of projecting rods or dowels.

2. **Disadvantages.**—There are, however, certain objections to reinforced-concrete footings that must not be overlooked. Since footings are usually constructed in damp, inaccessible places where moisture and mud are often present, there is danger of faulty construction. This circumstance is especially serious in reinforced-concrete footings because their strength is entirely dependent on the care with which the concrete is mixed and deposited and on the proper placing of the reinforcement. Furthermore, since the steel reinforcing rods are relied on to carry all the tensile stresses, and their total area is comparatively small, their possible deterioration by rust or electrolysis, where insufficient concrete protection is provided, might lead to serious consequences. However, these disadvantages only emphasize the great care that should be exercised in properly constructing reinforced-concrete footings; they do not constitute sufficient grounds for rejecting the obvious economy derived from the employment of reinforced concrete in the construction of footings.

3. **Use of Reinforced-Concrete Spread Column Footings.**—Reinforced-concrete spread footings may be used to support one or more columns. When used to support one column the footing is known as an *independent column footing*, and is usually made square, unless one of its dimensions is limited by the building line or adjoining structures, when it is made rectangular. When used to support two or more columns the footing is a *combined column footing*, and is made either rectangular or trapezoidal, depending on the conditions of loading and construction. Sometimes, when the bearing capacity of the soil is very small, the foundation is spread over the entire area of the building, forming what is known as a *raft foundation.*

INDEPENDENT COLUMN FOOTINGS

GENERAL DETAILS OF CONSTRUCTION

4. Types of Footings.—When a reinforced-concrete spread footing is employed to support one column, it is usually made in one of three forms: (1) *slab footing*, (2) *sloped footing*, or (3) *stepped footing*.

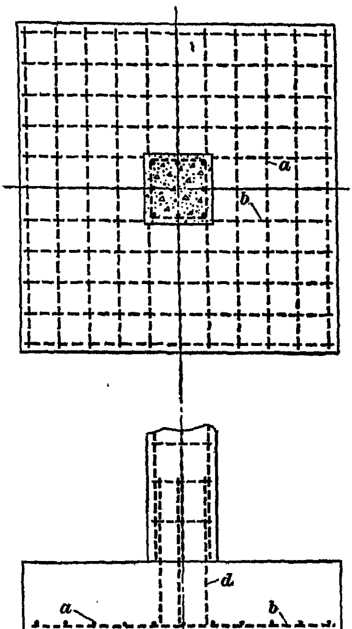


Fig. 1

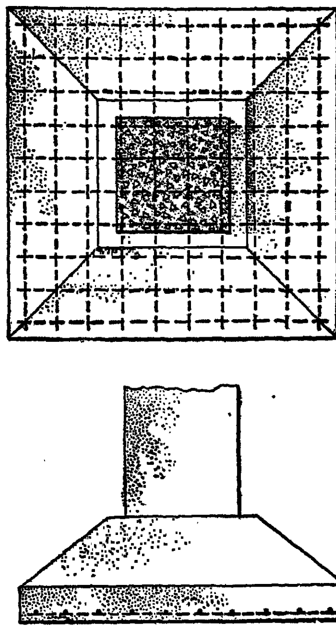


Fig. 2

In the slab footing, shown in Fig. 1, the column base rests directly on a reinforced-concrete slab. This type of footing is restricted to comparatively light column loads.

In the sloped footing the top of the footing slab slopes as shown in Fig. 2. This type of footing may be advantageously used to support both light and heavy loads.

The stepped footing, shown in Fig. 3 (*a*) and (*b*), consists of a reinforced-concrete slab and one or more steps poured monolithically. This type of footing is usually employed to support heavy loads on soils of low bearing capacity. The type of stepped footing that is most common in building construction consists of a slab and one step, as in Fig. 3 (*b*).

Another type of footing that is sometimes used is the *ribbed-slab footing*, which consists of a slab strengthened in various directions by reinforced-concrete ribs or beams. A ribbed-slab footing, used in the construction of the Atlanta Terminal Station of the Southern Railway, is illustrated in Fig. 4. The 3-inch slab *a* is strengthened by ribs *b* 6 inches wide on its four edges and by the tapered ribs *c* 8 inches wide, and *d*

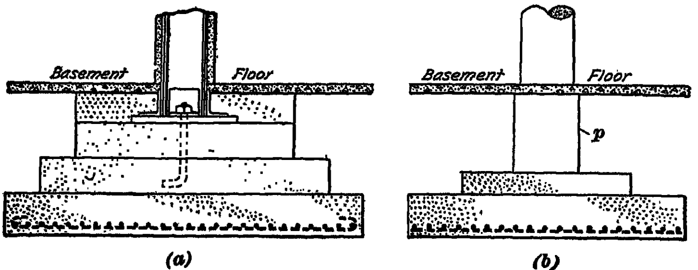


FIG. 3

10 inches wide, radiating from the pedestal in the center. This type of construction is conducive to a large saving in concrete, but it involves complicated form work and difficult construction, so that in the long run it is less economical than the other types. Its use is therefore not extensive.

5. Pedestals.—In the construction of reinforced-concrete footings, the columns are often supported on pedestals, *p* in Fig. 3 (*b*), which in turn rest on the footings. These pedestals constitute part of the footings, and may be reinforced or plain. The allowable compressive stress on the gross area of a concentrically loaded pedestal should not exceed 25 per cent. of the ultimate compressive stress of the concrete, which for 1:2:4 concrete is $.25 \times 2{,}000 = 500$ pounds per square inch. If the column load divided by the gross area of the pedestal

exceeds the allowable compressive stress, the pedestal should be reinforced as a section of a column.

6. Dowels.—The compressive stress carried by the longitudinal rods in the column can be effectively transferred to the footing by means of dowels, *d* in Fig. 1. At least one dowel for each longitudinal rod in the column should be used, and the total sectional area of the dowels should equal the total sectional area of the longitudinal column reinforcement. These dowels should project into the columns or into the footings a distance not less than fifty times the diameter of the column bars.

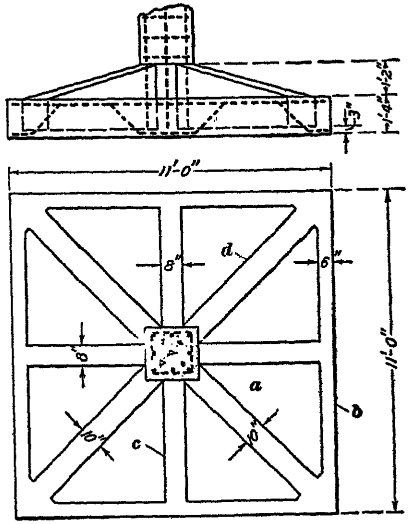


FIG. 4

Metal distributing bases, consisting of steel plates, cast-iron chairs, or steel beams, are sometimes employed to transfer the stress from the longitudinal column reinforcement to the footing, especially where the column loads are heavy.

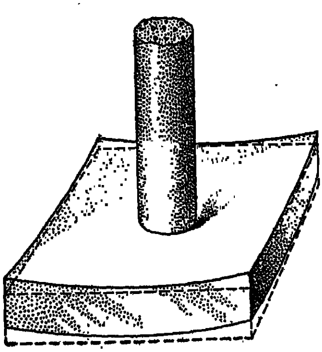


FIG. 5

7. Reinforcement.—Fig. 5 represents the general way in which a reinforced-concrete footing may be expected to deflect due to the upward soil pressure. The slab tends to bend upwards, assuming the shape of a bowl, which causes tensile stresses near its bottom and compressive stresses near its top.

The tensile stresses should be resisted by a sufficient number of reinforcing rods placed near the bottom of the slab. For the protection of the reinforcement 3 inches of concrete should be allowed below the rods.

8. Arrangement of Reinforcing Rods.—The usual arrangement of reinforcing rods in square footings is either in the *two-way system* or the *four-way system*.

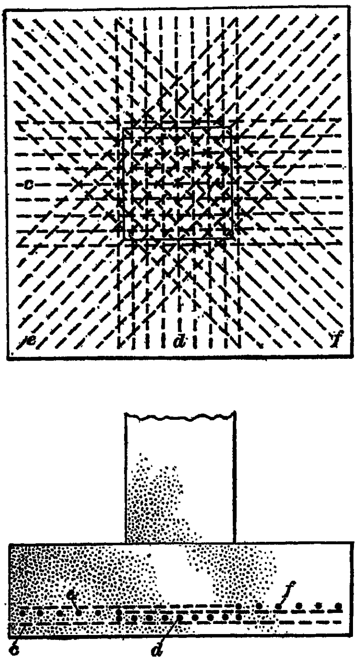


FIG. 6

In the **two-way system** the rods are placed in two layers, *a* and *b* in Fig. 1, at right angles to each other and parallel to the sides of the footing. Both layers consist of the same size rods usually placed at equal distances apart throughout the slab.

In the **four-way system,** shown in Fig. 6, the rods are placed in four belts or layers; two, *c* and *d*, are parallel to the sides of the slab, and two, *e* and *f*, are diagonal. In the four-way system nearly all the main rods pass under the pedestal or the column base.

Both systems of reinforcement have given good results in tests, but the two-way system is preferred by most engineers, because it involves rods of one or two lengths that are placed in two layers and distributed evenly throughout the slab; in the four-way system, a large proportion of steel is placed directly under the column where it is not fully utilized, and the rods used are of several lengths and placed in four layers.

METHODS OF DESIGN

9. Joint Committee Recommendations.—In slabs subject to deflection in two directions, as shown in Fig. 5, it is difficult to determine the exact stresses due to flexure and shear. Furthermore, it is not advisable to enter into excessive theoretical refinement where a non-homogeneous material like reinforced concrete is involved. The methods given in the following articles are mostly based on the recommendations of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete. These methods are founded on the assumption that the column base is securely tied to the footing so as to preclude the possibility of their separating before the footing develops its full strength. Ample provision should therefore be made for securely tying the column to the footing by means of anchor bolts, dowels imbedded in the column and footing, or other efficient means.

10. Weight of Footing.—In designing reinforced-concrete footings it is customary to proportion the footing area for the column load and an assumed weight of footing. The weights of independent column footings range from 8 to 20 per cent. of the column loads, depending on the type of footing used and the bearing capacity of the soil. The assumptions of weight made are usually according to past experience with similar footings and soil conditions. After the footing is designed its weight may be computed, and if it is found to differ much from the assumed weight the design is revised. However, the weight of the footing need not be considered in determining the bending moments and stresses in the footing because it is balanced by a corresponding upward soil reaction throughout the footing base. The upward reaction per unit area on the footing, which is considered effective in causing stresses, is therefore equal to the column load divided by the area of the base of the footing, and it is known as the *unit net soil pressure* on the footing.

11. Bending Moments in Footings With Two-Way Reinforcement.—In determining the bending moments in

square or rectangular column footings reinforced with two-way reinforcement and supporting concentric square or rectangular columns, it is customary to consider the footing composed of four cantilevers, each bounded by a face of the column, the corresponding edge of the footing slab, and portions of the two diagonals, as cantilever $ACBD$ in Fig. 7. The load on each cantilever is the net soil pressure on the footing, and the maximum bending moment is about the face of the column that forms one of its boundaries. Thus, in cantilever $ABCD$ the critical section for bending is the face AB

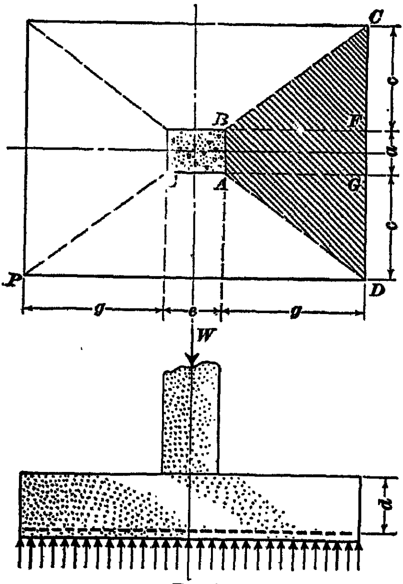


FIG. 7

For convenience in finding the moments, the trapezoid $ABCD$, Fig. 7, may be further subdivided into the triangles ADG and BCF and the rectangle $ABFG$. If w represents the unit net soil pressure on the footing, which is the column load W divided by the area of the footing, and c and g are respectively the projections of the footing beyond the faces of the column, then the area of each triangle is $\frac{1}{2}\,c\,g$ and the load on it is $\frac{w}{2}\,c\,g$, the distance of the center of gravity of the triangle from face AB is $\frac{2}{3}\,g$, and the moment of the load on the triangle about face AB is $\frac{w}{2}\,c\,g \times \frac{2}{3}\,g$. Similarly, the area of the rectangle is $a\,g$, the distance of its center of gravity from face AB is $\frac{g}{2}$, and the moment of the load on the rectangle about

face AB is $\frac{w}{2}ag^2$. The total bending moment M_1 of the trapezoid $ABCD$ about face AB is the sum of the moments of the loads on the two triangles and on the rectangle about face AB, or $M_1 = \frac{2}{3}wcg^2 + \frac{1}{2}wag^2$; whence,

$$M_1 = \frac{w}{2}\left(a + \frac{4}{3}c\right)g^2 \qquad (1)$$

In similar manner it can be shown that the maximum bending moment in cantilever $AEPD$ about face AE is

$$M_2 = \frac{w}{2}\left(e + \frac{4}{3}g\right)c^2 \qquad (2)$$

When the footing base and the column base are both square, as shown in Fig. 8, the bending moment in each cantilever is the same, and the formula for finding its value is

$$M = \frac{w}{2}\left(a + \frac{4}{3}c\right)c^2 \qquad (3)$$

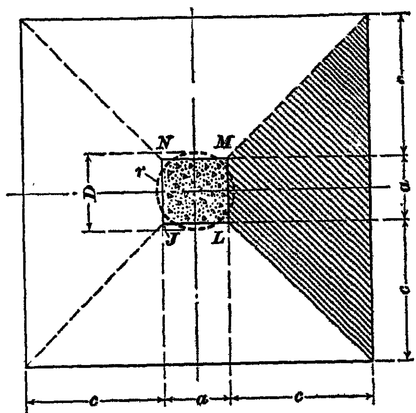


FIG. 8

The bending moments in square footings with round columns are found in the same manner as when they support square columns, the side a in formula **3** being the side of a square whose area is equivalent to the cross-sectional area of the column. Thus, in Fig. 8, the dotted circle r represents the perimeter of a round column the area of which is equal to the area of the square $JNML$. If the diameter of the circular column is D, the side of a square of equivalent area is

$$a = .886D \qquad (4)$$

EXAMPLE 1.—A 24″×30″ rectangular column pier, carrying a load of 300,000 pounds, is supported by a rectangular slab footing reinforced with two-way reinforcement. If one dimension of the footing is limited to

7 feet and the bearing capacity of the soil is $2\frac{1}{2}$ tons per square foot, what are the maximum bending moments in the slab?

SOLUTION.—If the footing is assumed to weigh 10 per cent. of the column load, the total load on the soil is 300,000+30,000=330,000 lb. The required bearing area is $\frac{330,000}{5,000}=66$ sq. ft., which may be supplied by a footing 7 ft. wide by 9.5 ft. long. The unit net soil pressure on the footing, $w=\frac{300,000}{7\times 9.5}=4,510$ lb. per sq. ft. The maximum bending moments in the slab are computed by means of formulas **1** and **2**, by substituting $a=2$ ft., $c=\frac{7-2}{2}=2.5$ ft., $e=2.5$ ft., $g=\frac{9.5-2.5}{2}=3.5$ ft., and $w=4,510$ lb. per sq. ft. Therefore,

$$M_1=\frac{w}{2}\left(a+\frac{4}{3}c\right)g^2=\frac{4,510}{2}\times\left(2+\frac{4}{3}\times 2.5\right)\times 3.5^2=147,000 \text{ ft.-lb. Ans.}$$

$$M_2=\frac{w}{2}\left(e+\frac{4}{3}g\right)c^2=\frac{4,510}{2}\times\left(2.5+\frac{4}{3}\times 3.5\right)\times 2.5^2=101,000 \text{ ft.-lb. Ans.}$$

EXAMPLE 2.—A 27-inch round column, carrying a load of 387,000 pounds, is supported by a square slab footing on a soil of 3 tons per square foot bearing capacity. What is the maximum bending moment in the slab?

SOLUTION.—If the footing is assumed to weigh about 10 per cent. of the column load, the total pressure on the soil is 387,000+39,000=426,000 lb., and the required bearing area is $\frac{426,000}{6,000}=71$ sq. ft., which is supplied by a footing 8.5 ft. square. In order to compute the maximum bending moment in the slab by means of formula **3**, it is first necessary to compute the values of a, c, and w, as follows:

$$a=.886\,D=.886\times 27=24 \text{ in.}=2 \text{ ft.,}$$

$$c=\frac{8.5-2}{2}=3.25 \text{ ft.,}$$

and $$w=\frac{387,000}{8.5\times 8.5}=5,360 \text{ lb. per sq. ft.}$$

Hence, $$M=\frac{w}{2}\left(a+\frac{4}{3}c\right)c^2=\frac{5,360}{2}\times\left(2+\frac{4}{3}\times 3.25\right)\times 3.25^2=179,200 \text{ ft.-lb.}$$

Ans.

12. Maximum Shear.—The maximum shear in each cantilever of the footing slab is at the critical section for bending, and it is equal to the total net soil pressure on the cantilever. In Fig. 7, the area of cantilever $A\,B\,C\,D$ is $\frac{g}{2}(a+2c+a)$

$=(a+c)g$, and if w represents the unit net soil pressure on the footing, the maximum shear in cantilever $ABCD$ is

$$V_1=w(a+c)g \qquad (1)$$

Similarly, if V_2 represents the maximum shear in cantilever $AEPD$,

$$V_2=w(e+g)c \qquad (2)$$

In square footings which support square or round columns, as in Fig. 8, the maximum shear in each cantilever is the same, and if V represents that maximum shear,

$$V=w(a+c)c \qquad (3)$$

EXAMPLE.—Determine the maximum shear in the footing of Example 2 in the preceding article.

SOLUTION.—As previously determined, $a=2$ ft., $c=3.25$ ft., and $w=5{,}360$ lb. per sq. ft. Hence, by formula 3,

$$V=w(a+c)c=5{,}360\times(2+3.25)\times 3.25=91{,}500 \text{ lb. Ans.}$$

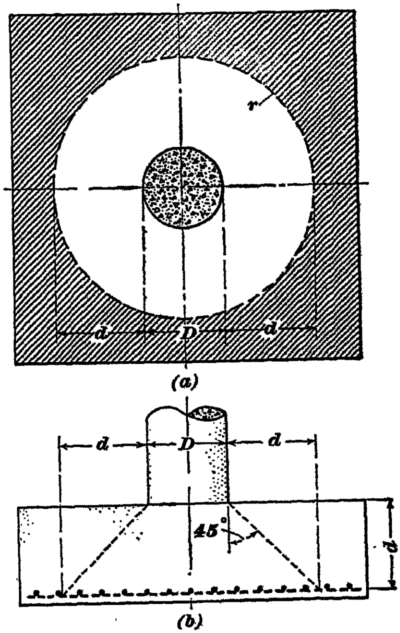


FIG. 9

13. Effective Depth. The effective depth of an independent column footing is the distance from the top of the footing to the center of the reinforcement. Thus, in Fig. 9, where two layers of rods are used, the effective depth d is the distance from the top of the slab to the under side of the top layer of rods. If four layers of rods were used, as in Fig. 6, the effective depth would be the distance from the top of the slab to the under side of the second layer of rods.

In independent column footings the effective depth is usually determined by the *punching shear* or *diagonal tension*

in the footing, which will be explained in the following articles.

14. Punching Shear.—The punching shear is the load tending to punch the column base or pedestal through the footing slab; it is equal to the upward net soil pressure between the edges of the footing and the edges of the column base. Thus, if the unit net soil pressure is w, the area of the footing slab is A, and the area of the column base is A', the punching shear is

$$V_p = w(A - A') \qquad (1)$$

The unit punching shear v_p is obtained by dividing the punching shear by the product of the perimeter s of the column base and the effective depth of the slab d, or

$$v_p = \frac{V_p}{sd} \qquad (2)$$

According to Joint Committee recommendations the unit punching shear should not exceed 6 per cent. of the ultimate unit compressive strength of the concrete. For 1:2:4 concrete the ultimate unit compressive strength is assumed to be 2,000 pounds per square inch and the allowable unit punching shear is 120 pounds per square inch.

When the punching shear V_p is known, the effective depth d of the footing slab which is required to resist the specified unit punching shear v_p may be found by the formula

$$d = \frac{V_p}{s\,v_p} \qquad (3)$$

EXAMPLE.—Find the effective depth required to resist a unit punching shear of 120 pounds per square inch in the 8.5 foot square footing of Example 2, in Art. **11.** The footing supports a 27-inch round column and the unit net soil pressure is 5,360 pounds per square inch.

SOLUTION.—Here $w = 5{,}360$ lb. per sq. in. The area of the footing, $A = 8.5^2 = 72.25$ sq. ft. The area of the column base, $A' = .7854 \times 2.25^2 = 3.98$ sq. ft. If the values for w, A, and A' are substituted in formula **1**, the punching shear is

$$V_p = w(A - A') = 5{,}360\ (72.25 - 3.98) = 366{,}000\ \text{lb.}$$

The perimeter of the column base, $s = 3.1416 \times 27 = 84.8$ in., and $v_p = 120$ lb. per sq. in.; hence, if these values are substituted in formula **3**, the required effective depth is

$$d = \frac{V_p}{s\,v_p} = \frac{366{,}000}{84.8 \times 120} = 36 \text{ in.} \quad \text{Ans.}$$

15. Diagonal Tension.—In a reinforced-concrete footing supporting a column load, the stresses produced in the concrete are tensile and compressive fiber stresses and horizontal and vertical shearing stresses. The combined effect of the fiber and shearing stresses is known as *diagonal tension*. The exact value of the diagonal tension at any point in the footing slab is generally indeterminate, and proper provision for it is based on experimental data. Tests indicate that as a means of providing resistance to diagonal tension, the footing should be designed to resist the vertical shear at a distance from the face of the column equal to the effective depth of the footing. In Fig. 9 (*a*), this vertical shear is along the perimeter of the circle *r*, the diameter of which is $D + 2d$, while in Fig. 10 (*a*) it is along the perimeter of the rectangle *ABCD*. The magnitude of this shear is equal to the upward soil pressure on the shaded area of the footing between the edges of the footing and the perimeter of the circle *r* or of the rectangle *ABCD*. If V' represents that vertical shear, b' the perimeter of the circle *r* or of the rectangle *ABCD*, and jd the usual notation for the arm of the stress couple in the reinforced slab then the formula for the unit shear v' is

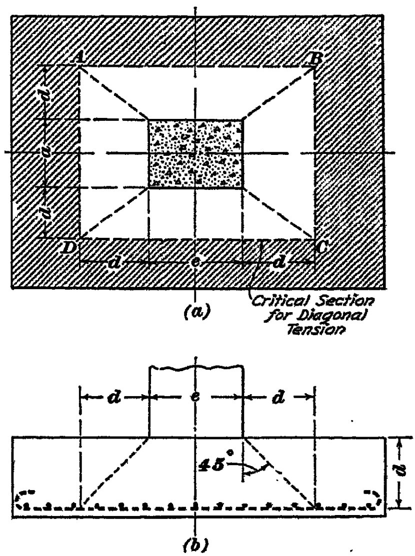


FIG. 10

$$v' = \frac{V'}{b' j d} \qquad (1)$$

If j is given the approximate value of $\frac{7}{8}$, which is sufficiently lose for all practical purposes, the formula for unit shear ecomes

$$v' = \frac{V'}{.875\, b'\, d} \qquad (2)$$

The value of v' should not exceed 2 per cent. of the ultimate nit compressive strength of the concrete when straight einforcement bars only are used, as in Fig. 9 (*b*), nor should it xceed 3 per cent. when bars anchored at both ends by adeuate hooks, as in Fig. 10 (*b*), are used. These hooks should e made by bending the ends of the bars in a full semicircle o a diameter not less than eight times the diameter of the ar, the total length of the bend being not less than sixteen iameters of the bar. When the allowable unit shear is xceeded, either shear reinforcement should be employed or he footing slab should be deepened so as to bring the unit hear within the specified limits. However, since the placing f bent bars and stirrups in foundation slabs is rather a roublesome process, deepening of the foundation slab in ndependent column footings will prove in nearly all cases he more economical procedure.

EXAMPLE.—The footing shown in Fig. 10 is 7 feet wide by 9.5 feet long nd it supports a 24″×30″ rectangular column. If the effective depth f the slab as determined by the punching shear is 23 inches, and the unit et soil pressure on the footing is 4,510 pounds, what is the unit shear easuring diagonal tension in the slab?

SOLUTION.—The critical section at which the diagonal tension is easured is along the perimeter of the rectangle $A\,B\,C\,D$, Fig. 10 (*a*). Since $=24$ in., $e=30$ in., and $d=23$ in., the length of $A\,D=24+(2\times23)$ $=70$ in., and the length of $A\,B=30+(2\times23)=76$ in. The length of the erimeter of $A\,B\,C\,D$ is

$$b'=(2\times70)+(2\times76)=292 \text{ in.}$$

nd its area equals $70\times76=5{,}320$ sq. in. $=37$ sq. ft. The area of the footing ab equals $7\times9.5=66.5$ sq. ft. Hence, the soil pressure on the shaded rea, or the shear along the perimeter of $A\,B\,C\,D$ is

$$V'=4{,}510\times(66.5-37)=133{,}000 \text{ lb.}$$

The unit shear along the perimeter of $A\ B\ C\ D$, as found by formula **2**, is

$$v' = \frac{V'}{.875\,b'\,d} = \frac{133{,}000}{.875 \times 292 \times 23} = 22.6 \text{ lb. per sq. in.} \quad \text{Ans.}$$

16. Effective Width.—After the effective depth of the footing has been determined, the question arises as to what width of the footing may be considered effective in resisting the maximum bending moment in each trapezoidal cantilever. This question is best decided by the result of tests performed at the Engineering Experiment Station of the University of Illinois. As indicated by these tests, for two-way reinforcement uniformly spaced over the footing, when the width of the footing does not exceed the width of the column base or pedestal plus twice the effective depth of the footing, the entire width of the footing may be considered effective in resisting the maximum bending moment in each cantilever, and the reinforcing rods should be spaced uniformly over the entire width of the footing. In the footing shown in Fig. 11, the width f of the footing slab is less than the width a of the column base plus twice the effective depth d of the slab; hence, the entire width of the footing slab may be considered effective, and the reinforcing rods necessary to resist the

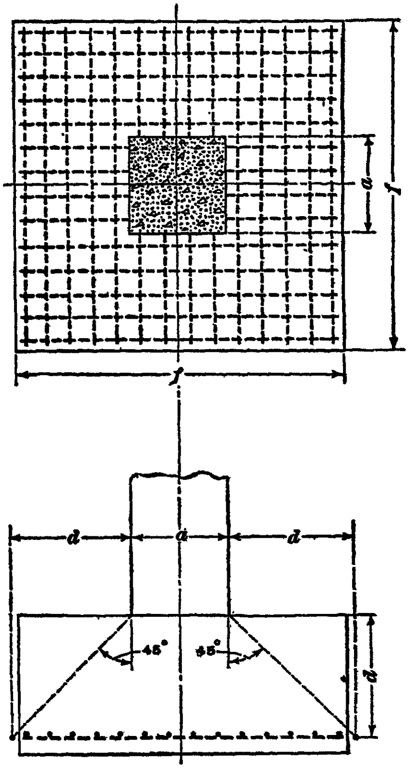


FIG. 11

bending moment in each cantilever may be spaced uniformly over the entire width. If, however, the width of the footing exceeds the width of the column base or pedestal plus twice the effective depth of the footing, as in Fig. 12, then the width of the footing to be considered effective in resisting the maximum bending moment is the width of the column base or pedestal plus twice the effective depth of the footing plus one-half the remaining width. Thus, if, as in Fig. 12, the width of the column base is a, the projection of the footing is c, and the effective depth of the slab is d, then the effective width, $b=a+2d+2\frac{(c-d)}{2}$, or

$$b=a+c+d$$

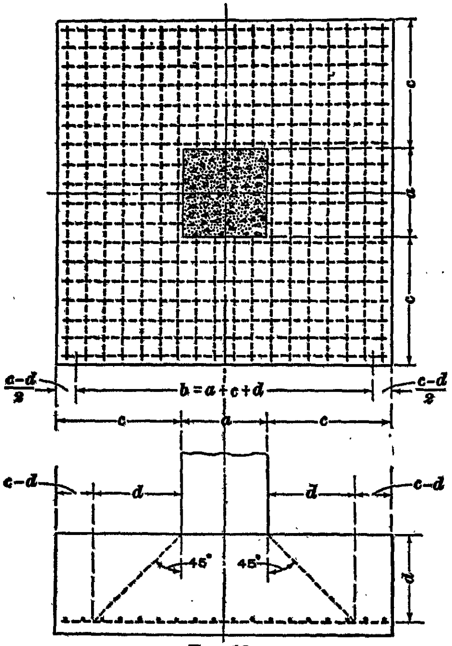


FIG. 12

The reinforcing rods which are found necessary to resist the maximum bending moment in the footing should be spaced uniformly across the effective width b. In order to avoid considerable unreinforced parts in the footing slab, additional rods should be placed outside the effective width, but they should not be considered effective in resisting the maximum bending moment. In Fig. 12, only the sixteen rods spaced within the effective width would be considered effective. The extra rods outside the effective width may be spaced twice as far apart as the rods within the effective width. The method of determining the effective width of a square footing is illustrated by the following example:

EXAMPLE.—The 8.5-foot square slab footing in example 2, Art. **11**, supports a 27-inch round column, and the effective depth of the footing is 36 inches. Determine the effective width of the footing.

SOLUTION.—As found in the solution of example 2, Art. **11**, $a = 24$ in., and $c = 3.25$ ft. $= 39$ in. Substituting these values and the value $d = 36$ in. in the formula for effective width,

$$b = a + c + d = 24 + 39 + 36 = 99 \text{ in. Ans.}$$

17. Compressive Stress in Concrete.—The effective depth and effective width of the footing slab being known, it is necessary to investigate whether the unit compressive stress in the extreme fiber of the concrete, f_c, exceeds the allowable unit compressive stress, which for 1 : 2 : 4 concrete may be assumed 650 pounds per square inch. For all practical purposes, the value of f_c may be computed by the approximate formula

$$f_c = \frac{6\,M}{b\,d^2}$$

in which M = bending moment in each trapezoidal cantilever of the slab;

b = effective width of the footing slab;

d = effective depth of the footing slab.

EXAMPLE.—What is the unit stress in the extreme fiber of the concrete in the footing under consideration in the example of the preceding article, the effective width being 99 inches, the effective depth 36 inches, and the maximum bending moment in each trapezoidal cantilever 179,200 foot-pounds?

SOLUTION.—Here $M = 179{,}200$ ft.-lb. $= 2{,}150{,}000$ in.-lb., $b = 99$ in., and $d = 36$ in. Hence, substituting in the formula,

$$f_c = \frac{6\,M}{b\,d^2} = \frac{6 \times 2{,}150{,}000}{99 \times 36^2} = 100 \text{ lb. per sq. in. Ans.}$$

18. Area of Reinforcement.—The next step in the design of a reinforced-concrete slab footing is to provide sufficient steel in the effective width of the slab, in each direction, to resist safely the tensile stresses due to the maximum bending moment in each cantilever. If A denotes the required steel area, $j\,d$ the lever arm of the stress couple, as before,

and f_s the allowable unit tensile stress in the steel, then the resisting moment of the steel is

$$M_s = f_s A_s j d \qquad (1)$$

If j is assumed $\frac{7}{8}$ as before,

$$M_s = \tfrac{7}{8} f_s A_s d \qquad (2)$$

The steel area A, which is required to render the resisting moment M_s equal to the bending moment M in the cantilever, may be found by the formula,

$$A_s = \frac{M}{.875 f_s d} \qquad (3)$$

The required number of rods is determined by dividing the required steel area by the area of one rod. The area of the various sizes of square and round rods which are generally used in reinforced-concrete construction are given in Table I.

TABLE I

AREAS AND PERIMETERS OF SQUARE AND ROUND RODS

Diameter of Rod Inches	Square		Round	
	Area Sq. In.	Perimeter Inches	Area Sq. In.	Perimeter Inches
$\frac{1}{4}$	.0625	1.00	.0491	.7854
$\frac{5}{16}$	.0977	1.25	.0767	.9817
$\frac{3}{8}$	.1406	1.50	.1104	1.1781
$\frac{7}{16}$	.1914	1.75	.1503	1.3744
$\frac{1}{2}$	.2500	2.00	.1963	1.5708
$\frac{5}{8}$	.3906	2.50	.3068	1.9635
$\frac{3}{4}$	.5625	3.00	.4418	2.3562
$\frac{7}{8}$	.7656	3.50	.6013	2.7489
1	1.0000	4.00	.7854	3.1416
$1\frac{1}{8}$	1.2656	4.50	.9940	3.5343
$1\frac{1}{4}$	1.5625	5.00	1.2272	3.9270

EXAMPLE.—If the maximum bending moment in each trapezoidal cantilever of the footing considered in the example of Art. 16 is 2,150,000

inch-pounds, what is (*a*) the number and (*b*) the spacing of $\frac{5}{8}$-inch round rods required to resist that bending moment when $f_s = 16{,}000$ pounds per square inch?

SOLUTION.—(*a*) In this example $M = 2{,}150{,}000$ in.-lb. and $f_s = 16{,}000$ lb. per sq. in. Substituting the known values in formula **3** to determine the area of reinforcement,

$$A_s = \frac{M}{.875 f_s d} = \frac{2{,}150{,}000}{.875 \times 16{,}000 \times 36} = 4.26 \text{ sq. in.}$$

According to Table I, the area of a $\frac{5}{8}$-in. round rod is .3068 sq. in. Hence, the number of $\frac{5}{8}$-in. round rods required is $\frac{4.26}{.3068} = 14$. Ans.

(*b*) The effective width of the slab as previously determined is 99 in., and therefore the required spacing of rods is $\frac{99}{14}$ = about 7 in. center to center. Ans.

19. Bond Stress.—After the number of reinforcing rods in the footing slab has been determined, it is necessary to compute the *unit bond stress* between the concrete and steel. If V represents the maximum shear in the footing slab, as determined by the formulas in Art. **12**, O the sum of the perimeters of their reinforcing rods within the effective width, $j\,d$ the lever arm of the stress couple, then the unit bond stress u may be found by the formula

$$u = \frac{V}{j d O} \qquad (1)$$

If j is assumed $\frac{7}{8}$, the formula becomes

$$u = \frac{V}{.875\, d O} \qquad (2)$$

Frequently, the maximum shear V and the unit bond stress u are given and it is required to find the number of rods n for that unit bond stress. If o is the perimeter of each rod, n can be determined by the formula

$$n = \frac{V}{.875\, d\, o\, u} \qquad (3)$$

According to Joint Committee recommendations, the unit bond stress u must not exceed 4 per cent. of the ultimate unit

compressive strength of the concrete for plain reinforcing bars and 5 per cent. for deformed bars. For 1:2:4 concrete the ultimate compressive strength is taken as 2,000 pounds per square inch, and a unit bond stress of 80 pounds per square inch is therefore allowable for plain bars and 100 pounds per square inch for deformed bars.

The allowable unit bond stress of 80 pounds per square inch for plain bars and 100 pounds per square inch for deformed bars applies only when the main reinforcing bars are laid in one direction. The allowable unit bond stress should be reduced 25 per cent. when the reinforcement is in two directions, and 10 per cent. for each additional direction. Accordingly, for footings reinforced with two-way reinforcement, the allowable unit bond stresses are 60 pounds per square inch for plain bars and 75 pounds per square inch for deformed bars. However, for bars adequately anchored at both ends with hooks, or otherwise, the bond stresses may be taken $1\frac{1}{2}$ times the specified values. For two-way reinforcement adequately anchored at the ends, the allowable bond stresses are therefore 90 pounds per square inch for plain bars and 112.5 pounds per square inch for deformed bars.

The use of smaller-size bars to reinforce footing slabs would ordinarily be preferred because they offer larger sums of perimeters for the same area of steel, but larger-size bars are less seriously injured by rust and electrolysis, which in foundation work are factors not to be overlooked. Deformed bars are more effective in resisting bond stress than plain bars, and they are therefore very advantageously used in spread footings where bond often determines the required amount of reinforcement. Only the bars within the effective width which are relied on to resist the maximum bending moment should be considered effective in resisting bond stress. Thus, in Fig. 12, only the sixteen rods within the effective width b should be considered. The perimeters of the various round and square rods commonly used in reinforced-concrete construction may be found in Table I.

EXAMPLE.—If the maximum shear in each trapezoidal cantilever of the footing considered in the example of the preceding article is 91,500 pounds,

what is the unit bond stress in the fourteen ⅝-inch round rods placed each way in the effective width?

SOLUTION.—In the footing under consideration $V=91{,}500$ lb., and $d=36$ in. According to Table I the perimeter of a ⅝-in. round rod is 1.9635 in. Hence, $O=14\times1.9635=27.5$ in. Substituting the known values in formula 2, the unit bond stress is

$$u=\frac{V}{.875\,d\,O}=\frac{91{,}500}{.875\times36\times27.5}=105.6 \text{ lb. per sq. in. Ans.}$$

It is evident that deformed bars adequately anchored at the ends should be used in this case.

20. Summary of Design Procedure.—In designing a reinforced-concrete slab footing it is first necessary to proportion the bearing area of the footing to support safely the column load and the assumed weight of the footing. The unit net soil pressure is then found by dividing the column load by the bearing area of the footing. The next step is to determine by the formulas in Art. 14 what effective depth of slab is necessary to resist safely the allowable punching shear. It is then necessary to investigate whether the slab offers sufficient resistance to diagonal tension, by applying the formulas in Art. 15. The weight of the slab may now be computed and compared with the assumed weight; if the computed weight differs by more than about 2 per cent. of the column load, the proportions of the footing should be revised.

The maximum bending moment in each cantilever of the footing slab is determined by the formulas in Art. 11, and the maximum shear by the formulas in Art. 12. The effective width of the footing is then found by the formula in Art. 16, and the unit compressive stress in the concrete by the formula in Art. 17. The steel area is next computed by formula 3 in Art. 18, the number and size of rods are determined, and the spacing is found by dividing the effective width by the number of rods. The unit bond stress between the concrete and steel is finally computed by formula 2 in Art. 19, and if found excessive the number of rods is increased so as to bring the unit bond stress within the allowable limits.

The following example illustrates the procedure in the design of a square slab footing:

EXAMPLE.—Design a square slab footing for a 32-inch diameter column which carries a load of 500,000 pounds on a soil the bearing capacity of which is 2 tons per square foot. Assume $f_s = 16{,}000$ pounds per square inch and the concrete mix 1:2:4.

SOLUTION.—*Bearing Area.*—If 15 per cent. of 500,000 lb., or 75,000 lb., is assumed for the weight of the footing, the total load on the soil is 575,000 lb., and the required bearing area is $\frac{575{,}000}{4{,}000} = 143.8$ sq. ft. A footing 12 ft. square offers an area of $12 \times 12 = 144$ sq. ft., and it will be adopted. The unit net soil pressure on the footing is therefore $w = \frac{500{,}000}{144} = 3{,}470$ lb. per sq. ft.

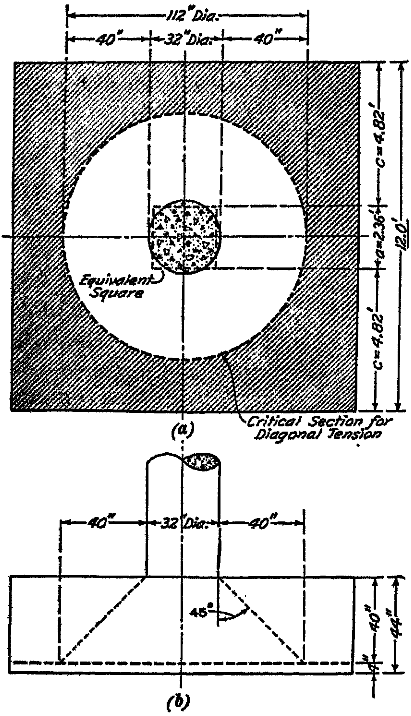


FIG. 13

Punching Shear.—The perimeter of the 32-in. diameter column is 100 in., and its area is 804 sq. in., or 5.58 sq. ft. According to formula **1** in Art. **14**, the punching shear is

$$V_p = w(A - A') = 3{,}470 \times (144 - 5.58) = 480{,}000 \text{ lb.},$$

and the effective depth required to resist a unit punching shear of 120 lb. per sq. in. is

$$d = \frac{V_p}{s\,v_p} = \frac{480{,}000}{100 \times 120} = 40 \text{ in.}$$

If 4 in. are added for the protection of the reinforcement, the total depth is 44 in., as shown in Fig. 13 (*b*).

Diagonal Tension.—The critical section for measuring diagonal tension in the footing, as specified in Art. **15**, is at a distance of the effective depth or 40 in. from the perimeter of the column, and is, therefore, along the perimeter of a circle with a diameter of $32 + (2 \times 40) = 112$ in., as in Fig. 13 (*a*). By *Smoley's Tables*, the area of the circle is 9,852 sq. in. = 68.4 sq. ft., and the circumference is 352 in. The soil pressure on the footing outside that circle, or the shear along its perimeter,

$$V' = 3{,}470 \times (144 - 68.4) = 262{,}000 \text{ lb.},$$

and the unit shear,

$$v' = \frac{V'}{.875\, b'\, d} = \frac{262{,}000}{.875 \times 352 \times 40} = 21.3 \text{ lb. per sq. in.,}$$

which is well below the permissible unit shear of 40 lb. per sq. in.

Weight of Footing.—Before proceeding further with the computations the weight of the footing should be computed and compared with the assumed weight. This weight is $144 \times \frac{44}{12} \times 150 = 79{,}000$ lb., which agrees closely with the assumed weight of 75,000 lb.

Bending Moment and Shear.—The side a of a square equivalent in area to the area of the column, by formula **4** in Art. **11**, is $a = .886 \times 32 = 28.35$ in. $= 2.36$ ft.; hence, $c = \frac{12 - 2.36}{2} = 4.82$ ft. As previously computed, the unit net soil pressure, $w = 3{,}470$ lb. per sq. ft. Substituting in formula **3**, Art. **11**,

$$M = \frac{w}{2}\left(a + \frac{4}{3}c\right)c^2 = \frac{3{,}470}{2} \times \left(2.36 + \frac{4}{3} \times 4.82\right) \times 4.82^2$$

$$= 354{,}000 \text{ ft.-lb.} = 4{,}250{,}000 \text{ in.-lb.}$$

By formula **3** in Art. **12**, the maximum shear in each cantilever,

$$V = w(a + c)c = 3{,}470 \times (2.36 + 4.82) \times 4.82 = 120{,}000 \text{ lb.}$$

Effective Width.—The width of slab to be considered effective in resisting the bending moment at the face of the column, as found by the formula in Art. **16**, is

$$b = a + c + d = 28.35 + 4.82 \times 12 + 40 = 126 \text{ in.}$$

Compressive Stress in Concrete.—The maximum bending moment in each cantilever is $M = 4{,}250{,}000$ in.-lb., the effective width $b = 126$ in., and the effective depth $d = 40$ in.; hence, by the formula in Art. **17**,

$$f_c = \frac{6M}{b\, d^2} = \frac{6 \times 4{,}250{,}000}{126 \times 40^2} = 126 \text{ lb. per sq. in.,}$$

which is well below the allowable 650 lb. per sq. in.

Reinforcement.—The required steel area, by formula **3** in Art. **18**, is

$$A_s = \frac{M}{.875\, f_s\, d} = \frac{4{,}250{,}000}{.875 \times 16{,}000 \times 40} = 7.59 \text{ sq. in.,}$$

which is supplied by the following combinations of bars:

No. of Bars	Section of Bar	Area of Bar Sq. In.	Perimeter of Bar Inches	Total Area Sq. In.	Total Perimeter Inches	Spacing of Bars Inches
14	¾″ Square	.5625	3.00	7.88	42.0	9.00
18	¾″ Round	.4418	2.36	7.95	42.5	7.00
20	⅝″ Square	.3906	2.50	7.81	50.0	6.30
25	⅝″ Round	.3068	1.96	7.67	49.0	5.04

If the twenty ⅝-in. square bars are chosen, their spacing is $\frac{126}{20}=6.30$ in., or about 6½ in. center to center. This spacing will be continued to the edges of the footing, as shown in Fig. 14, in plan in (*a*) and in elevation in (*b*)

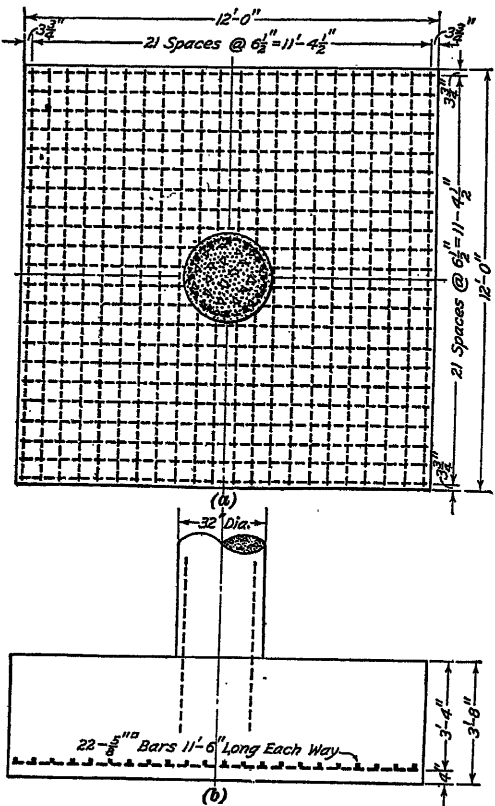


FIG. 14

Bond Stress.—The unit bond stress is

$$u=\frac{V}{.875\,d\,O}=\frac{120{,}000}{.875\times40\times50.0}=68.6 \text{ lb. per sq. in.,}$$

which is below 75 lb. per sq. in. for two-way reinforcement consisting of straight deformed bars, as specified in Art. **19**.

21. Footings Reinforced With Four-Way Reinforcement.—Four-way reinforcement is seldom used to reinforce

other than square footings. In computing the bending moments for square footings reinforced with four-way reinforcement, it is customary to consider the slab composed of eight cantilevers, four parallel to the sides of the column base, as cantilever *A B C D* in Fig. 15, and four along the diagonals of the footing, as cantilever *F M O N G*. The effective width of each cantilever which is parallel to the sides of the column base is assumed equal to the width of a side of the column base, as *A B* for cantilever *A B C D*; the effective width of each diagonal cantilever is assumed equal to the length of a diagonal of the column base, as *F G* = *A P* for cantilever *F M O N G*.

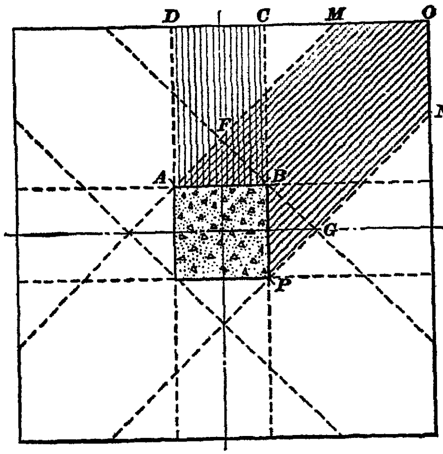


Fig. 15

To determine the maximum bending moment in each of the eight cantilevers of the footing, it is customary first to resolve the footing into four trapezoidal cantilevers, as was previously done with footings reinforced with two-way reinforcement, determine the maximum bending moment in each of the four trapezoidal cantilevers, and then assume one-half of that bending moment as the maximum bending moment in each of the eight cantilevers of the footing. The maximum shear in each of the eight cantilevers is found by a method similar to that used in finding the maximum bending moment; the maximum shear in each of the four trapezoidal cantilevers is first found, and one-half of that shear is assumed as the maximum shear in each of the eight cantilevers.

Provision for diagonal tension and punching shear in footings reinforced with four-way reinforcement is made in the same manner as for footings reinforced with two-way rein-

forcement. After the depth of the footing has been established, the number and size of rods required to resist the bending moment in each cantilever may be computed, and the composition of each belt of reinforcement thus determined. The belts parallel to the sides of the column base are often assumed wider than the column base, as shown in Fig. 6, in order to leave no parts of the footing unreinforced.

EXAMPLE.—Design a square footing with four-way reinforcement for the support of a 36-inch square pier which carries a load of 600,000 pounds on a soil good for 3 tons per square foot. Assume $f_s = 16{,}000$ pounds per square inch and the concrete mix 1:2:4.

SOLUTION.—*Bearing Area.*—Assuming the weight of the footing to be 10 per cent. of the load on the pier, the total load on the soil is 660,000 lb. The required footing area is therefore $\frac{660{,}000}{6{,}000} = 110$ sq. ft. Each side of the square footing is $\sqrt{110} = 10.5$ ft. The unit net soil pressure is therefore $w = \frac{600{,}000}{110} = 5{,}450$ lb. per sq. ft.

Punching Shear.—The perimeter of the 36-inch pier is $4 \times 36 = 144$ in., and the cross-sectional area is $3 \times 3 = 9$ sq. ft. Therefore,

$$V_p = w(A - A') = 5{,}450 \times (110 - 9) = 550{,}000 \text{ lb.}$$

and for a unit punching shear of 120 lb. per sq. in., the required effective depth is

$$d = \frac{V_p}{s\ v_p} = \frac{550{,}000}{144 \times 120} = 32 \text{ in.}$$

If 4 in. are added for protection of the reinforcement, the total depth is 36 in.

Diagonal Tension.—The diagonal tension in the footing is to be measured by the shear along the perimeter of a square each side of which is $36 + (2 \times 32) = 100$ in. $= 8$ ft. 4 in. The length of the perimeter of that square is $4 \times 100 = 400$ in., and its area is 69.4 sq. ft. The shear along the perimeter of the square is

$$V' = 5{,}450 \times (110 - 69.4) = 221{,}000 \text{ lb.}$$

and the unit shear is

$$v' = \frac{V'}{.875\ b'\ d} = \frac{221{,}000}{.875 \times 400 \times 32} = 19.7 \text{ lb. per sq. in.,}$$

which is below the permissible 40 lb. per sq. in.

Weight of Footing.—The weight of the footing may now be estimated, and it is found to be $110 \times 3 \times 150 = 49{,}500$ lb., which does not differ sufficiently from the assumed weight of 60,000 lb. to warrant a revision.

Bending Moment and Shear.—If the footing is considered resolved into four trapezoidal cantilevers as for two-way reinforcement, then in each cantilever, $c=\frac{10.5-3}{2}=3.75$ ft.; and, since $a=3$ ft., the maximum bending moment is

$$M=\frac{w}{2}\left(a+\frac{4}{3}c\right)c^2=\frac{5,450}{2}\times\left(3+\frac{4}{3}\times 3.75\right)\times 3.75^2$$
$$=307,000 \text{ ft.-lb.} = 3,680,000 \text{ in.-lb.}$$

By formula 3 in Art. 12, the maximum shear is

$$V=w(a+c)c=5,450\times(3+3.75)\times 3.75=138,000 \text{ lb.}$$

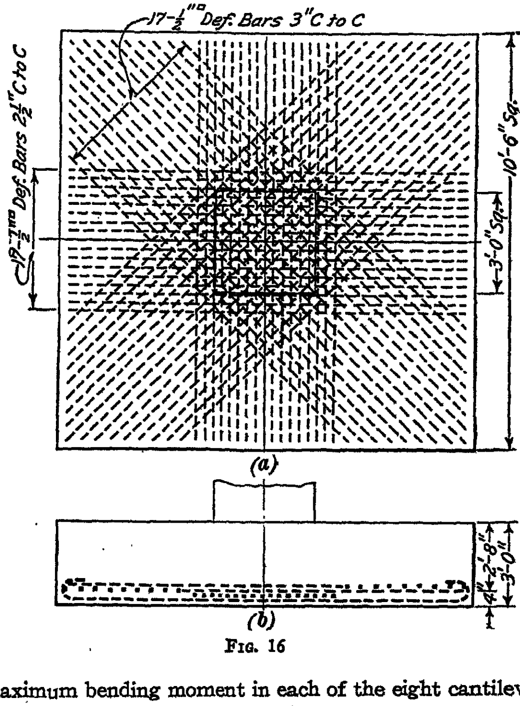


Fig. 16

The maximum bending moment in each of the eight cantilevers, shown in Fig. 15, may be assumed as $\frac{3,680,000}{2}=1,840,000$ in.-lb., and the maximum shear as $\frac{138,000}{2}=69,000$ lb.

Effective Width of Belts.—The effective width of a belt parallel to the sides of the pier is the same as the length of a side of the pier, or 36 in., while

the effective width of a diagonal belt is the same as the length of a diagonal of the pier base, or $36\sqrt{2}=50.9$ in.

Reinforcement.—The steel area required in each reinforcing belt is

$$A_s=\frac{M}{.875\,f_s\,d}=\frac{1{,}840{,}000}{.875\times16{,}000\times32}=4.12 \text{ sq. in.,}$$

which is supplied by seventeen $\frac{1}{2}$-in. square bars. These bars should be spaced about $\frac{36}{17}=2$ in. on centers in the belts parallel to the sides of the pier, and $\frac{50.9}{17}=3$ in. on centers in the diagonal belts. In the belts parallel to the sides of the pier the bars will arbitrarily be spaced further apart, so that no part of the footing will remain unreinforced; $2\frac{1}{2}$ in. spacing will therefore be used. The reinforcement employed is as shown in Fig. 16 (a) and (b).

Bond Stress.—As previously found, the maximum shear in each of the eight cantilevers is 69,000 lb. The unit bond stress in the bars of each belt at the points of maximum shear is

$$u=\frac{V}{.875\,d\,O}=\frac{69{,}000}{.875\times32\times17\times2}=72.5 \text{ lb. per sq. in.}$$

If deformed rods adequately anchored at the ends are used, the permissible unit bond stress for four-way reinforcement, according to Art. 19, is $100-25-(2\times10)=55$ per cent. of 150 lb. per sq. in., or 82.5 lb. per sq. in.

22. Sloped Footings.—For the same conditions of loading and soil, a sloped footing, shown in Fig. 2, requires less concrete than a slab footing and offers a better distribution of bond stress throughout the length of each reinforcing rod, but is somewhat more difficult to construct. It may be employed advantageously to support both heavy and light column loads and is widely used in practice.

In designing a sloped footing, the effective depth of the footing at the perimeter of the column base which can safely resist the punching shear is first computed as for slab footings. The depth required to resist safely the diagonal tension at the critical section is then determined. If V' is the shear at the critical section for diagonal tension along the perimeter b' of the square $ABCD$, Fig. 17 (a), and v' is the allowable unit shear for measuring diagonal tension, then the minimum effective depth d' at the critical section may be found by the formula

$$d' = \frac{V'}{.875\, b' v'} \qquad (1)$$

After the minimum effective depth has been determined the footing slab is sloped so as to maintain that depth. Thus, in Fig. 17 (*b*), the depth *k* should be equal to or greater than d'. The area of the level portion of the top of the footing, $EFGH$ in (*a*), is generally made about twice the area of the column base or pedestal on it. The edge *m* in (*b*) of the vertical portion of the footing should be at least 6 inches above the top of the reinforcement, and the vertical height *h* should therefore be at least 10 inches, and preferably 12 inches.

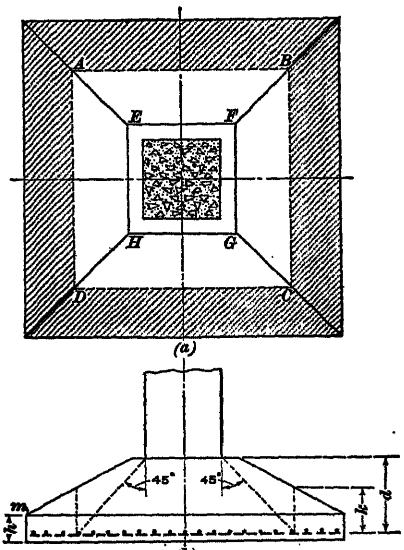


FIG. 17

The bending moment, shear, and bond stress for sloped footings are determined as for slab footings. However, in investigating the unit compressive stress in the concrete, the shape of the footing should be taken into consideration, and the unit compressive stress in the concrete should be increased by the formula

$$f'_c = f_c \sec^2 H \qquad (2)$$

in which f'_c = unit compressive stress in concrete for sloped surfaces;

f_c = unit compressive stress in concrete for level surfaces;

H = angle of inclination of sloped surface with horizontal.

The design of a sloped footing is illustrated in the following example:

EXAMPLE.—Design a sloped footing to support a 30-inch square column pedestal which carries a load of 500,000 pounds. The required bearing area on the soil is 140 square feet. Assume $f_s = 16{,}000$ pounds per square inch and the concrete to be of 1 : 2 : 4 mix.

SOLUTION.—*Bearing Area.*—As the required bearing area is 140 sq. ft., a footing 12 ft. square will be assumed. The unit net soil pressure on the footing is

$$w = \frac{500{,}000}{144} = 3{,}470 \text{ lb. per sq. ft.}$$

Punching Shear.—The perimeter of a 30-in. square pedestal is $4 \times 30 = 120$ in., and the area is 900 sq. in. = 6.25 sq. ft. By formula **1** in Art. **14**,

$$V_p = w(A - A') = 3{,}470 \times (144 - 6.25) = 478{,}000 \text{ lb.,}$$

and by formula **3**,

$$d = \frac{V_p}{s\,v_p} = \frac{478{,}000}{120 \times 120} = 33.2, \text{ say 34 in.}$$

If 4 in. are added for the protection of the reinforcement, a maximum total depth of 38 in., as in Fig. 18 (*b*), will be assumed.

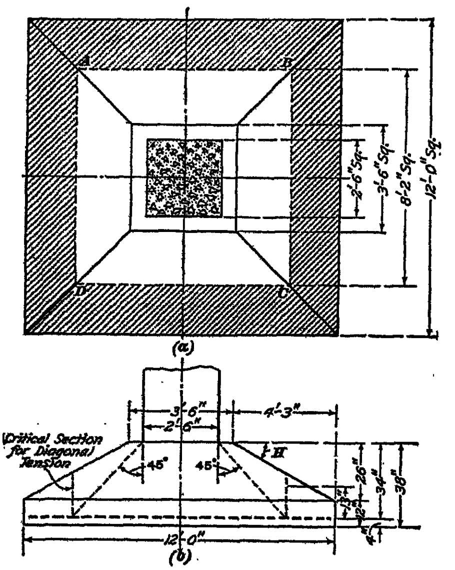


FIG. 18

Diagonal Tension.—The critical section for measuring diagonal tension in the footing is along the perimeter of the square *A B C D* in Fig. 18 (*a*), each side of which is equal to the sum of a side of the column base and twice the effective depth of the footing slab, or $30 + (34 \times 2) = 98$ in. = 8 ft. 2 in. The area of that square is $8.17^2 = 66.7$ sq. ft., and the perimeter is $4 \times 98 = 392$ in. The shear along the perimeter is

$$V' = 3{,}470 \times (144 - 66.7) = 268{,}000 \text{ lb.}$$

For an allowable unit shear of 60 lb. per sq. in., for 1 : 2 : 4 concrete with rods adequately anchored at the ends, the effective depth required at the critical section is

$$d' = \frac{V'}{.875\,b'\,v'} = \frac{268{,}000}{.875 \times 392 \times 60} = 13 \text{ in.,}$$

and if 4 in. are added for protection of reinforcement, a total depth of 17 in. is required.

The area of the column base is 900 sq. in., and if the level top of the footing is made about twice that area, or 1,800 sq. in., each side will be $\sqrt{1,800} = 42.4$ in., say 3 ft. 6 in. If the vertical ends of the footing are made 12 in. deep and the footing is sloped so as to make the level top 3 ft. 6 in. square, as in Fig. 18 (*b*), the depth at the critical section is considerably greater than the minimum required.

Maximum Bending Moment and Shear.—Each side of the column pier, $a = 2.5$ ft.; hence, $c = \frac{12-2.5}{2} = 4.75$ ft. By formula **3** in Art. **11**,

$$M = \frac{w}{2}\left(a + \frac{4}{3}c\right)c^2 = \frac{3,470}{2} \times \left(2.5 + \frac{4}{3} \times 4.75\right) \times 4.75^2$$

$$= 346,000 \text{ ft.-lb.} = 4,150,000 \text{ in.-lb.}$$

By formula **3** in Art. **12**,

$$V = w(a+c)c = 3,470 \times (2.5 + 4.75) \times 4.75 = 119,500 \text{ lb.}$$

Effective Width.—The effective width of the slab, as found by the formula in Art. **16**,

$$b = 30 + (4.75 \times 12) + 34 = 121 \text{ in.}$$

Compressive Stress in Concrete.—Since $b = 121$ in., $d = 34$ in., and $M = 4,150,000$ in.-lb., by the formula in Art. **17**,

$$f_c = \frac{6M}{b\,d^2} = \frac{6 \times 4,150,000}{121 \times 34^2} = 178 \text{ lb. per sq. in.}$$

However, since the footing is sloped, the unit compressive stress in the concrete should be increased by applying formula **2**. According to Fig. 18 (*b*), $\tan H = \frac{26}{51} = .51$. By the principles of trigonometry,

$$\sec^2 H = 1 + \tan^2 H = 1 + .51^2 = 1.26.$$

Substituting in formula **2**,

$$f'_c = f_c \sec^2 H = 178 \times 1.26 = 224 \text{ lb. per sq. in.,}$$

which is below the allowable 650 lb. per sq. in.

Reinforcement.—The required steel area,

$$A_s = \frac{M}{.875\,f_s\,d} = \frac{4,150,000}{.875 \times 16,000 \times 34} = 8.7 \text{ sq. in.}$$

which is best supplied by twenty-three $\frac{5}{8}$-in. square rods with an area of $.3906 \times 23 = 8.98$ sq. in. and a total perimeter of $2.50 \times 23 = 57.5$ in. The spacing of these rods within the effective width is $\frac{121}{23} = 5.27$, say 5 in.

Assuming that the same spacing will be maintained throughout the footing slab, the arrangement shown in Fig. 19 will be obtained.

Bond Stress.—The unit bond stress,

$$u=\frac{V}{.875\,dO}=\frac{119{,}500}{.875\times34\times57.5}=70 \text{ lb. per sq. in.},$$

which is below the permissible 75 lb. per sq. in. for two-way reinforcement securely anchored at the ends.

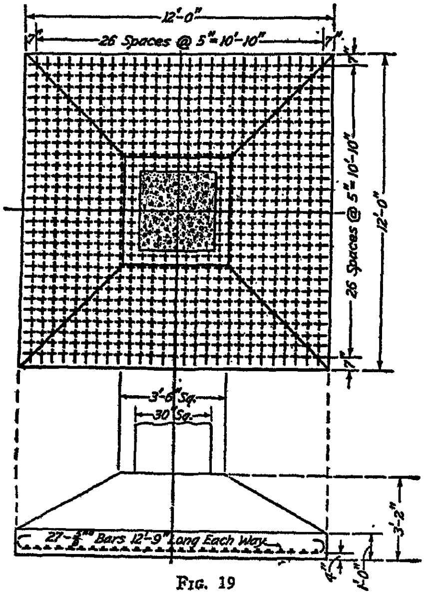


FIG. 19

Weight of Footing.—For the sake of providing a guide for estimating the weight of the footing in future problems, let the weight of the footing be determined, although it is not needed in this case.

The volume of the footing may be found by resolving the footing into two parts, namely, a prism the base of which is 12 ft. square and height 1 ft., and a frustum of a pyramid the lower plane of which is 12 ft. square, upper plane 3 ft. 6 in. square, and height 2 ft. 2 in. The volume of the prism is $144\times1=144$ cu. ft. The volume of the frustum of pyramid can be found by substituting in the formula,

$$\text{Volume}=\tfrac{1}{3}h\left(F+f+\sqrt{Ff}\right)$$

in which h = height of frustum of pyramid,
F = area of lower plane,
f = area of upper plane.

Hence, $\text{volume}=\frac{1}{3}\times2.17\times\left(12^2+3.5^2+\sqrt{12^2\times3.5^2}\right)=143$ cu. ft.

The volume of the footing is therefore, $144+143=287$ cu. ft., and the weight of the footing is $287\times150=43{,}100$ lb. which is about 9 per cent. of the column load of 500,000 lb.

23. Stepped Footings.—In order to facilitate construction the top of footings may be stepped instead of

sloped, as in Fig. 20. These steps should preferably be so constructed that the footing will have at all sections a depth not less than is required when the top is sloped. Stepped footings may be advantageously employed to support heavy column loads.

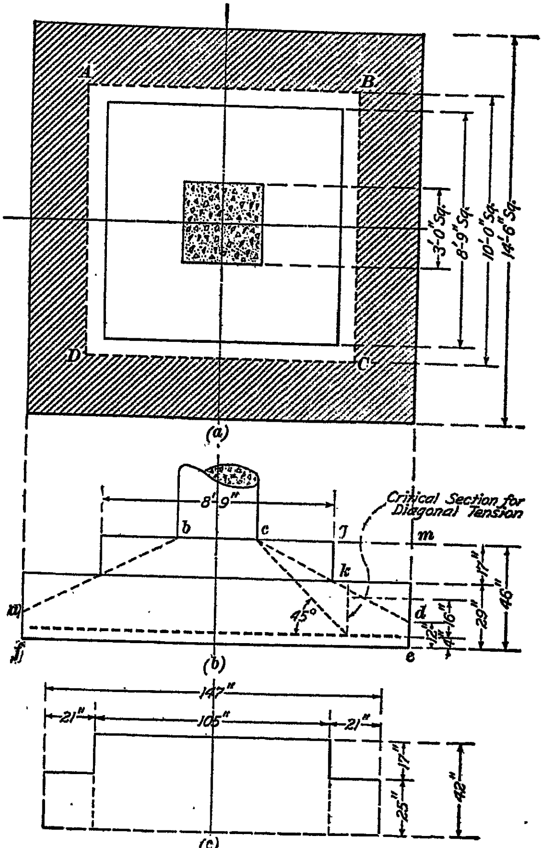


FIG. 20

Since the effective depth in a stepped footing changes abruptly, it is necessary to investigate at each step the strength of the footing in punching shear, bending, diagonal tension, and bond stress between the concrete and the steel. In such investigations, the step above the bottom slab and each

successive step is treated like the column base in slab footings, and the design is similar to that already explained for the latter. However, when the depth of the footing is at no section less than that required for a sloped footing, these investigations are superfluous in the usual cases. The design of a stepped footing is illustrated in the following example:

EXAMPLE.—Design a stepped footing to support a 36-inch square column pedestal carrying a load of 760,000 pounds. The bearing capacity of the soil is 2 tons per square foot. Assume a concrete mix of 1:2:4, and $f_s = 16{,}000$ pounds per square inch.

SOLUTION.—*Bearing Area.*—Assuming the weight of the footing about 12 per cent. of the column load, the total load on the soil is 760,000 +90,000=850,000 lb. The required footing area is $\frac{850{,}000}{4{,}000}=212.5$ sq. ft., and a footing slab 14 ft. 6 in. square will be adopted, as shown in Fig. 20 (*a*). The bearing area of the footing is $14.5^2=210$ sq. ft.; and, therefore,

$$w=\frac{760{,}000}{210}=3{,}620 \text{ lb. per sq. ft.}$$

Punching Shear in Footing.—The perimeter of the column pedestal is $s=4\times36=144$ in. and the area is 9 sq. ft. The total punching shear at the perimeter of the pedestal is

$$V_p=w(A-A')=3{,}620\times(210-9)=728{,}000 \text{ lb.,}$$

and the effective depth required to resist a unit punching shear of 120 lb. per sq. in. is

$$d=\frac{V_p}{s\,v_p}=\frac{728{,}000}{144\times120}=42 \text{ in.}$$

If 4 in. are added for protection of the reinforcement, a total depth of 46 in. is obtained.

Diagonal Tension.—The diagonal tension in the footing is measured by the shear along the perimeter of the square *A B C D* in Fig. 20 (*a*), each side of which is equal to the sum of a side of the column pedestal and twice the effective depth of the footing, or $36+(2\times42)=120$ in. = 10 ft. The area of that square is $10\times10=100$ sq. ft., and its perimeter is $4\times120=480$ in. The total shear along the perimeter *A B C D* is

$$V'=3{,}620\times(210-100)=398{,}000 \text{ lb.}$$

For a permissible unit shear of 60 lb. per sq. in., for plain rods adequately anchored at the ends,

$$d'=\frac{V'}{.875\,b'\,v'}=\frac{398{,}000}{.875\times480\times60}=16 \text{ in. (nearly)}$$

If the top of the footing were sloped so that the level portion would have the same area as the column pedestal and the vertical part of the footing were 12 in. high, the footing would be as shown by *a b c d e f* in Fig. 20 (*b*). If, instead, two steps are used, each step projecting an equal distance beyond the face of the column pedestal, then the projection of the top step beyond the face of the column pedestal is one-half the projection of the footing. Since the projections of the footing beyond the face of the column pedestal, $c=\frac{14.5-3}{2}=5.75$ ft., the projection of the top step is $\frac{5.75}{2}=2.875$ ft., and the width of the top step is $3+(2\times 2.875)=8.75$ ft. $=8$ ft. 9 in. The top step may now be laid off in Fig. 20 (*b*). Since triangle *c g k* is similar to triangle *c m d*, and *c g* is one-half of *c m*, the height of the top step *g k* is one-half of *m d*, or $g\,k=\frac{m\,d}{2}=\frac{46-12}{2}=17$ in. The bottom slab is therefore $46-17=29$ in. deep.

Weight of Footing.—The weight of the footing is

$$150\times(210\times 2.42+8.75^2\times 1.42)=92{,}600 \text{ lb.},$$

which is higher than the assumed 90,000 lb., but the difference, being well within the limit of 2 per cent. of the column load, is not sufficiently large to warrant a change in the design.

Bending Moment and Shear in Footing.—As previously determined, the projection of the footing beyond the face of the column pedestal, $c=5.75$ ft. and since $a=3$ ft., the maximum bending moment is

$$M=\frac{w}{2}\left(a+\frac{4}{3}c\right)c^2=\frac{3{,}620}{2}\times\left(3+\frac{4}{3}\times 5.75\right)\times 5.75^2$$
$$=639{,}000 \text{ ft.-lb.}=7{,}670{,}000 \text{ in.-lb.}$$

The maximum shear in each trapezoidal cantilever is

$$V=w(a+c)c=3{,}620\times(3+5.75)\times 5.75=182{,}000 \text{ lb.}$$

Effective Width of Footing.—The effective width of the footing is

$$b=a+c+d=36+(5.75\times 12)+42=147 \text{ in.}$$

Compressive Stress in Concrete.—In computing the compressive stress in the concrete, the actual shape of the footing should be taken into consideration. In the effective width of 147 in., the effective depth of the footing is 42 in. in a width of 8 ft. 9 in. $=105$ in., and 25 in. in the remaining 42 in., as illustrated in Fig. 20 (*c*). If the concrete in the width of 105 in. alone is assumed to resist the maximum bending moment in the footing, the compressive stress in the concrete, by the formula in Art. **17**, is

$$f_c=\frac{6M}{b\,d^2}=\frac{6\times 7{,}670{,}000}{105\times 42^2}=248 \text{ lb. per sq. in.},$$

which is considerably below the allowable 650 lb. per sq. in.

Reinforcement.—The required steel area is

$$A_s = \frac{M}{.875\ f_s d} = \frac{7{,}670{,}000}{.875 \times 16{,}000 \times 42} = 13 \text{ sq. in.},$$

which is supplied by the following combinations of bars:

No. of Bars	Section of Bar	Area of Bar Sq. In.	Perimeter of Bar Inches	Total Area Sq. In.	Total Perimeter Inches	Spacing of Bars Inches
22	⅞" Round	.6013	2.75	13.23	60.5	6.68
24	¾" Square	.5625	3.00	13.50	72.0	6.13
30	¾" Round	.4418	2.36	13.25	70.8	4.90

If the twenty-four ¾-in. square rods are assumed they will be spaced about 6 in. on centers, as shown in Fig. 21.

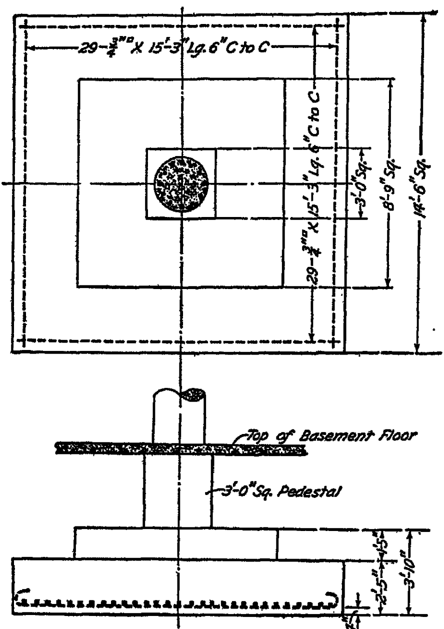


Fig. 21

Bond Stress.—The unit bond stress is

$$u = \frac{V}{.875\,d\,O} = \frac{182{,}000}{.875 \times 42 \times 72} = 68.8 \text{ lb. per sq. in.},$$

which is below 90 lb. per sq. in. allowed for two-way reinforcement, if securely anchored at the ends.

The footing illustrated in Fig. 21 is therefore amply safe.

EXAMPLES FOR PRACTICE

1. Find (*a*) the effective depth, and (*b*) the size and spacing of rods for an 8-foot square slab footing, reinforced with two-way reinforcement, that is to support a 24-inch round column which carries a load of 219,000 pounds, if $f_s = 16{,}000$ pounds per square inch and the concrete is to be of 1 : 2 : 4 mix.

Ans. $\begin{cases} (a) \text{ 23 in.} \\ (b) \text{ ½-in. sq. bars, spaced about 5½ in. c. to c.} \end{cases}$

2. A 33-inch square column which carries a load of 605,000 pounds is supported by a sloped footing on a soil with a bearing capacity of 3 tons per square foot. Determine (*a*) the size and (*b*) the effective depth of the footing, and (*c*) the size and spacing of the reinforcing rods.

Ans. (*a*) 10 ft. 6 in. square.
(*b*) 36 in.
(*c*) ⅝-in. sq. bars, spaced 5¾ in. c. to c.

3. If the footing in the preceding problem is stepped, find (*a*) the height and (*b*) the dimensions of the top step.

Ans. (*a*) 14 in.
(*b*) 6 ft. 7½ in. square.

COMBINED COLUMN FOOTINGS

24. Where building conditions make it necessary, combined footings of reinforced concrete are used to support two

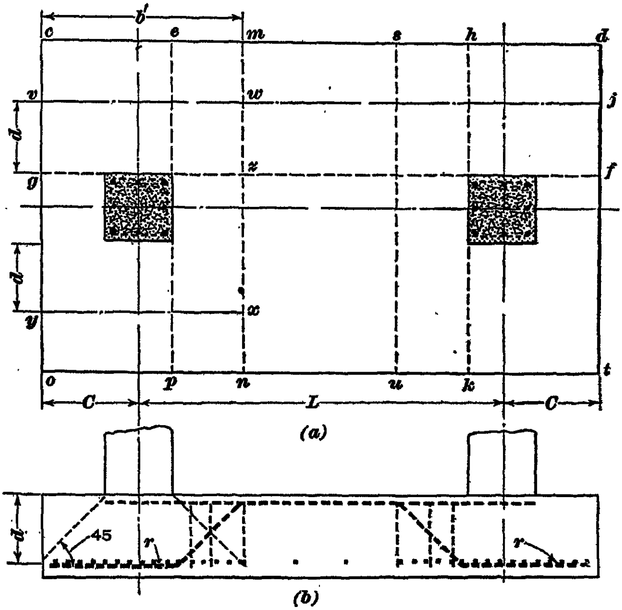


FIG. 22

or more columns. Such footings are usually reinforced slabs of uniform thickness. The columns rest either directly on the slab, as in Fig. 22, or on pedestals represented by *c* in Fig. 23.

Where two adjacent interior columns are so placed that if independent footings were designed for them they would touch or overlap, a combined footing for the support of the two columns is then designed, as in Fig. 22. However, the more usual condition that makes the design of a combined footing necessary is when an exterior column is so close to the building line that the construction of a symmetrical footing would encroach upon the adjacent property, as in Fig. 23 (*a*) and (*b*). In this case the exterior and the adjacent interior columns are supported on a combined footing which may be either rectangular or trapezoidal, according to the existing building conditions. In all cases, it is essential that combined column footings should be so proportioned that the resultant of the column loads will pass through the center of gravity of the footing base, in order that the pressure on the soil will be uniformly distributed.

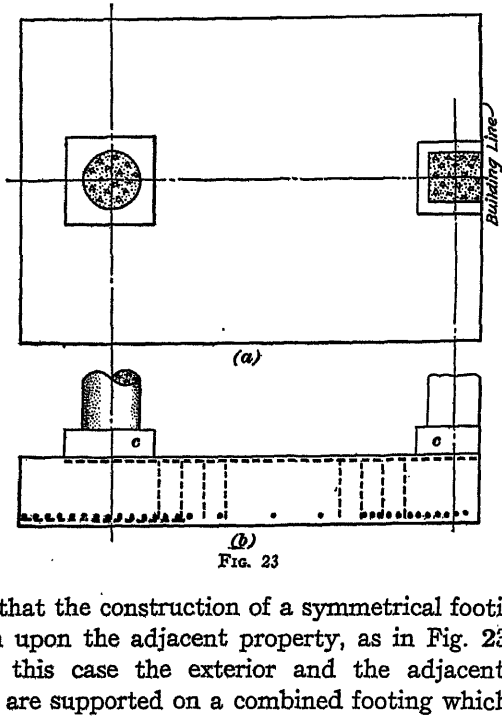


FIG. 23

FOOTINGS SUPPORTING TWO EQUALLY LOADED COLUMNS

25. When a combined footing is designed to support two equally loaded columns, it is usually made rectangular, as in Fig. 22, with projections C of equal length. The diagram of loading on such a footing, as shown in Fig. 24 (*a*), is the reverse of that of a slab loaded with a uniform load and supported on two columns. As previously explained, the weight of the slab is considered in proportioning the footing area but not in designing the footing slab. Such footings tend to bend longitudinally and transversely.

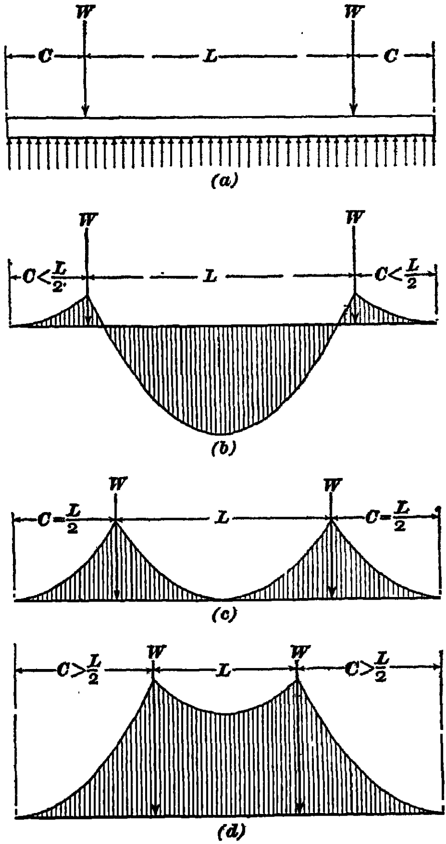


FIG. 24

26. Longitudinal Bending Moments.—The maximum longitudinal bending moments in the footing shown in Fig. 24 (*a*) occur at the center of the span L and under each column. If the load on each column is W, the net soil pressure per foot length of footing is $\frac{2W}{L+2C}$, and the maximum bending moment under each column is

$$M_1 = \frac{W C^2}{L+2C} \qquad (1)$$

The maximum bending moment at the center of the span L is

$M_2 = \frac{W(L+2C)}{4} - \frac{WL}{2}$; whence,

$$M_2 = \frac{W(2C-L)}{4} \qquad (2)$$

When C is less than $\frac{1}{2}L$, the bending moment at the center of span L is negative, causing tensile stresses near the top of the slab and compressive stresses near the bottom of the slab; the moment diagram for this condition is shown in (*b*). When C equals $\frac{1}{2}L$, the bending moment at the center of the span L is zero and elsewhere in the slab positive, as shown in (*c*), and when C is greater than $\frac{1}{2}L$, the bending moment is positive in the entire footing slab, as shown in (*d*).

In most combined footings dealt with in practice, the projections C are less than $\frac{1}{2}L$, and the footing must therefore be provided with longitudinal reinforcement to resist tensile stresses near the top of the slab in span L and near the bottom of the slab in projections C, as shown in Fig. 22 (*b*). The rods in the projections C are often provided by bending down some of the rods in span L, as rods r in Fig. 22 (*b*). Such bends should be made at points where the rods are no longer necessary to resist the tensile stresses due to the bending moments. However, a sufficient number of bars should be left to resist safely the bond stress between the concrete and steel. The inclined portions of the rods also help resist the diagonal tension in the slab.

In order to provide adequate anchorage in the bars relied on to resist the bond stress, they should either be hooked at the ends or carried a sufficient distance beyond the points of zero moment to develop by bond in that distance at least one-third of the allowable tensile stress in the steel. When $f_s = 16{,}000$ pounds per square inch, the allowable tensile stress in each bar t inches square is $16{,}000\ t^2$, and one-third of that tensile stress is $\frac{16{,}000}{3} t^2$. For a plain bar imbedded

in 1:2:4 concrete, the allowable unit bond stress is 80 pounds per square inch, and if the length of imbedment between the end of the bar and the point of zero moment is l', the bond stress developed in that length l' is $80 \times 4\,t\,l'$. When the bar develops by bond in the length l' one-third the allowable tensile stress in it, $\frac{16{,}000}{3}t^2 = 320\,t\,l'$; whence,

$$l' = \frac{16{,}000\,t}{3 \times 320} = 17\,t \text{ (about)} \qquad (3)$$

The same formula is obtained for round rods, one-third the allowable tensile stress in each rod being $\frac{16{,}000}{3 \times 4}\pi t^2$, and the bond stress in a length l' being $80\,\pi\,t\,l'$; then $\frac{16{,}000}{3 \times 4}\pi t = 80\,\pi\,t\,l'$, and $l' = 17\,t$ (about), as before. For deformed bars imbedded in 1:2:4 concrete the allowable unit bond stress is 100 pounds per square inch, and therefore the bond stress developed in each bar in a length l' is $400\,t\,l'$; when the bond stress thus developed equals one-third the allowable tensile stress in each bar, $400\,t\,l' = \frac{16{,}000}{3}t^2$; whence,

$$l' = \frac{16{,}000\,t}{3 \times 400} = 14\,t \text{ (about)} \qquad (4)$$

27. Transverse Bending Moments.—When the footing slab is much wider than the column base or pedestal, provision must be made to resist the transverse bending moments by means of transverse reinforcement near the bottom of the slab, designed to resist the tensile stresses developed there. The maximum transverse bending moment in the slab shown in Fig. 22 (*a*) is the moment of the net soil pressure on cantilever *cdfg* about line *gf*. However, since the portions of the slab directly under or near the column bases are stiffer transversely than the remainder of the slab, it is considered good practice to assume that all bending and shear is resisted by strips of slab directly under the column bases or pedestals increased on each side by about once the effective depth of

the slab, as strips $c\,m\,n\,o$ and $s\,d\,t\,u$. In no case, however, should the width of each strip exceed the total width of the slab. These strips of slab are known as *distributing beams.* It is customary to place all the required transverse reinforcing rods in the distributing beams, and to place a few rods outside those beams to resist the bending that may occur there. The width of distributing beams here recommended is not based on rigid theory, and practice may vary according to the judgment of the designer.

28. Depth of Footing.—The depth of the slab of a combined footing is determined by the punching shear at the perimeters of the column bases or pedestals, by the diagonal tension in the distributing beams, or by the bending moments in the footing slab. The diagonal tension in the distributing beams is measured by the unit shear at a distance of the effective depth of the footing slab from the sides of the column bases or caps. In the distributing beam $c\,m\,n\,o$, Fig. 22 (*a*), the diagonal tension would be measured by the unit shear along the dot-and-dash lines $v\,w$ and $y\,x$. The maximum shear at $g\,z$ in the distributing beam $c\,m\,n\,o$ is one-half the net soil pressure on $c\,d\,f\,g$. The shear V' at $v\,w$, which is at a distance d from $g\,z$, is one-half the net soil pressure on $c\,d\,j\,v$.

If b' is the width of the distributing beam, then the unit shear measuring diagonal tension in the distributing beam is

$$v' = \frac{V'}{.875\,b'd}$$

This unit shear v' is usually found to be less than the allowable unit shear for measuring diagonal tension, but when it exceeds that value the depth of the slab should be increased sufficiently to reduce v' to the allowable unit shear.

When the longitudinal bending moment in the center of the span L between the columns is negative, as is usually the case, the diagonal tension in that span is measured by the unit shear at the faces of the column bases or pedestals, that is, along $e\,p$ and $h\,k$ in Fig. 22 (*a*). This unit shear usually exceeds the allowable unit shear, and web reinforcement is employed to carry the excess shear, as explained in the following article.

29. Reinforcement for Diagonal Tension.—The reinforcement for diagonal tension between columns is provided as in simple reinforced-concrete beams. Let V represent the total shear at the face of the column, b the width of the footing slab, and .875 d the approximate length of the arm of the resisting stress couple, then the unit shear is

$$v=\frac{V}{.875\, b\, d} \qquad (1)$$

When plain longitudinal rods are used to reinforce the footing slab, the unit shear should not exceed 2 per cent. of the ultimate compressive strength of the concrete; for 1:2:4 concrete 40 pounds per square inch is allowable. When the longitudinal rods are adequately anchored at both ends, the unit shear should not exceed 3 per cent. of the ultimate compressive strength of the concrete or 60 pounds per square inch for 1:2:4 concrete. If the unit shear at the face of the column exceeds the specified limits, web reinforcement should be provided to resist the shear in excess of that safely carried by the concrete. Thus, if the unit shear that can safely be carried by the concrete is v_c, the unit shear carried by the web reinforcement is $v-v_c$.

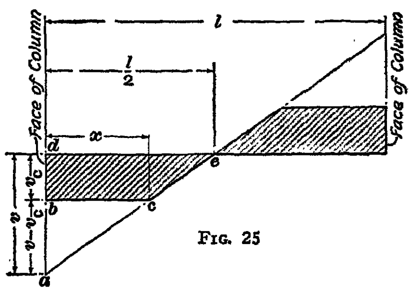


FIG. 25

Let the distribution of the unit shear between the faces of the column bases or pedestals be as shown in Fig. 25, then in the left half of the span the unit shear at the face of the column base or pedestal will be represented by the altitude of the triangle $a\,d\,e$, and the unit shear v_c that can be safely carried by the concrete will be represented by the altitude of the shaded trapezoid $b\,d\,e\,c$. The altitude of the triangle $a\,b\,c$ will represent the unit shear to be carried by the web reinforcement, or $v-v_c$. The distance from the face of the column

to the center of the span l is $\frac{l}{2}$, and the distance from the face of the column to the point c, or the point beyond which the concrete alone can safely carry the unit shear, is x. Since the triangles $a\,d\,e$ and $a\,b\,c$ are similar, their homologous sides are proportional, and $\frac{x}{\frac{l}{2}}=\frac{v-v_c}{v}$. Solving for x, the formula obtained is

$$x=\frac{(v-v_c)\,l}{2\,v} \qquad (2)$$

The total shear in a unit width of slab that is carried by the web reinforcement in the left half of the span is represented by the area of the triangle $a\,b\,c$ and is equal to $(v-v_c)\,\frac{x}{2}$. The total shear V_s in each half of the span, in the entire width of footing b, that is carried by the reinforcement is equal to the product of the area of triangle $a\,b\,c$ and the width of footing b, or

$$V_s=\frac{(v-v_c)\,x\,b}{2} \qquad (3)$$

The web reinforcement usually consists of vertical stirrups. If A_v represents the area of each vertical stirrup, and f_v the allowable unit tensile stress in the stirrup, then the total number of stirrups required in each half of the span is

$$n_v=\frac{V_s}{f_v A_v} \qquad (4)$$

The minimum spacing s of the stirrups, required near the face of the column, may be found by the formula,

$$s=\frac{f_v A_v}{(v-v_c)\,b} \qquad (5)$$

The distance of the first stirrup from the face of the column is, therefore, $\frac{s}{2}$. When the distance x, the total number of stirrups n_v and the minimum spacing s are known, it is good practice to arrange the stirrups over the distance x so that

their spacing gradually increases from the smallest spacing s to the maximum permissible spacing, which is one-half the effective depth of the slab. This method is approximate, but

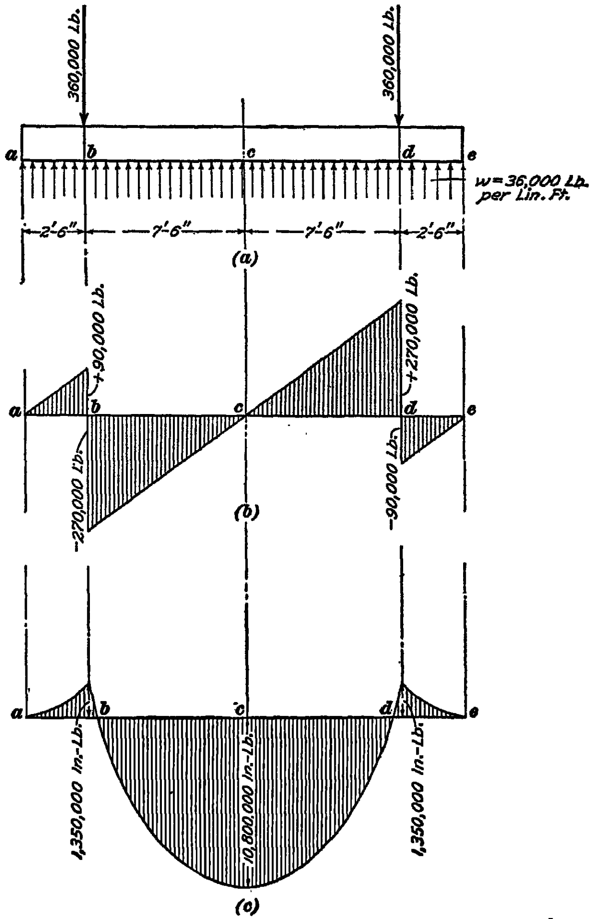


FIG. 26

it is considered sufficiently safe for the design of footings and is generally followed in practice. The first stirrup should never be placed more than one-quarter the effective depth of the footing from the face of the column.

In footing slabs with web reinforcement the maximum unit shear at the face of the column base should not exceed 6 per cent. of the ultimate strength of the concrete, nor should it be greater than 120 pounds per square inch. The maximum tensile stress f_v in the web reinforcement should not exceed 12,000 pounds per square inch.

30. Illustrative Example.—The design of a combined footing for the support of two equally loaded columns is illustrated in the following example:

EXAMPLE.—Design a combined reinforced-concrete footing to support two columns spaced 15 feet center to center. Each column is 2 feet square and carries a load of 360,000 pounds. Building conditions limit the projection of the footing beyond the center line of one column to 2 feet 6 inches. The safe bearing capacity of the soil is $2\frac{1}{2}$ tons per square foot. Assume $f_s = 16{,}000$ pounds per square inch, and $f_c = 650$ pounds per square inch.

SOLUTION.—*Bearing Area.*—The resultant of the two loads is $2 \times 360{,}000 = 720{,}000$ lb., and its point of application is midway between the two columns. To distribute the pressure on the soil uniformly the projections of the footing beyond the center lines of the columns are made equal, and a rectangular section projecting 2 ft. 6 in. beyond the center line of either column is chosen. If the weight of the footing is assumed to be about 10 per cent. of the column loads, the total pressure on the soil is $720{,}000 + 72{,}000 = 792{,}000$ lb. The required footing area is $\frac{792{,}000}{5{,}000} = 158.4$ sq. ft., and since the assumed length is $15 + (2 \times 2.5) = 20$ ft., the required width is $\frac{158.4}{20} = 8$ ft., as shown in Fig. 27.

Bending Moments and Shears.—The net soil pressure per foot length of footing is $\frac{720{,}000}{20} = 36{,}000$ lb., and per square foot of footing area it is $\frac{36{,}000}{8} = 4{,}500$ lb. The diagram of loading on the footing slab is shown in Fig. 26 (*a*). The maximum shear in each projection of the footing, *a b* or *d e*, is $V_1 = 36{,}000 \times 2.5 = 90{,}000$ lb.; the maximum shear in the center span *b d* is $V_2 = 360{,}000 - 90{,}000 = 270{,}000$ lb., and the shear diagram is as shown in (*b*). The maximum bending moment in each projection at *b* or *d*, by formula **1** in Art. **26**, is

$$M_1 = \frac{W C^2}{L + 2C} = \frac{360{,}000 \times 2.5^2}{20} = 112{,}500 \text{ ft.-lb.} = 1{,}350{,}000 \text{ in.-lb.};$$

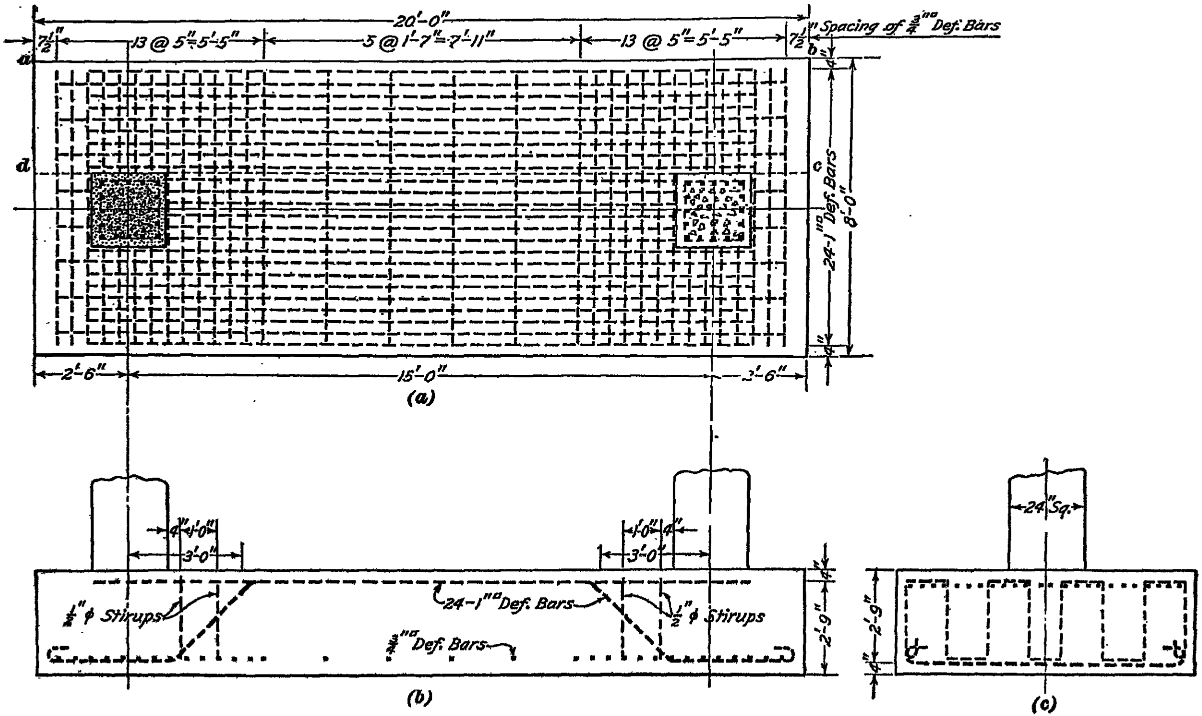

20'-0"
7½"
13 @ 5" = 5'-5"
5 @ 1'-7" = 7'-11"
13 @ 5" = 5'-5"
7½" Spacing of ¾"□ Def. Bars
a
b
c
d
4"
24-1"□ Def. Bars
8'-0"
2'-6"
15'-0"
2'-6"
(a)
4" 1'-0"
3'-0"
1'-0" 4"
3'-0"
½" φ Stirups
24-1"□ Def. Bars
¾"□ Def. Bars
½" φ Stirups
2'-9"
(b)
24" Sq.
2'-9"
4"
(c)

FIG. 27

the bending moment in the center span at c, by formula **2**, is

$$M_2 = \frac{W(2C-L)}{4} = \frac{360{,}000\times(2\times2.5-15)}{4}$$
$$= -900{,}000 \text{ ft.-lb.} = -10{,}800{,}000 \text{ in.-lb.},$$

and the bending moment diagram is as shown in (c).

Effective Depth.—The perimeter of a 24-in. square column is 96 in., and its area is 4 sq. ft. The net soil pressure directly under the column is $4{,}500\times4=18{,}000$ lb. The punching shear at each column is therefore $360{,}000-18{,}000=342{,}000$ lb., and the effective depth required to resist a unit punching shear of 120 lb. per sq. in. is $\frac{342{,}000}{96\times120}=30$ in.

The effective depth of slab required to resist a bending moment of 10,800,000 in.-lb. may be found by solving for d in the formula of Art. **17** and substituting the known values in the derived formula, the result being

$$d=\sqrt{\frac{6M}{f_c b}}=\sqrt{\frac{6\times10{,}800{,}000}{650\times96}}=32.2 \text{ in.}$$

An effective depth of 33 in. will be chosen, and if 4 in. are allowed for protection, a total depth of 37 in. will be used.

Weight of Footing.—The weight of footing may now be calculated and is found to be $8\times20\times3.08\times150=73{,}900$ lb., which does not differ materially from the assumed weight of 72,000 lb.

Longitudinal Reinforcement.—The reinforcement required near the top of the slab at the center of the span c in Fig. 26, $A'_s=\frac{M}{.875\,f_s d}$ $=\frac{10{,}800{,}000}{.875\times16{,}000\times33}=23.4$ sq. in., which is supplied by twenty-four 1-in. square bars.

The longitudinal reinforcement near the bottom of the slab required to resist the maximum bending moments in the projections is

$$A''_s=\frac{1{,}350{,}000}{.875\times16{,}000\times33}=2.92 \text{ sq. in.}$$

If every third bar in the center span is bent down at a distance of 3 ft. from the axes of the columns, where it is no longer necessary to resist the tensile stress near the top of the slab, and is carried to within 3 in. from the ends of the slab, eight 1-in. bars will be available for the reinforcement of the projections, offering an area of 8 sq. in. The unit bond stress,

$$u=\frac{V}{.875\,d\,O}=\frac{90{,}000}{.875\times33\times8\times4}=97.4 \text{ lb. per sq. in.}$$

Since the reinforcement in the projections is two-way, if deformed bars, hooked at the ends, are assumed, the permissible unit bond stress is 112.5 lb. per sq. in.

As shown in Fig. 27, the center span is reinforced with one-way reinforcement near the top of the slab, and hence if the bars are adequately anchored the permissible unit bond stress is 150 lb. per sq. in. Since sixteen bars remain after every third bar is bent down, the unit bond stress for a maximum shear of 270,000 lb. is

$$u=\frac{270{,}000}{.875\times33\times16\times4}=146 \text{ lb. per sq. in.}$$

Adequate anchorage of the reinforcement bars in this case is provided by carrying them 15 in. beyond the axes of the columns.

Transverse Reinforcement.—The width of the slab being much greater than the width of the column base, transverse reinforcement will be needed. The load to be considered in figuring transverse bending moments is the soil pressure on the cantilever *a b c d* in Fig. 27 (*a*), or $3\times20\times4{,}500 = 270{,}000$ lb. Its lever arm is $\frac{3}{2}\times12=18$ in., and the bending moment at line *c d* is

$$M_s=270{,}000\times18=4{,}860{,}000 \text{ in.-lb.}$$

The required transverse reinforcement is

$$A'''_s=\frac{4{,}860{,}000}{.875\times16{,}000\times33}=10.5 \text{ sq. in.}$$

The maximum shear in cantilever *a b c d* is 270,000 lb., and if ¾-in. square deformed bars well anchored at both ends are assumed, the number required to resist a unit bond stress of 112.5 lb. per sq. in. is

$$n=\frac{V}{.875\ d\ o\ u}=\frac{270{,}000}{.875\times33\times3\times112.5}=27.8, \text{ or 28 bars,}$$

which offer an area of $28\times.5625=15.7$ sq. in. One-half of the required number, or fourteen bars, will be placed under each column within distributing beams about 6 feet wide, the rod spacing being about 5 in. on centers. Between distributing beams the rods will be spaced about 19 in. on centers.

Diagonal Tension.—To complete the design, investigations for diagonal tension should be made. Transversely, this diagonal tension may be measured at 33 inches from the face of the column. The shear in cantilever *a b c d* in Fig. 27 (*a*) at 33 in. from the face of the column is $4{,}500\times20\times(3.00-2.75)=22{,}500$ lb., one-half of which, or 11,250 lb., is resisted by each distributing beam. The unit shear is

$$v'=\frac{V'}{.875\ b'd}=\frac{11{,}250}{.875\times72\times33}=5.41 \text{ lb. per sq. in.,}$$

which is amply safe. In a longitudinal direction, the diagonal tension is measured by the shear at the inside face of the column. This shear is $36{,}000\times6.5=234{,}000$ lb., and the corresponding unit shear is

$$v = \frac{234{,}000}{.875 \times 96 \times 33} = 84.4 \text{ lb. per sq. in.,}$$

which exceeds the allowable 60 lb. per sq. in.

Web Reinforcement.—Web reinforcement is therefore necessary between the face of the column and the section at which the unit shear is 60 lb. per sq. in. The distance from the face of the column to the point where the concrete alone can safely carry the shear, by formula **2** in Art. **29**, is

$$x = \frac{(v - v_c)l}{2v} = \frac{(84.4 - 60) \times 13}{2 \times 84.4} = 1.88 \text{ ft.} = 22.6 \text{ in.}$$

Let the effect of the bent rods be neglected, which is on the side of safety, and let the entire shear to be carried by web reinforcement be resisted by stirrups. The total shear to be resisted by the stirrups, according to formula **3** in Art. **29**, is

$$V_s = \frac{(v - v_c)x\,b}{2} = \frac{(84.4 - 60) \times 22.6 \times 96}{2} = 26{,}500 \text{ lb.}$$

If each stirrup is made of a ½-in. round rod, bent as shown in Fig. 27 (*c*), the number of vertical legs in each stirrup is 8, and since the area of each leg is .1963 sq. in., $A_v = 1.57$ sq. in. By formula **4** in Art. **29**, the number of stirrups required in each half of the center span is

$$n_v = \frac{V_s}{f_v A_v} = \frac{26{,}500}{12{,}000 \times 1.57} = 1.4, \text{ say } 2.$$

The minimum spacing of the stirrups, by formula **5** in Art. **29**, is

$$s = \frac{f_v A_v}{(v - v_c)\,b} = \frac{12{,}000 \times 1.57}{(84.4 - 60) \times 96} = 8 \text{ in.,}$$

and the first stirrup will, therefore, be placed 4 in. from the face of the column. The second stirrup will be spaced 12 in. from the first, as in Fig. 27 (*b*).

EXAMPLES FOR PRACTICE

1. A rectangular footing supports two columns each carrying a load of 390,000 pounds on a soil the bearing capacity of which is 2 tons per square foot. The columns are spaced 14 feet center to center and their bases are 27 inches square. If the building conditions limit the width of the footing to 12 feet, find (*a*) the length and (*b*) the effective depth of the footing when $f_c = 650$ pounds per square inch.

$$\text{Ans.} \begin{cases} (a)\ 18 \text{ ft.} \\ (b)\ 29 \text{ in.} \end{cases}$$

2. Determine (*a*) the number and spacing of 1-inch plain square bars anchored at the ends in the longitudinal reinforcement, and (*b*) the number

and spacing of $\frac{7}{8}$-inch square bars in the transverse distributing beams in the preceding example if $f_s = 16{,}000$ pounds per square inch.

Ans. $\begin{cases} (a) \text{ Twenty-nine 1-in. sq. bars, 5 in. c. to c.} \\ (b) \text{ Twenty } \frac{7}{8}\text{-in. sq. bars, 3 in. c. to c.} \end{cases}$

3. If the web reinforcement of the footing in Example 1 consists of stirrups made of $\frac{1}{2}$-inch round bars with 10 vertical legs, determine (*a*) the number of such stirrups required, and (*b*) the distance of the first stirrup from the face of the column to the nearest half inch, if the allowable unit shear in the concrete is 40 pounds per square inch, and $f_v = 12{,}000$ pounds per square inch.

Ans. $\begin{cases} (a) \text{ 3} \\ (b) \text{ 3 in.} \end{cases}$

FOOTINGS SUPPORTING TWO UNEQUALLY LOADED COLUMNS

31. Rectangular Footings.—Combined footings for the support of two unequally loaded columns are best constructed of rectangular shape. In such construction a uniform soil pressure over the footing area is obtained by making the projection of the footing beyond the heavier column sufficiently longer than the projection beyond the lighter column, as in Fig. 23, so that the resultant of the two unequal loads passes through the center of gravity of the rectangular slab. The footing is therefore unsymmetrical longitudinally, and different provisions for shear and bending should be made at each column. However, the design of such rectangular footings for the support of two unequally loaded columns is based on the same principles as the design of rectangular footings supporting two equally loaded columns, which were previously explained; therefore, no special treatment of these footings will be given here.

32. Trapezoidal Footings.—When the projections of the footing beyond both columns are limited, a trapezoidal shape must be adopted in order to distribute the pressure on the soil uniformly. The design of trapezoidal footings is illustrated by the following example:

EXAMPLE.—Two columns, 1 and 2, spaced 15 feet center to center, are to be supported by a footing which is to offer a bearing area of 150 square feet. Column 1 carries 475,000 pounds and is 28 inches square, while column 2 carries 350,000 pounds and is 24 inches square. If the footing must not project more than 2 feet beyond the axis of either column,

determine its dimensions, depth, and reinforcement for bending and diagonal tension. Assume $f_c = 650$ pounds per square inch and $f_s = 16{,}000$ pounds per square inch.

SOLUTION.—*Dimensions of Footing.*—The diagram of loading on the footing is shown in Fig. 28 (*b*). The resultant load $R = 350{,}000 + 475{,}000 = 825{,}000$ lb. Taking moments about the axis of column 1, the point of application of the resultant is $\frac{350{,}000 \times 15}{825{,}000} = 6.36$ ft. from the axis of column 1, and $15 - 6.36 = 8.64$ ft. from the axis of column 2. The length of the footing is $15 + (2 \times 2) = 19$ ft., and its center of gravity

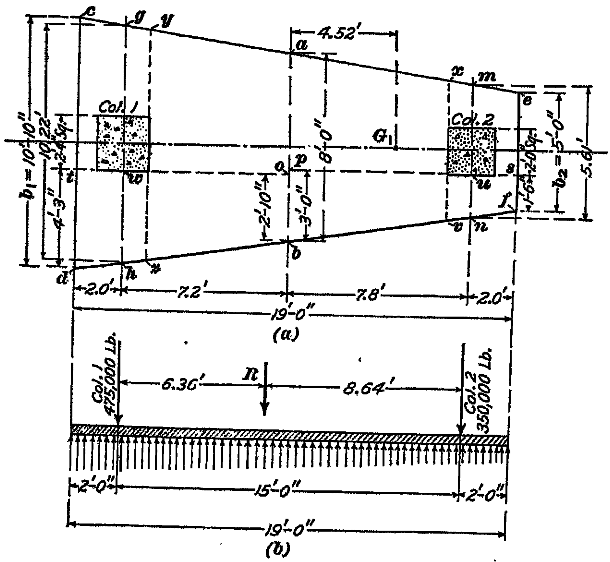


FIG. 28

is to be $6.36 + 2 = 8.36$ ft. from the wide end, and $8.64 + 2 = 10.64$ ft. from the narrow end. A trapezoidal shape will, therefore, be assumed.

The area of the footing is 150 sq. ft. Let the wide base be represented by b_1 and the narrow base by b_2, as in Fig. 28 (*a*), then since the altitude of the trapezoid is 19 ft., the area of the footing, 150 sq. ft., $= \frac{19}{2}(b_1 + b_2)$; hence,

$$b_1 + b_2 = 15.8 \text{ ft.} \qquad (1)$$

By the usual method for finding the distance x_1 of the center of a trapezoid from one of its bases, b_1,

$$x_1 = \frac{b_1 + 2b_2}{b_1 + b_2} \times \frac{h}{3}$$

In this case, x_1 has already been determined as 8.36 ft., and $h=19$ ft.; hence,

$$\frac{b_1+2b_2}{b_1+b_2}\times\frac{19}{3}=8.36 \text{ ft.,} \qquad (2)$$

Solving equations 1 and 2 simultaneously, $b_2=5$ ft., and $b_1=10.8$ ft., or 10 ft. 10 in. The dimensions of the footing are therefore as shown in Fig. 29 (*a*).

Longitudinal Shears and Bending Moments.—The unit net soil pressure on the footing is $\frac{825{,}000}{150}=5{,}500$ lb. per sq. ft. In every foot of its length,

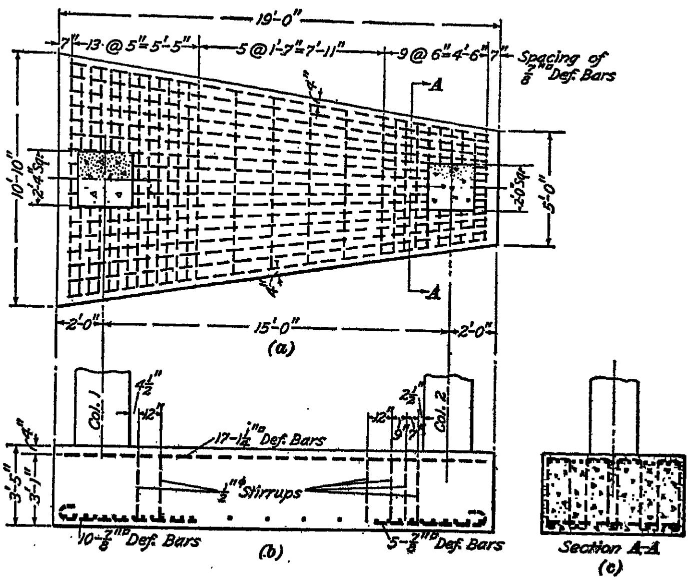


FIG. 29

from the wide end to the narrow end, the footing decreases in width $\frac{10.83-5}{19}=.307$ ft. At the axis of column 1, the width, $g\,h$ in Fig. 28 (*a*), is $10.83-(.307\times2)=10.22$ ft. The area of the trapezoid $c\,d\,h\,g$ is $\frac{10.83+10.22}{2}\times2=21.05$ sq. ft., and the maximum shear in the projection $c\,d\,h\,g$ at $g\,h$,

$$V_1=21.05\times5{,}500=116{,}000 \text{ lb.}$$

The point of application of the resultant pressure on trapezoid $c\,d\,h\,g$ is at the center of gravity of the trapezoid, which in this case may be

assumed as midway between the two bases, or 1 ft. from *g h*. Hence, the maximum bending moment in the projection *c d h g* is

$$M_1 = 116{,}000 \times 1 = 116{,}000 \text{ ft.-lb.} = 1{,}392{,}000 \text{ in.-lb.}$$

At the axis of column 2, the width $m\,n = 5 + (.307 \times 2) = 5.61$ ft. The area of the trapezoid *e f n m* is $\frac{5+5.61}{2} \times 2 = 10.61$ sq. ft., and the maximum shear in the projection *e f n m* at *m n*,

$$V_2 = 10.61 \times 5{,}500 = 58{,}400 \text{ lb.}$$

The resultant pressure may also in this case be assumed as midway between the two bases of the trapezoid, and hence the maximum bending moment,

$$M_2 = 58{,}400 \times 1 = 58{,}400 \text{ ft.-lb.} = 701{,}000 \text{ in.-lb.}$$

To find the maximum shear and bending moment in the center span *g h n m*, the line of zero shear, *a b*, is first located. If line *a b* is at a distance x from the right end of the footing, its width is $5 + .307\,x$. The net soil pressure on the trapezoid *a b f e* is $\frac{5+(5+.307x)}{2} \times 5{,}500\,x = 27{,}500\,x + 844\,x^2$, which is balanced by the column load 350,000 lb. Hence,

$$844\,x^2 + 27{,}500\,x = 350{,}000$$
$$x^2 + 32.6\,x = 415$$
$$x = \frac{-32.6 + \sqrt{32.6^2 + (4 \times 415)}}{2} = 9.8 \text{ ft.}$$

The width *a b* is therefore $5 + (.307 \times 9.8) = 8$ ft. (about).

The area of the trapezoid *a b n m* is $\frac{5.6+8}{2} \times 7.8 = 53$ sq. ft. Since the shear at *a b* is zero, the shear V_3 at *m n* is equal to the net soil pressure on the trapezoid *a b n m*, or

$$V_3 = 53 \times 5{,}500 = 291{,}500 \text{ lb.}$$

The area of the trapezoid *a b h g* is $\frac{8+10.22}{2} \times 7.2 = 65.6$ sq. ft., and the shear V_4 at *g h* is equal to the net soil pressure on the trapezoid, or

$$V_4 = 65.6 \times 5{,}500 = 361{,}000 \text{ lb.}$$

The resultant of the pressure on the trapezoid *a b f e* passes through the center of gravity G_1 of the trapezoid, which is located at a distance of $\frac{8+(5\times2)}{8+5} \times \frac{9.8}{3} = 4.52$ ft. from line *a b*. Therefore, the maximum bending moment about line *a b* is

$$M_3 = 350{,}000 \times (7.80 - 4.52) \times 12 = 13{,}800{,}000 \text{ in.-lb.}$$

Effective Depth.—The perimeter of column 1 is $4 \times 28 = 112$ in., and its area is 5.44 sq. ft. The net soil pressure directly under the column is

$5.44 \times 5{,}500 = 29{,}900$ lb., and hence the punching shear is $475{,}000 - 29{,}900 = 445{,}100$ lb. The effective depth required to resist a unit punching shear of 120 lb. per sq. in. is

$$d = \frac{445{,}100}{112 \times 120} = 33.1 \text{ in.}$$

The perimeter of column 2 is $4 \times 24 = 96$ in. and its area is 4 sq. ft. The net soil pressure directly under the column is $4 \times 5{,}500 = 22{,}000$ lb., and hence the punching shear is $350{,}000 - 22{,}000 = 328{,}000$ lb. The effective depth required is

$$d = \frac{328{,}000}{96 \times 120} = 28.5 \text{ in.}$$

When $f_c = 650$ lb. per sq. in., $b = 8$ ft. $= 96$ in., and $M = 13{,}800{,}000$ in.-lb., the required effective depth is

$$d = \sqrt{\frac{6M}{f_c\, b}} = \sqrt{\frac{6 \times 13{,}800{,}000}{650 \times 96}} = 36.4 \text{ in.}$$

An effective depth of 37 in. will therefore be assumed. Allowing 4 in. for protection, the total depth is 41 in.

Longitudinal Reinforcement.—The required steel area at $a\, b$, Fig. 28 (a), is

$$A_s = \frac{M}{.875 f_s\, d} = \frac{13{,}800{,}000}{.875 \times 16{,}000 \times 37} = 26.6 \text{ sq. in.}$$

If $1\frac{1}{4}$-in. square bars are assumed, the number required is $\frac{26.6}{1.56} = 17$, which will be spaced about $\frac{60-8}{16} = 3\frac{1}{4}$ in. at the narrow end and about $\frac{130-8}{16} = 7\frac{1}{2}$ in. at the wide end of the footing, as shown in Fig. 29 (a). This spacing is somewhat too close at the narrow end of the footing but it may be allowed in fan-shaped construction where the bars soon spread out. The unit bond stress in the seventeen bars for the maximum shear of 361,000 lb. at $g\, h$, Fig. 28 (a), is

$$u = \frac{V}{.875 d O} = \frac{361{,}000}{.875 \times 37 \times 5 \times 17} = 131 \text{ lb. per sq. in.,}$$

which is permissible for one-way reinforcement consisting of adequately anchored deformed bars. Adequate anchorage is here obtained by extending the bars 18 in. beyond the axes of the columns. In the projection $c\, d\, h\, g$, the maximum bending moment $M_1 = 1{,}392{,}000$ in.-lb., and the maximum shear $V_1 = 116{,}000$ lb. The steel area required to resist bending is

$$A_s = \frac{1{,}392{,}000}{.875 \times 16{,}000 \times 37} = 2.69 \text{ sq. in.}$$

If $\frac{7}{8}$-in. square deformed bars are assumed, the number required to resist a unit bond stress of 112.5 lb. per sq. in., which is permissible for anchored two-way reinforcement, is

$$n=\frac{V}{.875\ d\ o\ u}=\frac{116{,}000}{.875\times 37\times 3.5\times 112.5}=9.1,\text{ say } 10$$

The total area of ten $\frac{7}{8}$-in. square bars is 7.66 sq. in. These bars will be hooked at one end only, since at the other end they extend far enough beyond the point of zero moment to be considered adequately anchored.

In the projection $e\,f\,n\,m$, the maximum bending moment $M_2=701{,}000$ in.-lb. and the maximum shear $V_2=58{,}400$ lb. The required steel area is

$$A_s=\frac{701{,}000}{.875\times 16{,}000\times 37}=1.35\text{ sq. in.}$$

The number of anchored $\frac{7}{8}$-in. square deformed bars required is

$$n=\frac{58{,}400}{.875\times 37\times 3.5\times 112.5}=4.59$$

If five bars are used, their total area is $.766\times 5=3.83$ sq. in.

Transverse Reinforcement.—The distributing beam under column 2 will be assumed 5 ft. wide, which is the least width of the footing. The reinforcement in it will be designed to resist the stresses due to the soil pressure on the cantilever $p\,b\,f\,s$, Fig. 28 (*a*). The length of the base $p\,b$ is $\frac{8-2}{2}=3$ ft., and the length of the base $s\,f$ is $\frac{5-2}{2}=1.5$ ft. The area of the trapezoid $p\,b\,f\,s$ is $\frac{1.5+3}{2}\times 9.8=22.05$ sq. ft. The soil pressure on the trapezoid is $22.05\times 5{,}500=121{,}000$ lb. Hence, the maximum shear in the distributing beam is

$$V_5=121{,}000\text{ lb.}$$

Assume that the entire soil pressure on the trapezoid $p\,b\,f\,s$ is transferred to the distributing beam. For all practical purposes the lever arm of the load on the beam may be assumed one-half of the distance $u\,n$, or $\frac{5.61-2}{4}=.9$ ft. Therefore, the maximum bending moment in the beam is

$$M_5=121{,}000\times .9=108{,}900\text{ ft.-lb.}=1{,}310{,}000\text{ in.-lb.}$$

The required steel area is

$$A_s=\frac{1{,}310{,}000}{.875\times 16{,}000\times 37}=2.53\text{ sq. in.}$$

If adequately anchored $\frac{7}{8}$-in. square deformed bars are assumed, the number required for a unit bond stress of 112.5 lb. per sq. in. when the shear is 121,000 lb. is

$$n=\frac{121{,}000}{.875\times37\times3.5\times112.5}=9.5\text{, say 10,}$$

which offer an area of 7.66 sq. in. These bars will be spaced about 6 in. center to center, as in Fig. 29 (*a*).

The distributing beam under column 1 may be assumed about 6 ft. wide. The maximum shear in it is equal to the soil pressure on trapezoid *t d b o*, Fig. 28 (*a*). The area of the trapezoid is $\frac{4.25+2.83}{2}\times9.2=32.6$ sq. ft. Hence, the maximum shear in the distributing beam is

$$V_6=32.6\times5{,}500=179{,}000\text{ lb.}$$

The lever arm of the load on the distributing beam may be assumed as one-half $w\ h$, or $\frac{10.22-2.33}{4}=1.97$ ft., and the bending moment is

$$M_6=179{,}000\times1.97=352{,}000\text{ ft.-lb.}=4{,}220{,}000\text{ in.-lb.}$$

The required steel area is

$$A_s=\frac{4{,}220{,}000}{.875\times16{,}000\times37}=8.1\text{ sq. in.}$$

For a maximum shear of 179,000 lb. and a unit bond stress of 112.5 lb. per sq. in., the number of $\frac{7}{8}$-in. square deformed bars required is $\frac{179{,}000}{.875\times37\times3.5\times112.5}=14$, which offer a steel area of 10.7 sq. in. The fourteen bars will be spaced about 5 in. center to center.

Web Reinforcement.—The width of the footing at the face of column 2, $x\,v$ in Fig. 28 (*a*), is $5.61+.307\times1=5.92$ ft. $=71$ in. The area of the trapezoid $x\,v\,b\,a$ is $\frac{5.92+8}{2}\times6.8=47.3$ sq. ft., and the shear at $x\,v$,

$$V_7=47.3\times5{,}500=260{,}000\text{ lb.}$$

The unit shear,

$$v=\frac{260{,}000}{.875\times71\times37}=113\text{ lb. per sq. in.}$$

The longitudinal shear in trapezoidal footings does not vary uniformly with the length of the footing because the reaction of the soil per foot length of footing varies uniformly from a minimum at the narrow end of the footing to a maximum at the wide end, as shown in Fig. 28 (*b*). Nevertheless, all computations for web reinforcement may safely be based on a uniform shear variation, as will be assumed in this case. Therefore, formula **2** in Art. **29** may here be applied, and the distance from the face of the column to the section beyond which the concrete alone can safely carry the shear is

$$x=\frac{(v-v_c)l}{2v}=\frac{(113-60)\times12.83}{2\times113}=3.01\text{ ft.}=36\text{ in.}$$

The width of the footing at that point is $71+(36\times.307)=82$ in., and the average width of the footing in the distance x is $\frac{71+82}{2}=76.5$ in. Hence, according to formula **3** in Art. **29**, the total shear to be resisted by the stirrups is

$$V_s=\frac{(v-v_c)x\,b}{2}=\frac{(113-60)\times 36\times 76.5}{2}=73{,}000 \text{ lb.}$$

If each stirrup is made of $\frac{1}{2}$-in. round rods bent so as to give eight vertical legs, as in Fig. 29 (*c*), $A_v=.1963\times 8=1.57$ sq. in., and the number of stirrups required near column 2 is

$$n_v=\frac{V_s}{f_v\,A_v}=\frac{73{,}000}{12{,}000\times 1.57}=3.88, \text{ say } 4$$

The minimum spacing of the stirrups at the face of the column is

$$s=\frac{f_v\,A_v}{(v-v_c)b}=\frac{12{,}000\times 1.57}{(113-60)\times 71}=5 \text{ in.}$$

The first stirrup will, therefore, be placed $2\frac{1}{2}$-in. from the face of the column, and the remaining stirrups as shown in Fig. 29 (*b*).

The width of the footing at the face of column 1, yz in Fig. 28 (*a*), is $10.22-(.307\times 1.17)=9.86$ ft. $=118$ in. The area of trapezoid $yzba$ $=\frac{9.86+8}{2}\times(7.2-1.17)=53.8$ sq. ft., and the shear at yz is

$$V_s=53.8\times 5{,}500=296{,}000 \text{ lb.}$$

The unit shear is

$$v=\frac{296{,}000}{.875\times 118\times .37}=77.5 \text{ lb. per sq. in.}$$

The distance from the face of the column to the section beyond which the concrete alone can safely carry the shear is

$$x=\frac{(77.5-60)\times 12.83}{77.5\times 2}=1.45 \text{ ft.}=17.4 \text{ in.}$$

The width of the footing at that point is $118-(17.4\times .307)=113$ in., and the average width of the footing in the distance x is $\frac{118+113}{2}=115.5$ in. The total shear to be resisted by the stirrups is

$$V'=\frac{(77.5-60)\times 17.4\times 115.5}{2}=17{,}600 \text{ lb.}$$

Since $A_v=1.57$ sq. in., as before, the number of stirrups required near column 1 is

$$n_v=\frac{17{,}600}{12{,}000\times 1.57}=.93$$

Two stirrups will be used, which is the minimum that should ever be employed in such construction. The minimum spacing of the stirrups at the face of the column is

$$s = \frac{12{,}000 \times 1.57}{(77.5-60)\times 118} = 9.1 \text{ in.}$$

The first stirrup will, therefore, be spaced $4\frac{1}{2}$ in. from the face of the column and the second stirrup 12 in. from the first, as in Fig. 29 (*b*).

RAFT FOUNDATIONS

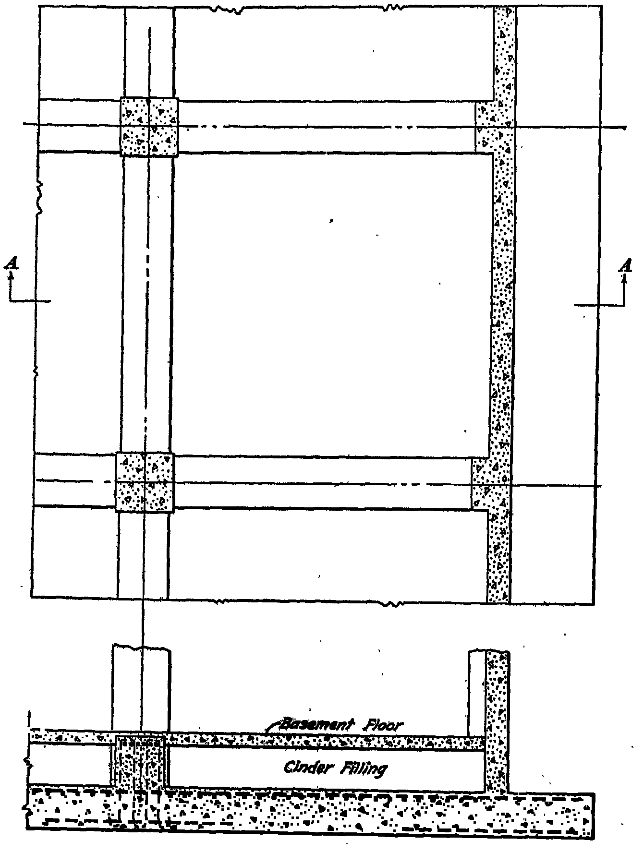
33. Types of Foundations.—In designing foundations for heavy buildings resting on a soil of low bearing capacity, *raft foundations* spreading continuously over the entire building site, or a large part of it, may be advantageously employed. Such foundations are either *flat slabs* of plain or reinforced concrete, or *slab-and-beam construction.*

34. Plain-Concrete Slabs.—In the early types of spread foundations for tall buildings, plain-concrete slabs, 3 to 5 feet thick, extending over the entire building site, were often used. However, due to the variation in the heavy concentrated loads coming on them, these foundations frequently cracked in various directions forming independent footings under the columns and walls that settled unevenly. Raft foundations of plain concrete should be used only for light buildings when reinforcement is not obtainable.

35. Reinforced-Concrete Flat Slabs.—Raft foundations constructed of reinforced-concrete flat slabs, if properly designed and constructed, give good results. Such foundations may be considered as inverted flat slabs loaded with the net soil pressure and supported by the columns, and they may be designed by the usual methods for designing flat-slab floors. The column bases should spread out sufficiently to prevent excessively large bending moments and shears in the slab, as in the case of columns supporting flat-slab floors.

36. Slab-and-Beam Construction.—When slab-and-beam construction is used for raft foundations, the design is similar to the usual slab-and-beam construction for building floors. Such foundations may be constructed either with the

slab above the beams and girders, or underneath them. When the beams are molded above the slab, the obvious economy of T-beam design is made possible, permitting lighter construction, but this is offset by the necessity of filling the spaces



Section A-A

Fig. 30

between beams with a compact fill of earth or cinders and of constructing a floor surface on top, as illustrated in Fig. 30. When the slab is constructed above the beams, heavier construction is required, but the extra fill and basement floor are eliminated.

REINFORCED-CONCRETE CANTILEVER FOUNDATIONS

THEORETICAL DESIGN

NECESSITY OF CANTILEVER FOUNDATIONS

1. In reinforced-concrete construction, as well as in other types of construction, it is frequently necessary to place a new building close against the walls of an adjoining building. In many instances the wall of the adjoining property rests entirely on its own lot and is not a party wall built half on each side of the party line; also, the adjoining building may be of inferior construction or may be occupied by tenants engaged in manufacture. Under such conditions it is undesirable to tear out the wall and build a party wall.

With buildings of ordinary height and load, provided the basement floor of the new building does not extend below that of the old, few difficulties are encountered in the design of the foundations for the new structure; but if the new building is to be many stories in height and requires extensive foundations along the wall lines, the problem of the design of the reinforced-concrete foundations becomes more complicated, because it is desirable to proportion the footings so that the center of action of the loads will coincide with the center of action from the pressure of the soil beneath. In order to accomplish this desired result in a steel structure, a cantilever-girder system of foundation construction would be employed, as explained in the Section on *Heavy Foundations*, and a similar system can be constructed in reinforced concrete.

TYPES OF CONSTRUCTION

2. The problem of cantilever-foundation construction in reinforced concrete may be solved by any of the three methods illustrated in Fig. 1. The choice of the method depends on the conditions and requirements of the building structure.

3. The construction shown in Fig. 1 (*a*) is not exactly a cantilever foundation, yet it is very similar to one and is used for the same purpose. It consists of reinforced-concrete beams and columns, as shown, carrying brick curtain walls. Under each basement column is a concrete footing. The basement column is placed directly under the columns of the stories above, but the footing cannot be placed under the basement column, because it would project on the adjoining property. The reinforcing rods of the basement column extend into the footing, so that the column and footing will act as one piece. If the pressure on the soil is uniform, the center of pressure will be at *d*. The line of action of the load on the column is at *a*. These two lines do not coincide. There is therefore a bending moment in the basement column equal to the load on the footing multiplied by the distance between the lines of action of *d* and *a*. The basement columns must therefore be reinforced with additional steel on the outside to resist this bending moment. This same moment is also transmitted from the column to the first-floor girder *c*, and produces in this beam a negative moment that tends to neutralize the positive moment due to the floor loads. The basement column must therefore be carefully tied to the beam *c* with steel reinforcement. Thus, the basement column is held rigidly at both ends.

The curtain wall in the basement carries no load except its own weight, and rests on its own separate foundation between columns not shown in the illustration.

4. Another method of solving the problem of cantilever-foundation construction in reinforced concrete is illus-

trated in Fig. 1 (*b*). Here, the construction is used in much the same manner as steel construction would be employed

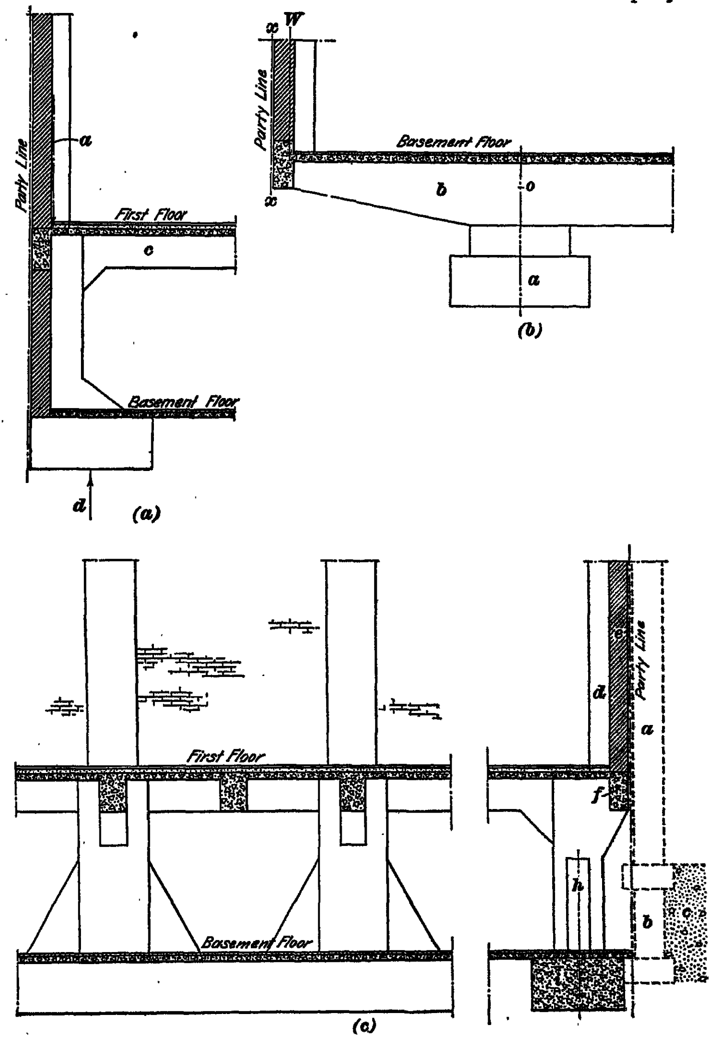


Fig. 1

for cantilever foundations and calculations similar to those explained in *Heavy Foundations* are made.

In this type of construction, the footing a is some distance within the party line $x\,x$, and if sufficient area cannot be secured by the adoption of isolated-pier construction, then these footings can be extended the full length of the building and reinforced for the bending moment between cantilevers. The weight W tends to create a bending moment about the point o, causing tensile stresses in the upper part of the beam b.

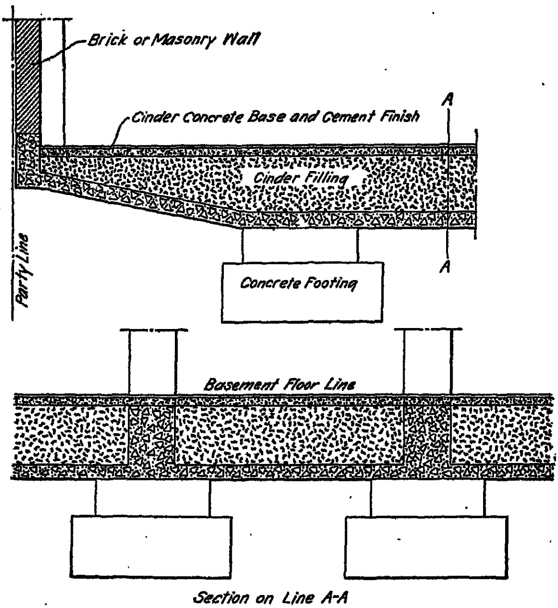


Fig. 2

If it is necessary to obtain compressive resistance at the bottom of the cantilever and throughout the length of the beam between supports, a slab construction can be adopted in conjunction with the cantilever beam. This may be used to carry the basement floor by filling in between with cinders and introducing the regular cement floor, as shown in Fig. 2.

5. In Fig. 1 (c) are indicated the conditions that require a typical cantilever-foundation construction in reinforced

ncrete. At *a* is shown the wall of the old building. The
lar bottom of this building is at a considerable distance
ove the cellar bottom of the new building. It is quite
ident, therefore, that the wall *a* must be underpinned, as
b. In this way, the danger of the bank *c* giving in during
e process of excavating for the new foundation is avoided.
The cantilever shown in the figure supports the new building,
ich is constructed with concrete wall columns extending
e full height of the building. These concrete wall columns
e shown at *d*, and between them are built brick curtain,
enclosure, walls 12 or 13 inches in thickness, as at *e*, the
rtain walls being supported at each story of the building by
eans of lintel beams, as at *f*. Often, the soil under the build-
g is such as to require a continuous footing, thus making
necessary to distribute the column loads.

In order to transmit the load from the wall columns over
e width of the continuous footing, the heavy concrete base-
ent column is often supplied with an inverted bracket or
eb, as at *h*. The continuous footing *i* must also be so pro-
rtioned and reinforced that it will be able to sustain the
veral column loads in the same way as a continuous beam.
nis ordinarily requires reinforcement at the top and the
ttom of the footing.

TRESSES PRODUCED IN REINFORCED-CONCRETE CANTILEVER FOUNDATIONS

GENERAL CONDITIONS

6. The type of reinforced-concrete cantilever foundation
nstruction most commonly employed is that shown in
ig. 1 (*c*), so that particular attention will be given to the
ction of the stresses in this construction. In order to under-
and these stresses, reference is had to Fig. 3 (*a*). Neglect-
g the load of the first floor, it may be said that the canti-
ver beam *b* is held in equilibrium by three forces, namely,
ne load W from the wall columns, the reaction R_1 from the
asement columns, and the load R_2 of the interior column.

The greatest bending moment occurs on the line $y\,y$, and is equal to $W\,x$. The beam b is a cantilever, and, being subjected to negative bending moment, it must be reinforced at the top. The load on the first floor will somewhat reduce this moment. The amount of this reduction of negative

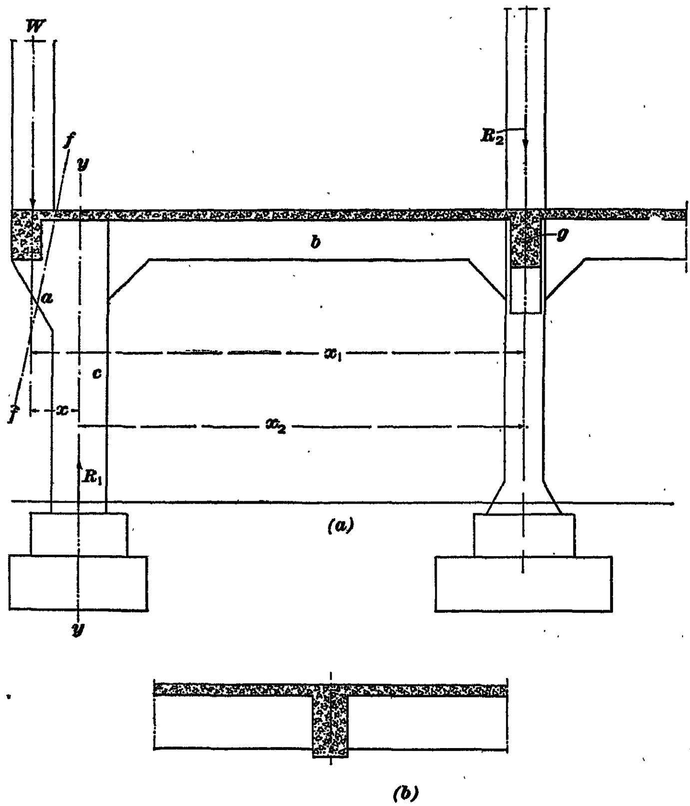


FIG. 3

bending moment will depend on the amount of live load acting at the time. The maximum negative moment at any point in the beam b is therefore equal to the negative moment at that point caused by the cantilever action, less the positive moment caused by the dead load of the beams and floor slab.

The minimum negative moment at any point is the negative moment caused by the cantilever action, less the positive moment caused by both the dead and live first-floor loads. It can be seen that if the floor loads that cause the positive moment are sufficiently large, the total moment may be positive, or at least vary between negative and positive as the live load varies. Great care must therefore be exercised to investigate carefully the bending moment at various points in the beam. The steel should then be placed in its correct position, either at the top or the bottom of the beam or at both the top and the bottom.

7. The easiest method of designing the beam is to plot the bending-moment diagram, first for the cantilever effect and then for the moment due to the first-floor load, using the method given in *Forces Acting on Beams*. These two diagrams, ordinate by ordinate, should then be subtracted from each other and the resultant diagram obtained. It will be found that the bending moment varies greatly along the length of the beam and is greater near the column *c*. The steel reinforcement in the beam *b* near the column *c* may be reduced in the other end of the beam by stopping off the rods at various points along its length.

If the beam *b*, instead of joining the girder *g* directly at the column, joins it in the middle of its span, as shown in (*b*), the conditions are somewhat different. Although the stresses in the beam *b* are exactly the same as before, the girder *g* is now affected. The end of the beam *b*, being no longer held down by the load on the central column, presses up on the girder *g* with a force R_2. This causes negative bending moment in the girder. The first-floor load causes positive bending moment in the girder. These two moments must be combined in the same manner as was suggested in regard to the beam *b* to find the resultant moment, which may be either positive or negative.

8. The projecting end of the cantilever beams, as at *a*, must be adequately reinforced to resist the tension and compression stress in the top and bottom of any section through

it. Its section must likewise be examined to determine whether the shearing stresses, as along the line $f f$, are adequately resisted by the rod reinforcement together with the shearing resistance of the concrete.

The column c forming the support of the cantilever must resist the reaction R_1, which is greater than the weight W, because the lever arm x_2 is smaller than the lever arm x_1, which is the leverage through which the weight W acts about the center column. The column should therefore be well reinforced.

Owing to the fact that the concrete in the bracket is likely to shear off, as indicated by the line $f f$, numerous stirrups should be introduced. These stirrups should extend across this line of possible shear, so as to tie in the concrete of the bracket and to provide against any such possible chance of failure. In all instances, the reinforcement of the girder b should have ample stirrups running through the mass of this structural member. However, in the placing of stirrups and light or secondary reinforcement, the matter of filling the forms should be considered; also, the reinforcement should be arranged so that there will be an ample body of concrete around it and that there will be sufficient space between the several bars and rods to make a solid concrete structure when completed.

SYMMETRICAL DESIGN

9. There is another way of solving the above problem if the building is symmetrical as regards its construction on both sides; that is, when the building is located in the center of a city block between adjoining buildings, the conditions at each party line being theoretically the same. This method of solution is described by means of Fig. 4.

In this figure the building is supported on the wall footings, as indicated at a and b, and on an intermediate interior row of column footings, as at c. The cantilevers d_1 and d_2 sustain the exterior wall loads. It is quite evident that the weights W_1 and W_2 tend to rotate about the points c_1 and c_2 of the

wall footings, and also that equilibrium of rotation is secured by the reactions R_1 and R_2 and the rigidity of the girders d_1 and d_2. The latter rigidity can be secured by placing sufficient reinforcement in such a position as to resist tensile stresses.

The solution is simple, as d_1 and d_2 may be considered as separate beams. If there is any likelihood of the reactions

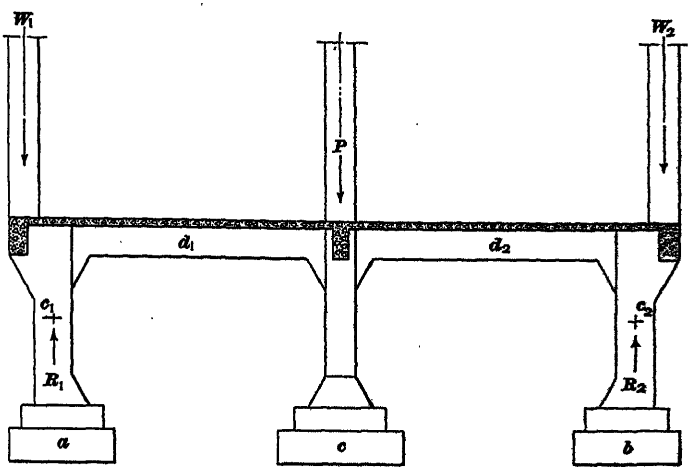


FIG. 4

of d_1 and d_2 being greater than the force P, that is, of lifting up the central column, they may be considered as one long beam with a central load and reinforced as such. In any case, the bending moments may be calculated by the rules given in *Forces Acting on Beams*.

CONTINUOUS FOOTINGS

10. Frequently, all three types of reinforced-concrete cantilever construction under discussion can be avoided, especially where the soil is excellent, by adopting the construction known as a **continuous footing**, which is shown in Fig. 5. In this construction, the necessary bearing area on

the soil is obtained by extending a longitudinal footing from column to column that has sufficient depth and is so reinforced as to transmit the load upon the soil. In this way area is gained by increasing the longitudinal dimension of the footing rather than the width. In adopting this construction, it is considered advisable by some engineers to cut clear through the concrete footing at the center of its length between each two columns, thus insuring that each column

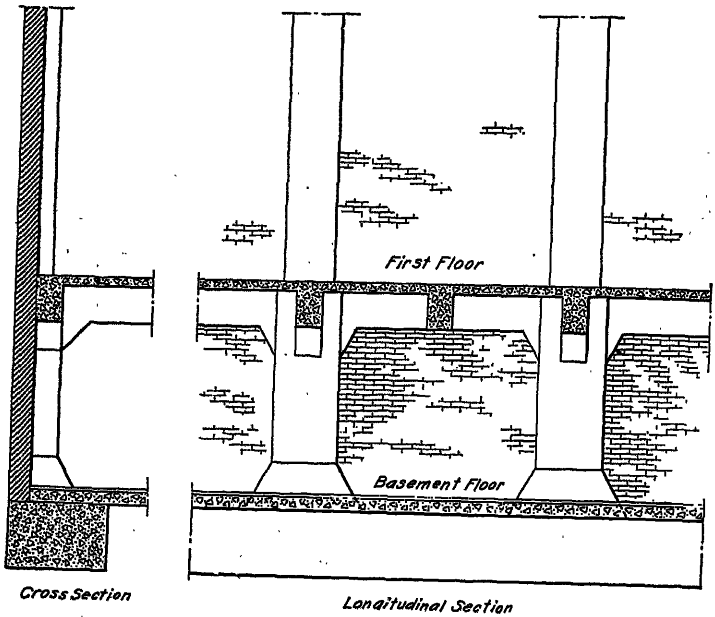


FIG. 5

will rest solidly on the soil under it and that, therefore, each section of footing will carry its calculated load. This is suggested because there may be soft places in the soil of such an extent that it may be impossible to determine the necessary reinforcement required to span such spaces. It is almost certain, however, that where this type of construction is well designed, no such precaution need be taken, especially if reinforcement is provided in both the top and the bottom

of the rectangular concrete beam that forms the footing and transmits the loads from the column to the soil.

In the design of all such special foundation construction, more than the ordinary amount of care and attention is required in order to reinforce the concrete successfully and in an economical manner.

PRACTICAL DESIGN

11. In Fig. 6 is shown the detail drawings of a reinforced-concrete cantilever-foundation construction for a six-story building, the floors of which are designed for light manufacturing purposes. In view (*a*) is shown a diagrammatic cross-section of the building, which illustrates the conditions of loading and the spans of the cantilever and other girders. The live load to be supported by the floor construction and the cantilever girder is 220 pounds per square foot, and the soil beneath the footings is capable of supporting safely 6,000 pounds to the square foot.

The distance between bents, or the distance center to center, of the reinforced-concrete wall columns is 8 feet, so that when the weight of the curtain wall *p* is added to the total dead and live load of 320 pounds for each floor and the total roof load of 120 pounds, the total load on each wall column that is transmitted to the end of the cantilever beams is almost 300,000 pounds. This load, together with the weight of the construction of the first floor, determines the size of the footings, and is the basis for the calculation to determine the size of the concrete cantilever girder, its reinforcement, and the size of the concrete column or pier supporting it.

12. The details of the construction and the reinforcement are shown in Fig. 6 (*b*). The footing is reinforced against failure from tranverse stress by bars *a*, placed near the bottom of the footing, and further by a mattress of rods or bars placed directly beneath the bearing of the foundation wall column, as at *b*. The column, or wall pier, *c* is reinforced with vertical reinforcing bars, which are tied at close intervals with wire

or loop ties *d*. Particular attention is called to the arrangement of the vertical column rods and the manner in which the outside rods of the foundation pier or column run straight through and up into the column above, being continued as the inside rods of the upper tiers. The outside rods of the first-floor wall column are bent so as to pass obliquely through the cantilever bracket of the foundation pier or column and the flare at the bottom, as shown at *e*. By arranging these rods in this manner the end of the cantilever is greatly strengthened to withstand compression through the lower part of its section; besides, these rods help to resist oblique shearing stresses through the end of the cantilever and the foundation column.

13. The reinforcing rods of the cantilever girder are arranged as shown at *f*, *g*, *h*, and *i*. The rods in the upper part of the girder are arranged in three rows, or layers, and are stopped off in length in practically the same manner as the flange plates of a built-up girder, because the bending moment is reduced toward the interior column. All the rods extend through to the end of the cantilever, the rods *f* and *g* being bent downwards at the end, so as to make a secure bond in the concrete and to give additional strength to the brackets in shearing resistance and in compression from the transverse, or bending, stress. Only one set of the reinforcing rods, as at *f*, extend entirely through into the interior column, because the bending moment is so reduced that this set is all that is required. The bottom rods, as at *i*, are inserted to improve the resistance to compression in the cantilever girder beam at the bottom.

Both the girder and the cantilever bracket are well supplied with stirrups properly arranged to resist any horizontal or oblique shearing stresses, as shown at *j*. The vertical reinforcement of the interior column is shown at *k*, and the ties and bracket reinforcement, at *l* and *m*, respectively. The lintels, or connecting beams, at the end of the cantilever are shown at *n*. These beams have the advantage of the added strength in compression imparted to them by the reinforced-

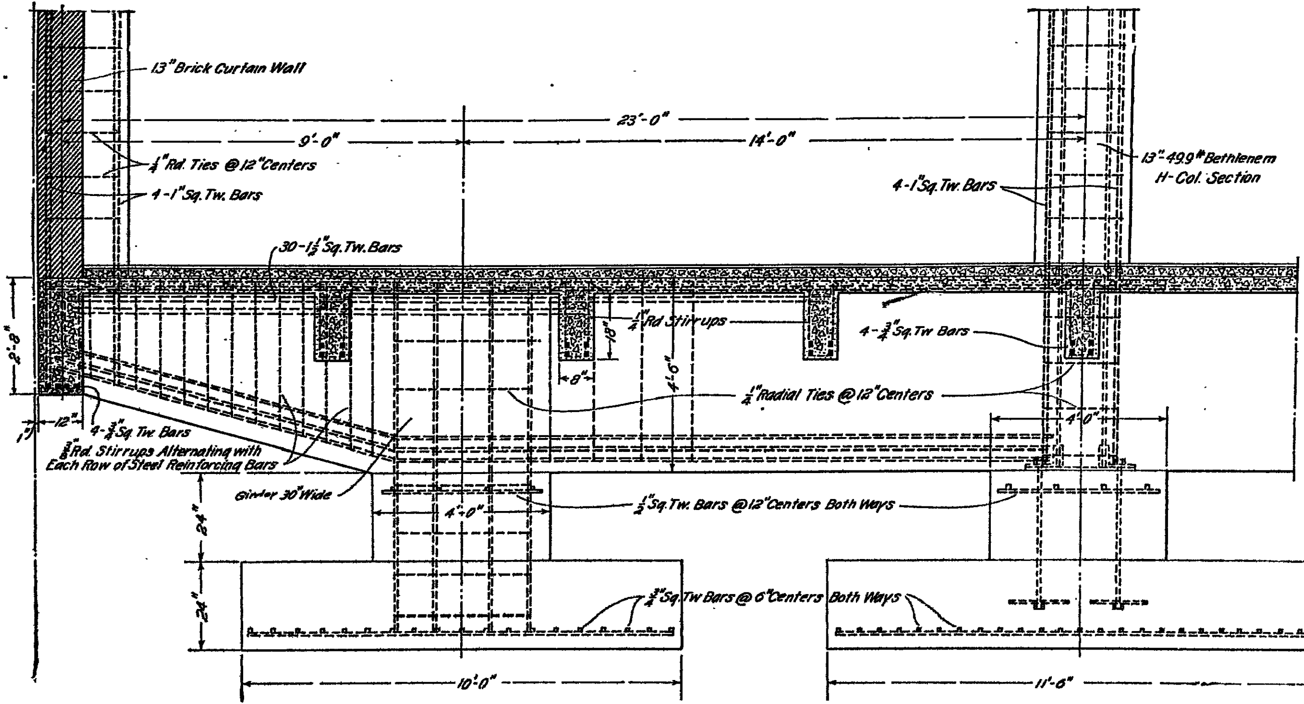

13" Brick Curtain Wall
23'-0"
9'-0"
14'-0"
1/4" Rd. Ties @ 12" Centers
4-1" Sq. Tw. Bars
13"-49.9# Bethlenem
H-Col. Section
4-1" Sq. Tw. Bars
30-1 1/2" Sq. Tw. Bars
2'-8"
1/4" Rd Stirrups
18"
8"
4'-6"
4-3/4" Sq. Tw Bars
1/4" Radial Ties @ 12" Centers
4'-0"
12"
1"
4-3/4" Sq. Tw Bars
3/8" Rd. Stirrups Alternating with
Each Row of Steel Reinforcing Bars
Girder 30" Wide
1/2" Sq. Tw. Bars @ 12" Centers Both Ways
4'-0"
24"
24"
3/4" Sq. Tw Bars @ 6" Centers Both Ways
10'-0"
11'-6"

FIG. 7

concrete slab. The steel reinforcement of these beams is shown at *o*. Some of these rods are bent upwards toward the ends of the lintel beams and well lapped over the top of the cantilever brackets, so as to give the beams the additional strength of a beam with the ends well fixed, or secured.

14. In Fig. 7 is shown a reinforced-concrete cantilever-foundation construction designed to carry the same load on the end of the cantilever as was required of the construction illustrated in Fig. 6. The several dimensions show the distance of the footings within the walls, and as the cantilever girders have a considerable projection beyond the foundation footings, they must be very strongly reinforced in both the top and the bottom, as shown. The several details of this construction are worked out in the illustration, and an analysis of these will show that the several principal and secondary stresses created in the structure are amply provided for.